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ALTERNATIVE ANALISYS: SOME NEW METHODOLOGY AND EXTENSION OF THE RIAL NUMBERS SET

АЛЬТЕРНАТИВНЫЙ АНАЛИЗ: ЭЛЕМЕНТЫ МЕТОДОЛОГИИ И РАСШИРЕНИЕ МНОЖЕСТВА ДЕЙСТВИТЕЛЬНЫХ ЧИСЕЛ

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Abstract. Positive definitions made it possible to prove new theorems on well-known objects of analysis: numerical sequences and series. The estimates of the quantity of all primes and the largest of them are obtained. With lineal function $\mathrm{f(x) = kx + b}$ at $0 < \mathrm{k} \leq 1$ we define Alternative extension of the real numbers set.

Аннотация. Позитивные определения позволили доказать новые теоремы о хорошо известных объектах анализа: числовых последовательностях и рядах. Получены оценки количества всех простых чисел и набольшего из них. С помощью линейной функции определено Альтернативное расширение множества действительных чисел.

Keywords: C–pair, Euclidian Axiom 8-th, continuum–hypothesis, e–divergence, w–convergence, infinite larger number, Prime Number, dogmas, Alternative extension of the real numbers set.

Ключевые слова. С–пара, е-расходимость и w-сходимость, аксиома 8 Евклида, бесконечно большие числа, простые числа, догмы, Альтернативное расширение множества действительных чисел.

We use well known mathematical texts and follow to Paul Cohen's forecast about continuum–hypothesis (CH) [1: IV.13]: "A point of view which the author feels may eventually come to be accepted is that CH is obviously false". As well as following [2, Chapter 1], we consider the functional series $\sum u(x,t)$ to be meaningless if it diverges for all values of the parameter *t*.

1. Let (A,B) , $\boldsymbol{S}(A,B)$ and $\boldsymbol{B}(A,B)$ be the set of injections, surjection and bijections $f\colon A \to B$, respectively. Then as it well known

$$\boldsymbol{B}(A,B) = \boldsymbol{In}(A,B) \cap \boldsymbol{S}(A,B) .$$

It is obvious that $C \subset A$ and $\varphi \in \boldsymbol{In}(A,B)$ hold $\varphi_{|\,c} \in \boldsymbol{B}(C,D)$ at $D \triangleq \varphi(C)$, by default. A term one-to-one correspondence has some everyday nuance and can be easily verified either for finite sets or in the case of some specification by any formula. We will not use this term for a more correct understanding of our texts by readers.

Let $\bar{R}$ =$R \cup \{\pm\infty\}$ be extended real numbers set. Properties of infinity (∞), surprising and not clear from the point of view of all final, were incentive motive of our research. Really, properties of infinity in the analysis:$a + \infty = \infty, a \times \infty = \infty, \infty + \infty = \infty, \infty \times \infty = \infty, \infty^{\infty} = \infty$ and others are not intelligible in the finite arithmetic. Moreover, the equalities as $\sum(1)^n = \sum(n)^{-1}$ deprive concept of infinity of any definiteness and structure that increases a risk of any mistakes occurrence in proofs of statements about the infinite. In the beginning of XVII century G. Galilei has opened as if quantities of natural numbers and their squares are equal. On this basis he approved, that «…properties of equality, and also greater and smaller size have no place there where it is a question of infinity, and they are employ only to finite quantities» [3, p. 140–146].

2. We define new alternative concept using positive properties of subject matter. Let $E \triangleq A \cup B \subseteq N, \Psi \triangleq \{(m,k)\} \subset (A,B)$ be the set of neighboring in E pairs of elements $m \in A$ and $k \in B$.

Definition 1. The pair (m, k) of natural variables $m \in A$, $k \in B$ is said to be C–pair, if there exists such number C that every pair $(m,k) \in \Psi$ holds the inequality

$$|m\text{-}k|<C.$$

Definition 2.The number sequence (***a***) is named *e-divergent* one if there are such two infinite subsequences $A, B \subset N, A \cap B = \emptyset$, which hold:

$$\exists(\delta > 0, n^*(\delta) \epsilon N): \forall(m,k) \in \Psi \subset (A,B), m,k > n^*(\delta)\ |a_m - a_k| \geq \delta.$$

Definition 3. The number sequence (***a***) is said to be *w-convergent* if the following condition holds $\forall\varepsilon > 0\ \exists n(\varepsilon) \in N: (\forall n \geq n(\varepsilon)|a_{n+1} - a_n| < \varepsilon)$.

3. Using new notions we prove alternative Theorems which contradict some traditional dogmas [4]. At the first we prove [5, (6.2.7)]

Theorem 1. $\varphi(N) = N \Rightarrow \lim_{n \in N}(\varphi(n): n) = 1$.

With Theorem 1 we prove [5, Th. 6.2.3; 6, Th. 6.2.5] following

*Theorem 2. There does not exist any bijection between set **N** of natural numbers and its own subset **A**.*

Then we divide all injective mappings $\varphi: N \to N$ onto six not crossed classes.

In common case we prove Euclidian 8th Axiom as [6, Th. 3.8]

Theorem 3. $B \subset A \Rightarrow \{\forall \varphi: A \to B \exists (a,q) \in (A,A): a \neq q \& \varphi(a) = \varphi(q)\}$.

Theorem 3 has brief form: $B \subset A \Rightarrow \neg(A \sim B)$, which confirms the Paul Cohen's forecast: some false hypothesis had implicated an incorrect Problem (Continuum Hypothesis); thus we prove following statement which contrary this dogma:

Theorem 4. The infinite sets are divided into classes of equivalence as well as the finite sets to within of one element.

Ignoring Theorem 4 often leads to either incorrect formulations or false statements. We give the simplest, but the traditional passage with divergent series as an illustration to what has been said

Example 1. $s \triangleq 1-1+1-1+\cdots \Rightarrow s = 1-(1-1+1-1+\cdots) \Rightarrow_{tr}$

$$s = 1-(s) \Rightarrow s = 1/2. \quad (1)$$

Really we have

$s_k \triangleq 1-1+1-1+\cdots+(-1)^{k+1} \Rightarrow s = 1-(1-1+1-1+\cdots+(-1)^k) \Rightarrow_{Alt}$ $s_k = 1-(s_{k-1}) \Rightarrow s_k \neq 1/2 \Rightarrow_{Alt} s \triangleq s_\infty \neq 1/2.$

By analogy with (1) there is

Example 2: Let $s_k(x) \triangleq 1+x+\cdots+x^k$=1+$x(1+x+\cdots+x^{k-1})$. Then $s_k(x) = 1+x(s_{k-1}(x) \pm x^k) = 1+xs_k(x)-x^{k+1} \Rightarrow_{Alt} s(x) \triangleq s_\infty(x) \neq 1/(1-x)$, at $|x| \geq 1$.

4. Now we need following statements:

Theorem 5.There exist a set of unlimited with finite number Cauchy sequence (***a***), *everyone of them converges to corresponding infinity large number* (*ILN* (***a***)), [6, Th. 7.1.3])

Theorem 6.Unlimited differentiated in the ∞ function $f: R \to R$ converges to corresponding ILN Ω(*f*) *if and only if $f'(\infty) = 0$.* [6, Th. 7.2.1])

Theorem 7. Any permutation of alternative series addends does not change its convergence. [6, Th. 8.2.1])

Example 3. With Theorem 6 we have proved [7, P. 229-230] the convergence of three sequences: $\{ln(n)\}$, $\{\sin(lnn)\}$, $\{cos(lnn)\}$: $(ln\infty) \triangleq \Omega_e$, $\{\sin(\varphi_0) \triangleq a\}$, $\cos(\varphi_0) \triangleq b$ at $\varphi_0 \triangleq mod_{0,5\pi}(\Omega_e)$ and $ab \neq 0$. Thus, $\lim(ctg(lnn)) = ctg(\varphi_0) = b/a$.

Example 4. Now following [8, formulae (1)] we consider

$$V \triangleq lim\,(\sin\varphi(n+1)/\sin(n\,\varphi)). \quad (2)$$

Let $n = lnt$, then $t = e^n$. Then we have *V*= $\cos(\varphi) + \sin\varphi\,(cos(\varphi_0)/\sin(\varphi_0))$.

A brief solution of this problem is presented in the note [8]. The interested reader will find a great many details and the history of the investigation of continued fractions and their applications in a thorough monograph [9].

5. Now let ***P*** be the set of all prime numbers p_k, $k \in \pi \subset N$. Let farther ***P***(*x*)≜ $\{p_k: p_k \leq x > 1\}$. Now let $\pi(x) \triangleq |\boldsymbol{P}(x)|$ by [10], then $\lim_{x\to\infty} \pi(x) = |\boldsymbol{P}|$, what is generally accepted. That is obvious that the graph $y = \pi(x)$ of function $\pi(x)$ has a consecutive form and the function $\pi(x)$ is a step-function with $\forall k\ \pi(p_{k+1}) - \pi(p_k) = 1$. Let $g(x)$ be a differentiable function which has following complementary properties: $\forall k \in \pi\ \ g(p_k) = \pi(p_k)$. A limiting equality $lim\ g'(x) = 0$ holds

Theorem 8. There exists the ILN $\Omega(\pi)$ which defined the $|\boldsymbol{P}|$ as the quantity of all Prime Numbers.

Consequence of Theorem 8. There exists the max ILN $(\pi))\triangleq \lim_{n\in\pi}(p_n)$.

A finely by Theorem 7–8 *we prove that Hardy–Littlewood's Hypothesis* [10: 1.2.4] *has the positive decision.*

6. Now we can use lineal function $f(x) = kx + b$ at $0 < k \leq 1$ for extension of the real numbers set. Let f_{max} be largest value of function $f(x)$ at $k = 1$ $f_{max} = f(x)_{k=1} \triangleq \omega$. By analogy we will define $\omega_\alpha \triangleq f_{max} = f(x)_{k=\alpha}$. Using traditional symbols we can write $\omega_\alpha \triangleq \infty_\alpha$, $0 < \alpha < 1$, $\omega_1 \triangleq \infty$.

It is obvious there exist some α *for every ILN=* $\omega(\alpha)$.

We conclude this article with the assertion that every infinite element can be defined in modern technology only from accuracy to a constant addend.

7. Note in small print. Sometimes a traditional mathematical thinking happens in the captivity either of a formula or any dogma.

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