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Complementary Nil Eccentric Domination Number of a Graph

M. Bhanumathi[1], **Sudhasenthil**[2]

[1] Government Arts College for Women (Autonomous)
Pudukkottai - 622 001, India.
bhanu_ksp@yahoo.com

[2] S.D.N.B. Vaishnav College for Women (Autonomous)
Chennai - 600 044, India.
sudhasenthilmaths@gmail.com

Abstract

A subset D of the vertex set $V(G)$ of a graph G is said to be a dominating set if every vertex not in D is adjacent to atleast one vertex in D. A dominating set D is said to be an eccentric dominating set if for every $v \in V - D$, there exists atleast one eccentric point of v in D. An eccentric dominating set D of G is a complementary nil eccentric dominating set if the induced subgraph $< V - D >$ is not an eccentric dominating set for G. The minimum of the cardinalities of the complementary nil eccentric dominating sets of G is called the complementary nil eccentric domination number $\gamma_{cned}(G)$ of G. In this paper, bounds for $\gamma_{cned}(G)$, its exact value for some particular classes of graphs and some results on complementary nil eccentric domination number are obtained.

Keywords: Domination, eccentric domination, complementary nil domination, complementary nil eccentric domination.
AMS Subject Classification(2010): 05C69.

1 Introduction and preliminaries

Let G be a finite, simple undirected graph on p vertices and q edges with vertex set $V(G)$ and edge set $E(G)$. For graph theoretic terminology refer Harary [4], Buckley and Harary [2].

In 2010, Janakiraman, Bhanumathi and Muthammai defined eccentric domination in graphs [5] and Bhanumathi and Muthammai studied eccentric domination in trees and various bounds of eccentric domination in graphs [1, 5]. Kulli and Janakiram introduced the maximal domination number in graphs [6]. This maximal domination is also termed as complementary nil domination. Tamizh Chelvam and Robinson Chellathurai studied the concept of this domination number [7]. Motivated by these, we have defined the complementary nil eccentric domination number of a graph and studied its bounds.

Let G be a connected graph and u be a vertex of G. The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus $e(v) = \max\{d(u, v); u \in V\}$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity. For any connected graph G, $r(G) \leq diam(G) \leq 2r(G)$, v is a central vertex if $e(v) = r(G)$. The center $C(G)$ is the set of all central vertices. The central subgraph $< C(G) >$ of a graph G is the subgraph induced by the center. v is a peripheral vertex if $e(v) = diam(G)$. The periphery $P(G)$ is the set of all peripheral vertices.

For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex of v. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G)/d(u,v) = e(v)\}$. The open neighborhood $N(u)$ of a vertex u is the set of all vertices adjacent to u in V. $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood of v. For a vertex $v \in V(G)$, $N_i(v) = \{u \in V(G); d(u,v) = i\}$ is defined to be the i^{th} neighborhood of v in G.

A set $D \subseteq V$ is said to be a dominating set in G, if every vertex in $V - D$ is adjacent to some vertex in D. A dominating set D is an independent dominating set, if no two vertices in D are adjacent.

A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists atleast one eccentric point of v in D. If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But if $D'' \subseteq D$ then D'' is not necessarily an eccentric dominating set. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set. The eccentric domination number $\gamma_{ed}(G)$ of a graph G is the minimum cardinality of an eccentric dominating set.

An eccentric dominating set D of G is a complementary nil eccentric dominating set if the induced subgraph $< V - D >$ is not an eccentric dominating set for G. The minimum of the cardinalities of the complementary nil eccentric dominating sets of G is called the complementary nil eccentric domination number $\gamma_{cned}(G)$.

In this paper, we have studied the complementary nil eccentric domination number of graphs.

2 Prior Results

Theorem 2.1. [5] An eccentric dominating set D is a minimal eccentric dominating set if and only if for each vertex $u \in D$, one of the following is true.

(i) u is an isolated vertex of D or u has no eccentric vertex in D.

(ii) There exists some $v \in V - D$ such that $N(v) \cap D = \{u\}$ or $E(v) \cap D = \{u\}$.

Theorem 2.2. [7] For any graph G, $\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq \gamma(G) \leq p - \Delta(G)$.

Theorem 2.3. $\gamma_{ed}(K_{1,n}) = 2$ for $n \geq 2$.

Theorem 2.4. $\gamma_{ed}(K_{m,n}) = 2$ for $m, n \geq 2$.

Theorem 2.5. $\gamma_{ed}(W_n) = 3$ for $n \geq 7$.

3 Main Results

In this paper, we define a new domination parameter known as complementary nil eccentric domination as follows.

Definition 3.1. An eccentric dominating set D of G is a complementary nil eccentric dominating set (cned-set) if the induced subgraph $< V - D >$ is not an eccentric dominating set for G.

The complementary nil eccentric domination number $\gamma_{cned}(G)$ of a graph G equals the minimum cardinality of a complementary nil eccentric dominating set. That is $\gamma_{cned}(G) = \min |D|$, where the minimum is taken over D in $\mathcal{D}$, where $\mathcal{D}$ is the set of all minimal complementary nil eccentric dominating sets of G. $V(G)$ is the complementary nil eccentric dominating set for any graph G. Hence, $\gamma_{cned}(G)$ is a well defined parameter. Obviously, $\gamma_{ed}(G) \leq \gamma_{cned}(G)$.

Example 3.2.

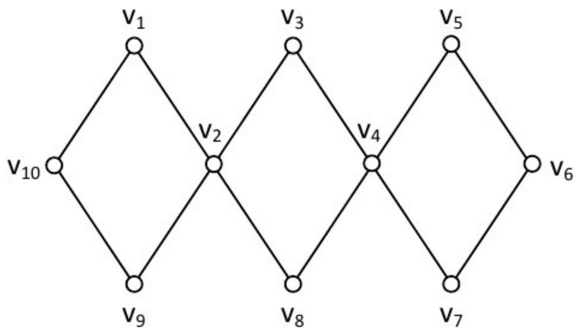

Figure 1

$D = \{v_2, v_4, v_6, v_{10}\}$ is a minimum eccentric dominating set.
$D_1 = \{v_2, v_4, v_6, v_{10}\}$ is a minimum complementary nil eccentric dominating set.
$D_2 = \{v_2, v_4, v_6, v_{10}, v_9\}$ is a minimum complementary nil dominating set.
Therefore, $\gamma_{cned}(G) = 4$, $\gamma_{cnd}(G) = 5$, $\gamma_{ed}(G) = 4$.
Here, $\gamma_{cned}(G) < \gamma_{cnd}(G)$.

Example 3.3.

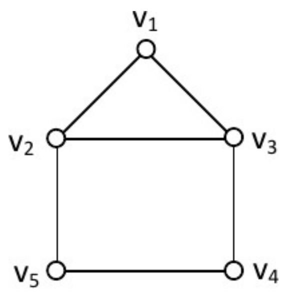

Figure 2

Here $D = \{v_2, v_4, v_5\}$ is a minimum complementary nil eccentric dominating set.
$D_1 = \{v_2, v_4, v_5\}$ is a minimum eccentric dominating set.
$D_2 = \{v_2, v_4\}$ is a minimum dominating set.
$D_3 = \{v_2, v_4, v_5\}$ is a minimum complementary nil dominating set.
Therefore, $\gamma(G) = 2$, $\gamma_{ed}(G) = 3$, $\gamma_{cnd}(G) = 3$, $\gamma_{cned}(G) = 3$, $\gamma_{cned}(G) = \gamma_{cnd}(G)$.

Example 3.4.

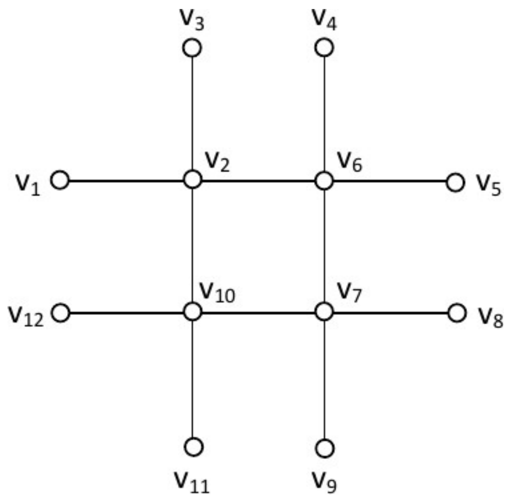

Figure 3

$D = \{v_1, v_3, v_4, v_5, v_8, v_9, v_{11}, v_{12}\}$ is an eccentric dominating set and also complementary nil eccentric dominating set.
$D_1 = \{v_2, v_6, v_7, v_8, v_{10}\}$ is a complementary nil dominating set.
$\gamma_{ed}(G) = 8$, $\gamma_{cnd}(G) = 5$, $\gamma_{cned}(G) = 8$, $\gamma_{cned}(G) > \gamma_{cnd}(G)$.

Obviously, $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{cned}(G)$. But, sometimes $\gamma_{cned}(G) < \gamma_{cnd}(G)$, otherwise $\gamma_{cned}(G) \geq \gamma_{cnd}(G)$ depending upon the graph G. So, the parameters $\gamma_{cned}(G)$ and $\gamma_{cnd}(G)$ are incomparable.

Observation 3.5.

(1) $\gamma_{cned}(P_n) = \gamma_{ed}(P_n)$.

(2) $\gamma_{cned}(K_p - \{e\}) = p - 1$, where, e is an edge in K_p.

(3) $2 \leq \gamma_{cned}(G) \leq p - 1$, for $p \geq 3$.

Let $S \subseteq V$. Then a vertex $v \in S$ is said to be an enclave of S if $N[v] \subseteq S$.

Theorem 3.6. Let S be a cned-set of a graph G. Then S contains atleast one enclave of S or S contains atleast one vertex whose eccentric vertices are in S.

Proof: Let S be a cned-set of a graph G. By the definition of cned-set, $V-S$ is not an eccentric dominating set, this implies that there exist a vertex $v \in S$ such that v has no eccentric point in $V-S$, [Therefore v has all its eccentric vertices in S] or there exist $v \in S$ such that v is not adjacent to any of the vertices in $V-S$. That is $N[v] \subseteq S$. That is S contains atleast one enclave of S. ∎

Theorem 3.7. Let S be a cned-set of a graph G. Then S is minimal if and only if for each $u \in S$ one of the following conditions is satisfied.

1. u is an isolated vertex of S or u has no eccentric vertex in S.
2. There exists some $v \in V-S$, such that $N(v) \cap S = \{u\}$ or $E(v) \cap S = \{u\}$.
3. $V-[S-\{u\}]$ is an eccentric dominating set.

Proof: Suppose S is minimal. On the contrary, if there exist a vertex $u \in S$ such that u does not satisfy any of the given conditions (i), (ii), (iii), then $S_1 = S-\{u\}$ is an eccentric dominating set of G. Also by (iii) $V-[S-\{u\}]$ is not eccentric dominating set. This implies that S_1 is a cned-set of G, which is a contradiction to the minimality of S.

Conversely, Suppose S is a cned-set and for each $u \in S$, one of the conditions holds, we show that S is a minimal complementary nil eccentric dominating set.

Suppose that S is not a minimal complementary nil eccentric dominating set, that is there exist a vertex $u \in S$ such that $S-\{u\}$ is a complementary nil eccentric dominating set. Hence, u is adjacent to atleast one vertex v in $(S-\{u\})$ and u has an eccentric point in $D-\{u\}$. Therefore, condition (i) does not hold.

Also if, $S-\{u\}$ is a complementary nil eccentric dominating set, every element x in $V-[S-\{u\}]$ is adjacent to atleast one vertex in $S-\{u\}$ and x has an eccentric point in $S-\{u\}$. Hence condition (ii) does not hold. Since $S-\{u\}$ is a cned-set, $V-[S-\{u\}]$ is not ac eccentric dominating set, that is condition (iii) does not hold. Therefore, there exists $u \in S$ such that u does not satisfy conditions (i), (ii), (iii) which is a contradiction to our assumption. ∎

Theorem 3.8.

1. $\gamma_{ed}(K_{1,n}) = \gamma_{cned}(K_{1,n}) = 2$.
2. $\gamma_{cned}(K_{m,n}) = \min\{m,n\}+1$, for $m,n \geq 2$, $m \geq n$.
3. $\gamma_{cned}(W_n) = 4$, for $n \geq 7$.

Proof: (1) When $G = K_{1,n}$. Let $D = \{u,v\}$. Here v is the central vertex. The central vertex dominate all vertices in $V-D$, u is an eccentric point vertices of $V-D$ and also $V-D$ is not a dominating set. Therefore D is a complementary nil eccentric dominating set. Thus,

$$\gamma_{cned}(G) \leq 2. \tag{1}$$

But $\gamma_{ed}(G) \leq \gamma_{cned}(G)$. Therefore,

$$2 \leq \gamma_{cned}(G). \tag{2}$$

From (1) and (2) $\gamma_{cned}(G) = 2$.

(2) When $G = K_{m,n}$. $V(G) = V_1 \cup V_2$. $|V_1| = m$ and $|V_2| = n$ such that each element of V_1 is adjacent to every vertex of V_2 and vice versa. Let $D = V_2 \cup \{u\}$, $u \in V_1$ is a complementary nil eccentric dominating set. $|D| = n + 1 = \min\{m, n\} + 1$. Therefore,

$$\gamma_{cned}(G) \leq \min\{m, n\} + 1. \tag{3}$$

$D_1 = \{y, z\}$, $y \in V_1$, $z \in V_2$ is a eccentric dominating set. Hence $S \subseteq V(G)$ is a complementary nil dominating set if and only if $V - S$ does not have vertices from both V_1 and V_2. So if D is a complementary nil dominating set it must contain V_1 or V_2. Also V_1 and V_2 are dominating sets. But V_1 is not an eccentric dominating set. V_2 is also not an eccentric dominating set. Therefore,

$$\gamma_{cned}(G) \geq \min\{m, n\} + 1. \tag{4}$$

From (3) and (4) $\gamma_{cned}(G) = \min\{m, n\} + 1$. Therefore $\gamma_{cned}(K_{m,n}) = \min\{m, n\} + 1$.

(3) When $G = W_n$ for $n \geq 7$. Let $D = \{u, x, v, w\}$, where u and v are any two non adjacent non central vertices, x is adjacent to both u and v, and w is the central vertex. D is a minimum eccentric dominating set of G. The complement $V - D$ is not a dominating set. Therefore, D is a complementary nil eccentric dominating set. Hence

$$\gamma_{cned}(G) \leq 4, \tag{5}$$

but $\gamma_{ed}(G) = 3$ by Theorem 2.5 and no γ_{ed}-set is a complementary nil eccentric dominating set. Therefore,

$$\gamma_{cned}(G) \geq 4. \tag{6}$$

From (5) and (6) $\gamma_{cned}(G) = 4$. ■

Theorem 3.9. For any graph G, every γ_{cned}-set intersects with every γ_{ed}-set of G.

Proof: Let S_1 be a γ_{cned}-set and S be a γ_{ed}-set of G. Suppose that $S_1 \cap S = \phi$ then $S \subseteq V - S_1$, $V - S_1$ contains eccentric dominating set S. Therefore $V - S_1$ itself is an eccentric dominating set, which is a contradiction. Thus, $S_1 \cap S \neq \phi$. ■

4 Bounds for Complementary Nil Eccentric Domination Number

In this section, we obtain some bounds for the cned-number of graphs.

Theorem 4.1. If G is a graph with a pendent vertex, then $\gamma_{cned}(G) = \gamma_{ed}(G)$ or $\gamma_{ed}(G) + 1$.

Proof: Let D be a γ_{ed}-set of G. Let u be a pendent vertex in G. If u and its support vertex is in D, then $V - D$ is not a dominating set. Therefore, $\gamma_{cned}(G) = \gamma_{ed}(G)$. If u or its support vertex v is in D, then $D_1 = D \cup \{v\}$ or $D_1 = D_1 \cup \{u\}$ is an eccentric dominating set and $V - D_1$ is not a dominating set. Therefore, $\gamma_{cned}(G) = \gamma_{ed}(G) + 1$. ■

Theorem 4.2. If G is of diameter two, then $\gamma_{cned}(G) \leq 1 + \delta(G)$.

Proof: $Diam(G) = 2$. Let $u \in V(G)$ be such that $degu = \delta(G)$. Now take $D = \{u\} \cup N(u) = N[u]$. Every point in $N_2(u) = V - D$ is adjacent to elements of $N(u)$ and are eccentric to u. This implies that D is an eccentric dominating set, and $V - D = N_2(u)$, this $V - D$ has no dominating set. Since u cannot be dominated by any element of $N_2(u)$. Therefore, D is an cned-set. Hence $\gamma_{cned}(G) \leq 1 + \delta(G)$. ■

Theorem 4.3. If $\gamma_{ed}(G) > \frac{p}{2}$, then $\gamma_{ed}(G) = \gamma_{cned}(G)$.

Proof: Let $\gamma_{ed}(G) > \frac{p}{2}$, and let D be a minimum eccentric dominating set of G. Therefore, $|D| > \frac{p}{2}$. Now $|V - D| < \frac{p}{2}$. $V - D$ has atmost $\frac{p}{2} - 1$ elements and every γ_{ed}-set has at least $\frac{p}{2} + 1$ elements. Hence $V - D$ cannot have an eccentric dominating set. Therefore, $\gamma_{ed}(G) = \gamma_{cned}(G)$. ■

Theorem 4.4. If $\gamma_{ed}(G) = \frac{p}{2}$, then $\gamma_{cned}(G) = \frac{p}{2}$ or $\frac{p}{2} + 1$ where p is even.

Proof: Let D be a minimum eccentric dominating set. By the given hypothesis $|D| = \frac{p}{2}$. Now $V - D$ has $\frac{p}{2}$ elements. Suppose $V - D$ itself is an eccentric dominating set, then $\gamma_{cned}(G) = \frac{p}{2} + 1$, otherwise $\gamma_{cned}(G) = \frac{p}{2}$. ■

Theorem 4.5. For any graph G, $\gamma_{ed}(G) \leq \gamma_{cned}(G) \leq \gamma(G) + t$, where t is the number of all eccentric vertices of G.

Proof: Obviously $\gamma_{ed}(G) \leq \gamma_{cned}(G)$. Let D be a minimum dominating set. Let $S = \{u \in V(G)/u$ is an eccentric vertex of some $v \in V(G)\}$. Then clearly $D \cup S$ is an eccentric dominating set. Also $V - (D \cup S)$ has no eccentric vertices. So $D \cup S$ is a complementary nil eccentric dominating set. Hence, $\gamma_{cned}(G) \leq |S| + |D| = \gamma(G) + t$. ■

Theorem 4.6. Let n be a even integer. Let G be obtained from the complete graph K_n by deleting edges of a linear factor. Then $\gamma_{cned}(G) = \frac{n}{2} + 1$.

Proof: Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices of G, let $G = K_n - \{v_1v_2, v_3v_4, \ldots, v_{n-1}v_n\}$. Then $D = \{v_1, v_3, \ldots, v_{n-1}\}$ and $V - D = \{v_2, v_4, \ldots, v_n\}$ are eccentric dominating sets, and we know that $\gamma_{ed}(G) = \frac{n}{2}$. Therefore, $\gamma_{cned}(G) = \frac{n}{2} + 1$. ■

Theorem 4.7. For any graph G, $\gamma_{cned}(G) \leq \gamma_{ed}(G) + \delta(G)$.

Proof: Let S be the γ_{ed}-set of G and $u \in V$ such that $d(u) = \delta$. If $u \in V - S$, there exists $v \in N(u)$ such that $v \in S$, $|N(u)| = \delta$. Now $S_1 = S \cup N[u]$ is a $\gamma_{ed}(G)$-set and $V - S_1$ is not a dominating set, which implies that $\gamma_{cned}(G) \leq \gamma_{ed}(G) + \delta(G)$. ■

Theorem 4.8. For any connected graph G which is not complete, with $p > 1$, $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma_{cned}(G) \leq 2q - p + 1$. Also, if $\gamma_{cned}(G) = 2q - p + 1$ then G is a tree.

Proof: Since $\left\lceil \frac{p}{\Delta+1} \right\rceil \leq \gamma(G) \leq \gamma_{cned}(G)$, the first inequality follows. For any graph G, $\gamma_{cned}(G) \leq p - 1 = 2(p-1) - p + 1 = 2q - p + 1$. Therefore, $\gamma_{cned}(G) \leq 2q - p + 1$. Also, if $\gamma_{cned}(G) = 2q - p + 1$, then $2q - p + 1 \leq p - 1$ and so $q \leq p - 1$. Therefore, G must be a tree. ■

Theorem 4.9. Let G be a graph such that both G and its complement $\overline{G}$ are connected. Then $\gamma_{cned}(G) + \gamma_{cned}(\overline{G}) \leq (p-1)(p-2)$ equality holds for $G = P_4$.

Proof: By the above result $\gamma_{cned}(G) \leq 2q - p + 1$ and $\gamma_{cned}(\overline{G}) \leq 2q - p + 1$, then $\gamma_{cned}(G) + \gamma_{cned}(\overline{G}) \leq 2(q + q) - 2(p-1) = p(p-1) - 2(p-1) = (p-1)(p-2)$. ■

Theorem 4.10. For any tree T, $\gamma_{cned}(T) + \epsilon(T) \leq p + 2$, where $\epsilon(T)$ is the number of pendent vertices in T.

Proof: All the non pendent vertices together with atmost two pendent vertices form a complementary nil eccentric dominating set. Therefore, $\gamma_{cned}(T) \leq p - \epsilon(T) + 2$, and so $\gamma_{cned}(T) + \epsilon(T) \leq p + 2$. ■

Theorem 4.11. If G is a caterpillar such that each non pendent vertex is of degree three, then $\gamma_{cned}(G) = \frac{p}{2} + 1$.

Proof: Since degree of each non pendant vertex is three, G is of the following form, and $\gamma_{ed}(G) = \frac{p}{2} + 1$.

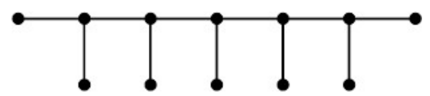

Figure 4

It is clear that $\gamma_{cned}(G) = \frac{p}{2} + 1$. ■

Theorem 4.12. Let T be a tree with $diam(T) > 2$, then $\gamma_{cned}(\overline{T}) \leq p + 1 - \Delta(T)$. Equality holds for P_4.

Proof: Since $\gamma(\overline{T}) = 2$, $\gamma_{cned}(\overline{T}) \leq \gamma(\overline{T}) + \delta(\overline{T}) \leq 2 + p - 1 - \Delta(T)$. Hence, $\gamma_{cned}(\overline{T}) \leq p + 1 - \Delta(T)$. ■

Theorem 4.13. Let T be a tree such that every non end vertex is adjacent to atleast one end vertex. Then $\gamma_{cned}(T) \leq s + 2$, where s is the number of support vertices.

Proof: Let s be the number of support vertices. All the non end vertices form a dominating set. To form an eccentric dominating set we have to add atmost two peripheral (end) vertices. Therefore, $\gamma_{cned}(T) \leq s + 2$. ■

Theorem 4.14. Let G be a graph with $rad(G) \geq 3$, then $\gamma_{cned}(G) \leq p - \delta$.

Proof: Let $v \in V$ with $d(v) = \delta$. Since $rad(G) \geq 3$, there exists a vertex $u \in V - N[v]$ but u is not adjacent to any vertex in $N[v]$, and every vertex in $N[v]$ has an eccentric point in $V - N(v)$. Now, $V - N(v)$ is an eccentric dominating set, but vertices in $N(v)$ has no eccentric points in $V - N(v)$. So, $V - N(v)$ is an eccentric dominating set, but $N(v)$ is not an eccentric dominating set. Therefore, $\gamma_{cned}(G) \leq |V - N(v)| \leq p - \delta$. ■

Theorem 4.15. If G is of radius 2 with a unique central vertex u, then $\gamma_{cned}(G) \leq p - deg(u)$.

Proof: If G is of radius 2 with a unique central vertex u then u dominates $N[u]$ and the vertices in $V - N[u]$ dominate themselves and each vertex in $N[u]$ has eccentric vertices in $V - N[u]$ only. Therefore, $D = V - N(u)$ is an eccentric dominating set. Therefore, $\gamma_{cned}(G) \leq p - deg(u)$. ■

5 Conclusion

Here we have evaluated the complementary nil eccentric domination number of some families of graphs and also studied some bounds for the complementary nil eccentric domination number of a graph.

References

[1] M. Bhanumathi and S. Muthammai, *Eccentric domination in trees*, International Journal of Engineering Science, Advanced Computing and Bio-Technology, 2(1) (2011), 38–46.

[2] F. Buckley and F. Harary, *Distance in graphs*, Addison-Wesley Publishing Company, 1990.

[3] E.J. Cockayne and S.T. Hedetniemi, *Towards a theory of domination in graphs*, Networks, 7 (1977), 247–261.

[4] F. Harary, *Graph theory*, Addison-Wesley Publishing Company Reading Mass, 1972.

[5] T.N. Janakiraman, M. Bhanumathi and S. Muthammai, *Eccentric domination in graphs*, International Journal of Engineering Science, Advanced Computing and Bio Technology, 1(2) (2010), 55–70.

[6] V.R. Kulli and B. Janakiram, *The maximal domination number of graph*, Graph Theory Notes of New York, New York Academy of Sciences, XXXIII (1997), 11–13.

[7] Tamizh Chelvam and S. Robinson Chellathurai, *Complementary nil domination number of a graph*, Tamkang Journal of Mathematics, 40(165-172) (2009).

[8] Teresa W. Haynes, Stephen T. Hedetnimi and Peter J. Slater, *Fundamentals of domination in graphs*, Marcel Decker, 1998.