International Journal of Mathematics and Soft Computing
Vol.7, No.1 (2017), 65 - 73.

ISSN Print : 2249 - 3328
ISSN Online : 2319 - 5215

Lucky edge neighborhood labeling of graphs

S. Ragavi[1], R. Sridevi[2]

[1]M. Phil Scholar
PG and Research Department of Mathematics
Sri S.R.N.M.College, Sattur-626 203.
stragavi22@gmail.com

[2]PG and Research Department of Mathematics
Sri S.R.N.M.College, Sattur-626 203.
r.sridevi_2010@yahoo.com

Abstract

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$ respectively. Vertex set $V(G)$ is labeled arbitrary by positive integers and $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e. A lucky edge neighborhood labeling of G is an assignment of positive integers to the vertices of G so that edge neighborhood labelings are distinct for every edge e.The least integer for which a graph G has a lucky edge labeling from the set $\{1, 2, ..., k\}$ is called the lucky neighborhood number and is denoted by $\eta_N(G)$. In this paper, we prove that P_n, C_n, $T_{m,n}$, $S(P_n^+)$ and $S(C_n^+)$ are lucky edge neighborhood labeled graphs.

Keywords: Lucky edge labeling, lucky edge labeled graph, lucky edge neighborhood labeling, lucky edge neighborhood labeled graph.
AMS Subject Classification(2010): 05C78.

1 Introduction

In 1967, Rosa [4] introduced the concept of labeling and Golomb[2] called the labeling as graceful. Gallian [1] maintains a dynamic survey of graph labeling. Many graphs are constructed from standard graphs by using various operations. Nellai Murugan [3] introduced the concept of lucky edge labeling and proved that the path P_n, cycle C_n, comb $S(P_n^+)$ and the crown $S(C_n^+)$ are lucky edge labeled graphs.

In this paper, we define lucky edge neighborhood labeling of a graph and prove that P_n, C_n, $T_{m,n}$, $S(P_n^+)$ and $S(C_n^+)$ are lucky edge neighborhood abeled graphs.

2 Preliminaries

Definition 2.1. [3] Let G be a simple graph with vertex set V(G) and edge set E(G) respectively. The vertex set V(G) is labeled arbitrary by positive integers and E(e) denotes the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be *lucky edge labeling* if the edge E(G) is a proper coloring of G, that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_1 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, ..., k\}$ is the *lucky number* of G denoted by $\eta(G)$.

A graph which admits a lucky edge labeling is called an *lucky edge labeled graph.*

Definition 2.2. Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$ respectively. The vertex set $V(G)$ is labeled arbitrary by positive integers and $E(e)$ denotes the edge label such that it is the sum of labels of vertices incident with edge e. A *lucky edge neighborhood labeling* of G is an assignment of positive integers to the vertices of G so that each edge neighborhood labels are distinct. The least integer for which a graph G has a lucky edge neighborhood labeling from the set $\{1, 2, ..., k\}$ is the *lucky neighborhood number* and is denoted by $\eta_N(G)$.

The graph which admits a lucky edge neighborhood labeling is called a *Lucky edge neighborhood labeled graph.*

Definition 2.3. A graph obtained by joining each u_i of a path P_n to a vertex v_i is called a *comb* and denoted by P_n^+.

Definition 2.4. C_n^+ is a graph obtained from C_n by attaching a pendent vertex from each vertex of the graph $C_n$is called *crown.*

Definition 2.5. The *tadpole graph* $T_{m,n}$ also called *dragon graph* is the graph obtained by joining a cycle C_m to a path P_n with a bridge.

Definition 2.6. If $e = uv$ is an edge of G and w is not a vertex of G, the edge e is said to be *subdivided* if it is replaced by the edges uw and wv.

Definition 2.7. Let G be a graph. A *subdivision* graph $S(G)$ of G is obtained by subdividing each edge of G only once.

3 Main Results

Theorem 3.1. Path P_n has $\{a, b\}$ lucky edge neighborhood labeling for any $a, b \in N$.

Proof: Let $v_1, v_2, ..., v_n$ be the vertices of P_n. Assign label 2 to the vertices v_i for $i \equiv 0$ (mod 4) and 1 to0 the remaining vertices.
Then the induced edge neighborhood labeling are distinct. Hence, lucky edge neighborhood labeling of P_n is $\{2, 3\}$ and the lucky neighborhood number of P_n is $\eta_N(P_n) = 3$. ■

Illustration 3.2. A lucky edge neighborhood labeling of P_7 is shown in Figure 1.

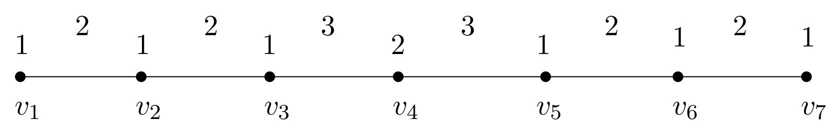


Figure 1: A lucky edge neighborhood labeling of P_7.

Theorem 3.3. Cycle C_n has

1. $\{a, b\}$ lucky edge neighborhood labeling if $n \equiv 0 \pmod 4$.

2. $\{a, b, c\}$ lucky edge neighborhood otherwise.

Proof: Let $V[C_n] = \{v_i : 1 \leq i \leq n\}$ and $E[C_n] = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n v_1\}$.
Case(i): $n \equiv 0 \pmod 4$.
Let $f : V[C_n] \to \{1, 2\}$ be defined by
$$f(v_i) = \begin{cases} 1 & i \equiv 1, 2, 3 \pmod 4 \\ 2 & i \equiv 0 \pmod 4 \end{cases} \quad \text{for } 1 \leq i \leq n.$$
Then the induced edge labeling is given by
$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1, 2 \pmod 4 \\ 3 & i \equiv 0, 3 \pmod 4 \end{cases} \quad \text{for } 1 \leq i \leq n-1.$$
It is clear that the lucky edge neighborhood labeling of C_n is $\{2, 3\}$ and the lucky neighborhood number of C_n is $\eta_N(C_n) = 3$.
Case(ii): $n \equiv 1, 3 \pmod 4$.

A lucky edge neighborhood labeling of C_3 is shown in Figure 2.

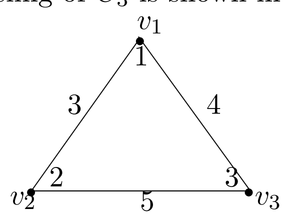


Figure 2: A lucky edge neighborhood labeling of C_3.

From Figure 2, the lucky neighborhood number of C_3 is $\eta_N(C_n) = 5$.
For $n \geq 5$, let $f : V[C_n] \to \{1, 2, 3\}$ be given by
$$f(v_i) = \begin{cases} 1 & i \equiv 0, 1, 2 \pmod 4 \\ 2 & i \equiv 3 \pmod 4 \end{cases} \quad \text{for } 1 \leq i \leq n-1 \text{ and}$$
$f(v_n) = 3$.
Then the induced edge labeling is given by
$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 0, 1 \pmod 4 \\ 3 & i \equiv 2, 3 \pmod 4 \end{cases} \quad \text{for } 1 \leq i \leq n-2 \text{ and}$$
$f^*(v_{n-1} v_n) = f^*(v_n v_1) = 4$.

It is clear that lucky edge neighborhood labeling of C_n is $\{2, 3, 4\}$ and the lucky neighborhood number of C_n is $\eta_N(C_n) = 4$.
Case(iii): $n \equiv 2 \pmod 4$.
Let $f : V[C_n] \to \{1, 2, 3\}$ be defined by
$$f(v_i) = \begin{cases} 1 & i \equiv 1, 2, 3 \pmod 4 \\ 2 & i \equiv 0 \pmod 4 \end{cases} \quad \text{for } 1 \leq i \leq n-2 \text{ and}$$

$f(v_{n-1}) = f(v_n) = 2.$

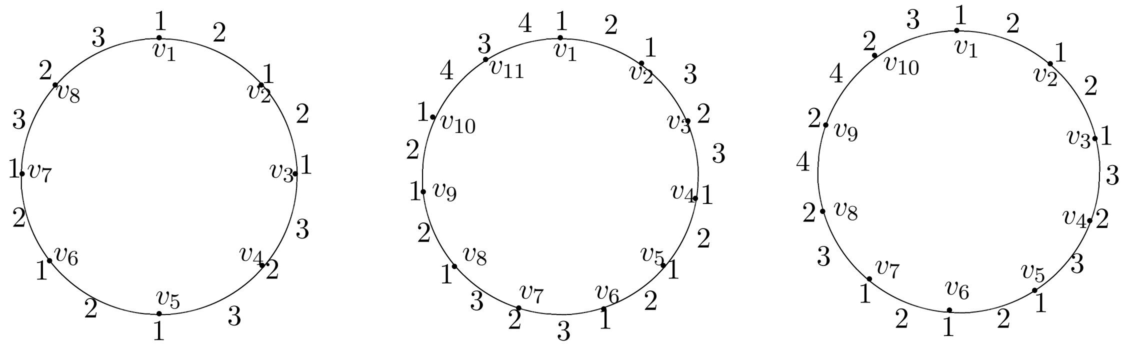
Then the induced edge labeling is given by

$$f^*(v_iv_{i+1}) = \begin{cases} 2 & i \equiv 1, 2 \pmod 4 \\ 3 & i \equiv 0, 4 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le n-3 \text{ and}$$

$f^*(v_{n-2}v_{n-1}) = f^*(v_{n-1}v_n) = 4, f^*(v_nv_1) = 3.$

It is clear that lucky edge neighborhood labeling of C_n is $\{2, 3, 4\}$ and the lucky neighborhood number of C_n is $\eta_N(C_n) = 4$. ■

Illustration 3.4. Lucky edge neighborhood labelings of cycles C_8, C_{10} and C_{11} are given in Figure 3.



3(a) 3(b) 3(c)

Figure 3: Lucky edge neighborhood labelings of C_8, C_{11} and C_{10}.

Theorem 3.5. The tadpole graph $T_{m,n}$ has $\{a, b, c\}$ lucky edge neighborhood labeling for any $a, b, c \in N$.

Proof: Let $v_1, v_2, ..., v_m, v_{m+1}, ..., v_{m+n}$ be the vertices of $T_{m,n}$.

Case(i): $m \equiv 0 \pmod 4$.

Let $f : V[T_{m+n}] \to \{1, 2, 3\}$ be defined by

$$f(v_i) = \begin{cases} 1 & i \equiv 0, 1, 2 \pmod 4 \\ 2 & i \equiv 3 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m,$$

$f(v_{m+1}) = 3,$

$$f(v_{m+i}) = \begin{cases} 1 & i \equiv 0, 2, 3 \pmod 4 \\ 2 & i \equiv 1 \pmod 4 \end{cases} \quad \text{for } 2 \le i \le n.$$

Then the induced edge labeling is given by

$$f^*(v_iv_{i+1}) = \begin{cases} 2 & i \equiv 0, 1 \pmod 4 \\ 3 & i \equiv 2, 3 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m-1,$$

$f^*(v_mv_1) = 2,\ f^*(v_mv_{m+1}) = f^*(v_{m+1}v_{m+2}) = 4,$

$$f^*(v_{m+i}v_{m+i+1}) = \begin{cases} 2 & i \equiv 2, 3 \pmod 4 \\ 3 & i \equiv 0, 1 \pmod 4 \end{cases} \quad \text{for } 2 \le i \le n-1.$$

Case(ii): $m \equiv 2 \pmod 4$.

Let $f : V[T_{m+n}] \to \{1,2,3\}$ be defined by

$$f(v_i) = \begin{cases} 1 & i \equiv 1,2,3 \pmod 4 \\ 2 & i \equiv 0 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m-2,$$

$f(v_{m-1}) = f(v_m) = 2$ and $f(v_{m+1}) = 3$,

$$f(v_{m+i}) = \begin{cases} 1 & i \equiv 0,1,3 \pmod 4 \\ 2 & i \equiv 2 \pmod 4 \end{cases} \quad \text{for } 2 \le i \le n.$$

Then the induced edge labeling is given by

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 1,2 \pmod 4 \\ 3 & i \equiv 0,3 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m-3,$$

$f^*(v_m v_1) = 3,\ f^*(v_{m-2}v_{m-1}) = f^*(v_{m-1}v_m) = 4,\ f^*(v_m v_{m+1}) = f^*(v_{m+1}v_{m+2}) = 5,$

$$f^*(v_{m+i} v_{m+i+1}) = \begin{cases} 2 & i \equiv 0,3 \pmod 4 \\ 3 & i \equiv 1,2 \pmod 4 \end{cases} \quad \text{for } 2 \le i \le n-1.$$

Case(iii): $m \equiv 1,3 \pmod 4$.

Let $f : V[T_{m+n}] \to \{1,2,3\}$ be defined by

$$f(v_i) = \begin{cases} 1 & i \equiv 0,1,2 \pmod 4 \\ 2 & i \equiv 3 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m-2,$$

$f(v_{m-1}) = 3, f(v_m) = 1,$

$$f(v_{m+i}) = \begin{cases} 1 & i \equiv 0,2,3 \pmod 4 \\ 2 & i \equiv 1 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le n.$$

Then the induced edge labeling is given by

$$f^*(v_i v_{i+1}) = \begin{cases} 2 & i \equiv 0,1 \pmod 4 \\ 3 & i \equiv 2,3 \pmod 4 \end{cases} \quad \text{for } 1 \le i \le m-3,$$

$f^*(v_m v_1) = 2,$

$f^*(v_{m-2}v_{m-1}) = 5,$

$f^*(v_{m-1}v_m) = 4,$

$$f^*(v_{m+i} v_{m+i+1}) = \begin{cases} 2 & i \equiv 0,1 \pmod 4 \\ 3 & i \equiv 2,3 \pmod 4 \end{cases}, \qquad 0 \le i \le n-1.$$

It is clear that the lucky edge neighborhood labeling of $T_{m,n}$ is $\{2,3,4,5\}$ and the lucky neighborhood number of $T_{m,n}$ is $\eta_N(T_{m,n}) = 5$. ■

Illustration 3.6. Lucky edge neighborhood labelings of $T_{4,6}$, $T_{10,6}$ and $T_{5,7}$ is shown in Figure 4, 5 and 6.

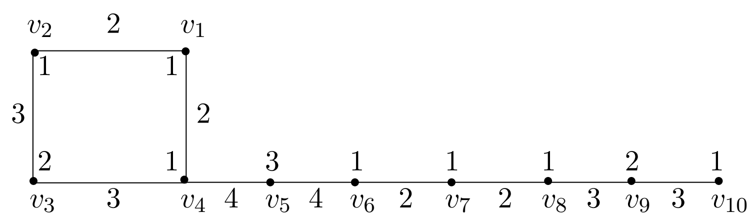


Figure 4: Lucky edge neighborhood labeling of $T_{4,6}$.

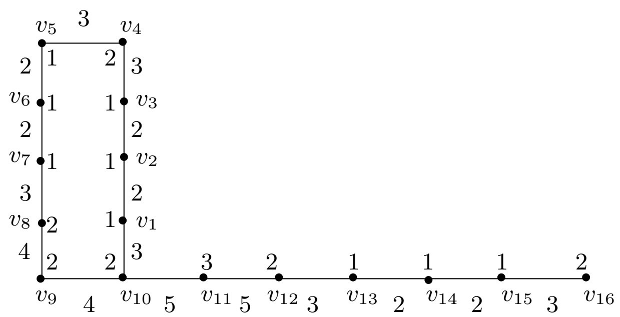


Figure 5: A lucky edge neighborhood labeling of $T_{10,6}$.

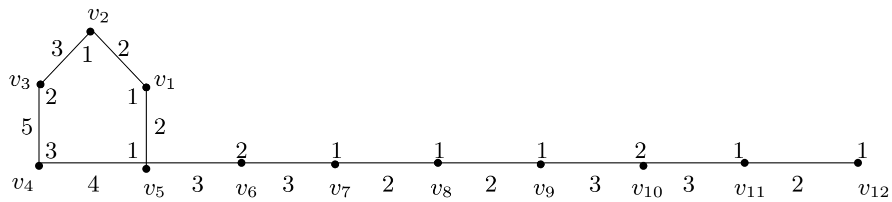


Figure 6: A lucky edge neighborhood labeling of $T_{5,7}$.

Theorem 3.7. $S(P_n^+)$ has $\{a,b,c\}$ lucky edge neighborhood labeling for any $a,b,c \in N$.

Proof: Let $V[S(P_n^+)]$=$\{\{u_i, v_i, v_i'; 1 \leq i \leq n\} \cup \{u_i'; 1 \leq i \leq n-1\}\}$ and $E[S(P_n^+)] = \{\{u_iu_i'; 1 \leq i \leq n-1\} \cup \{u_i'u_{i+1}; 1 \leq i \leq n-1\} \cup \{u_iv_i', v_i'v_i; 1 \leq i \leq n\}\}$.

Let $f : V[S(P_n^+)] \to \{1, 2, 3\}$ be defined by

$f(u_i) = 1$ for $1 \leq i \leq n$,

$$f(u_i') = \begin{cases} 2 & i \equiv 1 \pmod 2 \\ 3 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n-1,$$

$f(v_i) = f(v_i') = 1$, $1 \leq i \leq n$.

Then the induced edge labeling is given by

$$f^*(u_iu_i') = f^*(u_i'u_{i+1}) = \begin{cases} 3 & i \equiv 1 \pmod 2 \\ 4 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n-1,$$

$f^*(u_iv_i') = f^*(v_i'v_i) = 2$, $1 \leq i \leq n$.

It is clear that the lucky edge neighborhood labeling of $S(P_n^+)$ is $\{2, 3, 4\}$ and the lucky neighborhood number of $S(P_n^+)$ is $\eta_N(S(P_n^+))$=4. ■

Illustration 3.8. A lucky edge neighborhood labeling of $S(P_5^+)$ is shown in Figure 7.

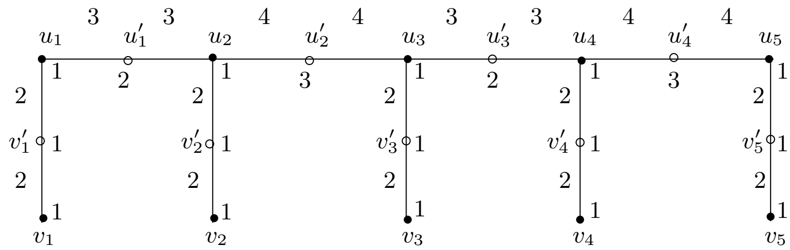


Figure 7: A lucky edge neighborhood labeling of $S(P_5^+)$.

Theorem 3.9. $S(C_n^+)$ has $\{a, b, c\}$ lucky edge neighborhood labeling for any $a, b, c \in N$.

Proof: Let $V[S(C_n^+)] = \{u_i, u_i', v_i, v_i' : 1 \leq i \leq n\}$ and $E[S(C_n^+)]$=$\{\{u_iu_i' ; 1 \leq i \leq n\} \cup \{u_i'u_{i+1} ; 1 \leq i \leq n-1\} \cup \{u_n'u_1\} \cup \{u_iv_i', v_i'v_i ; 1 \leq i \leq n\}\}$.

Case(i): n is odd.

Let $f : V[S(C_n^+)] \to \{1, 2, 3\}$ be defined by

$f(u_i) = 1,\ 1 \leq i \leq n,$

$$f(u_i') = \begin{cases} 2 & i \equiv 1 \pmod 2 \\ 3 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n-1,$$

$f(u_n') = 1,\ f(v_1') = 3,\ f(v_n') = 2,$

$f(v_i') = 1$ for $2 \leq i \leq n-1,$

$f(v_i) = 1,\ 1 \leq i \leq n.$

Then the induced edge labeling is given by

$$f^*(u_iu_i') = f^*(u_i'u_{i+1}) = \begin{cases} 3 & i \equiv 1 \pmod 2 \\ 4 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n-1,$$

$f^*(u_nu_n') = f^*(u_n'u_1) = 2,$

$f^*(u_1v_1') = f^*(v_1'v_1) = 4,$

$f^*(u_nv_n') = f^*(v_n'v_n) = 3,$

$f^*(u_iv_i') = f^*(v_i'v_i) = 2$ for $2 \leq i \leq n-1.$

Case(ii): n is even.

Let $f : V[S(C_n^+)] \to \{1, 2, 3\}$ be defined by

$f(u_i) = 1$ for $1 \leq i \leq n,$

$$f(u_i') = \begin{cases} 2 & i \equiv 1 \pmod 2 \\ 3 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n,$$

$f(v_i') = f(v_i) = 1,\ 1 \leq i \leq n.$

Then the induced edge labeling is given by

$$f^*(u_iu_i') = f^*(u_i'u_{i+1}) = \begin{cases} 3 & i \equiv 1 \pmod 2 \\ 4 & i \equiv 0 \pmod 2 \end{cases} \quad \text{for } 1 \leq i \leq n-1,$$

$f^*(u_nu_n') = f^*(u_n'u_1) = 4,$

$f^*(u_iv_i') = f^*(v_i'v_i) = 2$ for $1 \leq i \leq n.$

It is clear that the lucky edge neighborhood labeling of $S(C_n^+)$ is$\{2, 3, 4\}$ and the lucky neighborhood number of $S(C_n^+)$ is $\eta_N(S(C_n^+)) = 4$. ■

Illustration 3.10. Lucky edge neighborhood labelings of $S(C_5^+)$ and $S(C_4^+)$ are given in Figure 8 and 9 respectively.

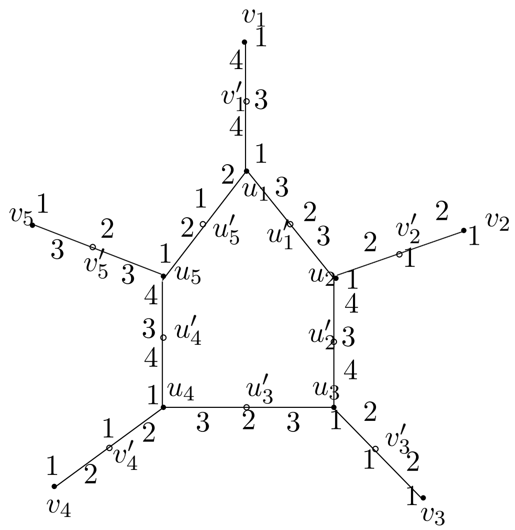


Figure 8: Lucky edge neighborhood labelings of $S(C_5^+)$.

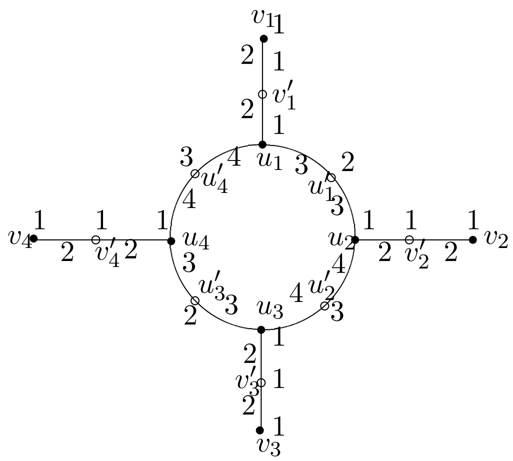


Figure 9: Lucky edge neighborhood labelings of $S(C_4^+)$.

References

[1] J. A. Gallian, *A Dynamic Survey of Graph Labeling.* The Electronic Journal of Combinatorics, 18(2015), #DS6.

[2] S.W. Golomb, *How to number a graph.* In: Graph Theory and Computing(R. C. Read.Ed.) Academic Press, New York (1972), 23 - 37.

[3] A. Nellai Murugan and R. Maria Irudhaya Aspin Chitra,*Lucky Edge Labeling of P_n, C_n, Corona of P_n, C_n*, International Journal of Scientific and Innovative Mathematical Research, Volume 2, Issue 8 (2014), 710-718.

[4] A. Rosa, *On Certain Valuations of the vertices of a Graph*, In: Theory of Graphs, (International Symposium, Rome, July 1966), Gordan and Breach, N. Y. and Dunod Paris(1967), 349 - 355.