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THE IMPACT OF VOLUME FRACTION METAL/CERAMIC CONSTITUENTS ON CRITICAL BUCKLING TEMPERATURE OF FUNCTIONALLY GRADED PLATE

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Abstract:
A thermal buckling analysis of functionally graded rectangular plates is presented. Mechanical and thermal properties of the functionally graded material, except Poisson's ratio, are assumed to vary continuously through the thickness of the plate according to a power-law distribution of the metal and ceramic volume fractions. Formulations of equilibrium and stability equations are based on high order shear deformation theory including shape function. An analytical method for determination of critical buckling temperature is developed. Numerical results were obtained in MATLAB software using combinations of symbolic and numeric values. The effects of power-law index and temperature gradient on mechanical responses of the plates are discussed and appropriate conclusions are given.

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Functionally graded plate, thermal buckling, power-law distribution, high order shear deformation theory

1. INTRODUCTION

Failure and delamination between two layers is the biggest problem of the conventional composite laminates. Delamination of layers due to high local inter-laminar stresses causes a reduction of stiffness and a loss of structural integrity of a construction. In order to eliminate these problems, improved materials such as functionally graded materials (FGM) are used for innovative engineering constructions. FGM, as well as nanocomposite [1], are modern materials in the family of engineering composite materials, in which there is a continuous and discontinuous variation of chemical composition through a defined geometric distance. After many years' research and development, FGM represent a class of materials that can form a property which is impossible to achieve with any other homogeneous material or composite laminates. The main aim of FGM is to use the properties of available materials in the best possible way by combining their potentials. Most frequently used FGM is metal/ceramics, where ceramics have a good temperature resistance, fine antioxidant properties, low thermal conductivity, while metal is superior in terms of mechanical strength, toughness and high thermal conductivity. Due to continuous change between properties of the constituents, delamination between layers should be avoided (Fig.1). By varying a percentage of volume fraction content of metal/ceramic constituents, FGM can be formed so that it achieves a desired gradient property in specific directions.

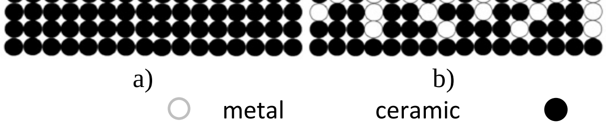

Fig.1. Traditional composite laminate (a) compared to FGM (b)

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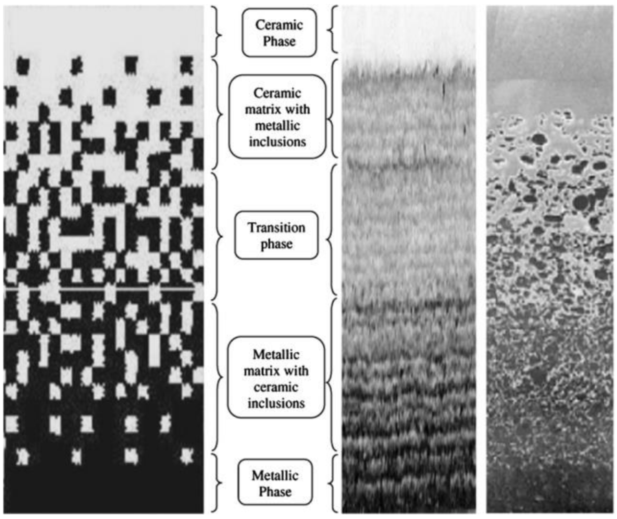
A continuously graded microstructure with metal/ceramic constituents is represented in Fig.2 schematically for illustration.



a) b) c)

Fig.2. Schematic of continuously graded microstructure with metal-ceramic constituents (a) Smoothly graded microstructure (b) Enlarged view and (c) Ceramic–Metal FGM [2].

Many authors have studied stability of a rectangular plates which is elastically buckled using different theories during their research [3]. In case it wants to be determined the thermomechanical properties of new materials, such as FGM, it is necessary to study the behaviour of the material at high temperature. Thermoelastic behaviour of a FG rectangular ceramic-metal plate using a four-node rectangular finite element based on the first order shear deformation theory (FSDT), including von Karman's non-linear effect is studied [4]. Using and expanding the adopted FSDT formulation, the static analysis of FG rectangular plates is done based on the third order shear deformation theory (TSDT) [5]. Stability and equilibrium equations for rectangular FG plates under thermal load, by using classical plate theory (CPT) is derived in [6]. Thermoelastic buckling behaviour of thick rectangular plates on the basis of Reddy's higher order shear deformation theory (HSDT) is studied by using the energy method [7,8].

This paper presents the methodology of the application of the HSDT theory based on the shape functions. The results have been verified through comparison with the results in literature obtained with other theories. In order to determine a procedure for the analysis and the prediction of behaviour of FGM plates, theories developed in this paper have been implemented into the software written in the program package MATLAB (MATrix LABoratory).

2. MATHEMATICAL MODEL OF FGM

FGM rectangular plate of $a \times b \times h$ dimensions where the x-y plane coincides with the midsurface of the plate while z-axis has a direction of thickness *h* were studied in this paper (Fig.3).

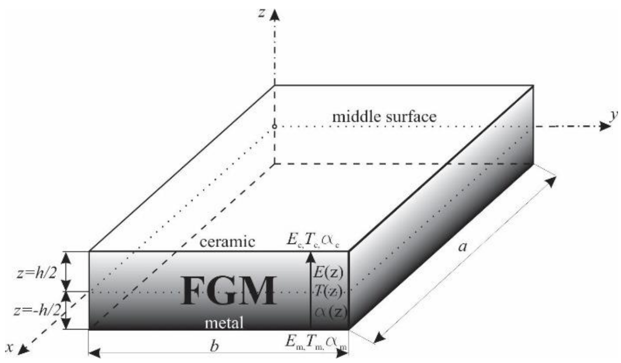


Fig.3. Geometry of a functionally graded plate

Young's modulus of elasticity, thermal expansion coefficient and changes in temperature are defined according to the power law distribution [9]:

$$
\begin{aligned}
E(z) &= E_{\mathrm{m}} + E_{\mathrm{cm}}\left(\frac{1}{2}+\frac{z}{h}\right)^{\mathrm{p}}, \qquad E_{\mathrm{cm}} = E_{\mathrm{c}} - E_{\mathrm{m}}, \\
\alpha(z) &= \alpha_{\mathrm{m}} + \alpha_{\mathrm{cm}}\left(\frac{1}{2}+\frac{z}{h}\right)^{\mathrm{p}}, \qquad \alpha_{\mathrm{cm}} = \alpha_{\mathrm{c}} - \alpha_{\mathrm{m}}, \\
T(z) &= T_{\mathrm{m}} + \Delta T_{\mathrm{cr}}\left(\frac{1}{2}+\frac{z}{h}\right)^{\mathrm{s}}, \qquad \Delta T_{\mathrm{cr}} = T_{\mathrm{c}} - T_{\mathrm{m}}.
\end{aligned} \tag{1}
$$

This law defines the change of the mechanical properties as the function of the volume fraction of the FGM constituents in the thickness direction of the plate. As *h* represents total thickness of the plate, *E(z)*, $\alpha(z)$, *T(z)* represents a material property in an arbitrary cross-section "*z*". E_c, α_c, T_c represents a material property at the top of the plate *z=h/2* - ceramics, and E_m, α_m, T_m represents a material property at the bottom of the plate *z=-h/2* - metal. In the equation (1), index *p* is the exponent of the equation which defines the volume fraction of the constituents in FGM (Fig.4.). Practically, by varying the index *p*, homogenous plates of precisely determined and specific gradient structure could be obtained:

- when *p* = 0 the plate is homogenous, made of ceramics,
- when 0 < *p* < ∞ the plate has a gradient structure,
- theoretically, when *p* = ∞ the plate becomes homogenous again, made of metal, although the plate can be considered homogenous even when *p* > 10.

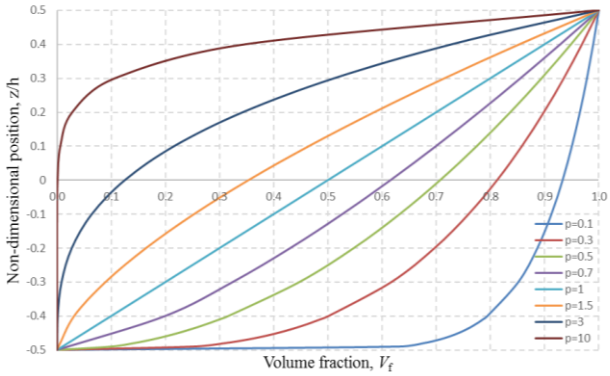


Fig.4. Variation of the volume fraction through the thickness of a plate

3. EQUILIBRIUM AND STABILITY EQUATIONS

Disadvantages of the classical lamination theory, and first order shear deformation theory which require correctional factors, are eliminated by introducing high order shear deformation theory based on shape functions.

Here assumed form of the displacement field is:

$$\begin{aligned} u &= u_0(x,y) - z w_{0,x} + f(z)\theta_x, \\ v &= v_0(x,y) - z w_{0,y} + f(z)\theta_y, \\ w &= w_0(x,y), \end{aligned} \tag{2}$$

where *f(z)* is shape function (Table 1).

Table 1. Shear deformation shape functions

Num. of shape function (SF)	Name of authors	Shape function *f(z)*
SF 1	Reissner [10]	$\frac{5z}{4}\left(1-\frac{4}{3h^2}z^2\right)$
SF 2	Mantari [11]	$z\cdot 2.85^{-2\left(\frac{z}{h}\right)^2}+0.028z$

In order to define components of unit loads, it is necessary to apply the relations between displacements and strains in accordance with von Karman's non-linear theory of elasticity. Using a generalized Hooke's law as well as stiffnes matrix and taking into consideration the effect of the change in temperature (1) and thermal expansion which cause a strain $\alpha\Delta T$, the components of unit loads are obtained.

In order to get an stability equation, it is necessary to define the strain energy in the following form:

$$\begin{aligned} U = \int_{h^-}^{h^+}\int_A &(\sigma_{xx}\left[\varepsilon_{xx}-\alpha(z)T(z)\right]+\sigma_{yy}\left[\varepsilon_{yy}-\alpha(z)T(z)\right] \\ &+\sigma_{zz}\varepsilon_{zz}+\tau_{xy}\gamma_{xy}+\tau_{xz}\gamma_{xz}+\tau_{yz}\gamma_{yz})dAdz. \end{aligned} \tag{3}$$

Using the principles of minimum potential energy equilibrium equation is derived:

$$\begin{aligned} &\delta u_0: N_{xx,x}+N_{xy,y}=0, \\ &\delta v_0: N_{yy,y}+N_{xy,x}=0, \\ &\delta w_0: M_{xx,xx}+2M_{xy,xy}+M_{yy,yy} \\ &\qquad +N_{xx}w_{0,xx}+2N_{xy}w_{0,xy}+N_{yy}w_{0,yy}=0, \\ &\delta\theta_x: P_{xx,x}+P_{xy,y}-R_x=0, \\ &\delta\theta_y: P_{xy,x}+P_{yy,y}-R_y=0. \end{aligned} \tag{4}$$

Stability equation for a thick FG plate is derived on the basis of the equilibrium equation. The stability equation of the plate under thermal load can be defined using the displacement components $u_0, v_0, w_0, \theta_{x0}$ and θ_{y0} . Displacement components of the next stable configuration are the following:

$$\begin{aligned} &u=u_0+u_1, \quad v=v_0+v_1, \quad w=w_0+w_1, \\ &\theta_x=\theta_{x0}+\theta_{x1}, \quad \theta_y=\theta_{y0}+\theta_{y1}, \end{aligned} \tag{5}$$

Therefore, the stability equations of the functionally graded rectangular plate are the following:

$$\begin{aligned} &\delta u_0: N^1_{xx,x}+N^1_{xy,y}=0, \\ &\delta v_0: N^1_{yy,y}+N^1_{xy,x}=0, \\ &\delta w_0: M^1_{xx,xx}+2M^1_{xy,xy}+M^1_{yy,yy} \\ &\qquad +N^0_{xx}w_{1,xx}+2N^0_{xy}w_{1,xy}+N^0_{yy}w_{1,yy}=0, \\ &\delta\theta_x: P^1_{xx,x}+P^1_{xy,y}-R^1_x=0, \\ &\delta\theta_y: P^1_{xy,x}+P^1_{yy,y}-R^1_y=0, \end{aligned} \tag{6}$$

In order to obtain analytical solutions of equation (6), assumed solution forms and boundary conditions are adopted in accordance to Navier's methods applied in [12].

4. NUMERICAL RESULTS

The aim of this section is to check the accuracy and the effectiveness of the given theory in determining critical buckling temperature of FG plates for linear (*s*=1) change temperature across thickness. This present the results obtained for the FG plates made of metal and ceramic constituents. Obtained results were compared with the results available in the literature. Material properties that are used for numerical examples:

Table 2. Material characteristics of FGM constituents

Material	Material characteristics		
	Elasticity modulus $E[GPa]$	Poisson's ratio ν	Thermal expansion coefficient $\alpha[°C^{-1}]$
Aluminum (Al)	$E_m = 70$	$\nu = 0.3$	$\alpha_m = 23 \cdot 10^{-6}$
Alumina (Al_2O_3)	$E_c = 380$	$\nu = 0.3$	$\alpha_c = 7.4 \cdot 10^{-6}$

Table 3. shows comparative results of the critical buckling temperatures of square and rectangular plate for two different ratios of length and width of the plate (*a/b* = 1 and *a/b* = 2) and for different values of the index *p*. Verification of the results has been conducted by comparing them to the results obtained in [7] when *a/b* = 1, and then the results when *a/b* = 2 are provided for different values of the index *p*, i.e. different volume fraction of the constituents in FGM. Using HSDT theory with the shape function, the obtained results are compared to the results obtained using two different shape functions (SF1 and SF2).

As can be seen it Table 3 there was a good match of results for given shape functions with the results from the papers [7]. Analyzing results, we can conclude that critical buckling temperatures decrease with increase power law index p. Changing the power law index from 1 to 5 (increase of metal volume fraction) leads to decrease of critical buckling temperatures approximately 17% depending from ratio a/b.

Table 3. Comparison of critical buckling temperatures (ΔT_{cr}) of square and rectangular FGM plates under a linear increase of temperature across their thickness *(a/h = 20,* $T_m = 5°C$*)*

p	Source	ΔT_{cr}	
		a/b = 1	*a/b* = 2
1	[7]	358.695	------
	SF 1	358.711	894.942
	SF 2	358.740	895.120
5	[7]	298.693	------
	SF 1	298.705	740.461
	SF 2	298.635	740.053
10	[7]	315.677	------
	SF 1	315.683	779.786
	SF 2	315.671	779.721

While analyzing the diagrams, it should be considered that when *p* = 0 the plate is homogenous made of ceramics, when *p* = 10 the plate is homogenous made of metal, and when 0 < *p* < 10 the plate is made of FGM.

Fig.5. shows the decrease of the difference between the obtained results with the increase of the value *p*, so when p>5, the constant ratio *a/h*=20 and a variable ratio (*a/b*), the curves merge. Shape functions do not have a significant effect on this behavior because the curves obtained by the use of SF1 and SF2 completely overlap.

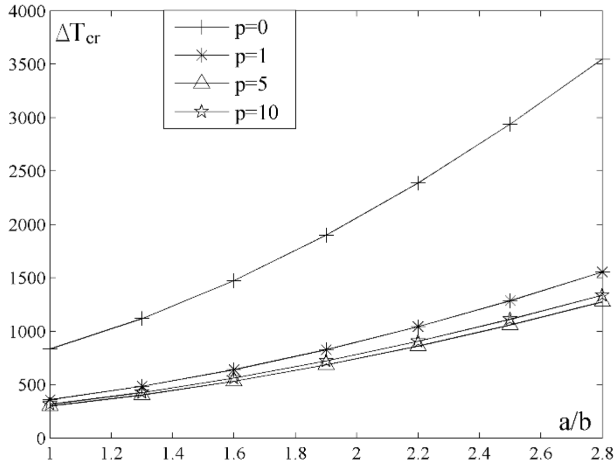


Fig.5. Effect of the aspect ratio *a/b* and the power law index *p* on the critical buckling temperature ΔT_{cr} under a linear change of temperature

5. CONCLUSION

Based on the given results, it can be concluded that the shape functions given in the Table 1 are acceptable for thermomechanical analysis of the functionally graded plates. The accuracy of the presented formulation and obtained numerical results is verified by comparing the results available in the literature. The diagram of the critical buckling temperature shows the difference in behaviour between a homogenous plate (ceramic or metal) and FGM plate. Analyzing results, it is concluded that critical buckling temperatures decrease with increase of power law index *p,* i.e. with increase of metal volume fraction.

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