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Watson-Crick local languages and Watson-Crick two dimensional local languages

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Abstract

Watson-Crick finite automata are language recognizing devices similar to finite automata introduced in DNA computing area. Local languages are of great interest in the study of formal languages using factors of length two or more. We define Watson-Crick local languages using double stranded sequences where the two strands relate to each other through a complementary relation inspired by the DNA complementarity. We also define tiling recognizable Watson-Crick local languages and prove some closure properties. We also extend Watson-Crick local languages to finite arrays and prove some closure properties.

Keywords: Watson-Crick local languages, Watson-Crick tiling system.

AMS Subject Classification (2010): 68Q45.

1 Introduction

The remarkable progress made by molecular biology and biotechnology in the last couple of decades, particularly in sequencing, synthesizing and manipulating DNA molecules, gave rise to the possibility of using DNA as a support for computation. The computer science community quickly reacted to this challenge and many computational models were built to exploit the advantages of nano-level biomolecular computing. One of them is Watson-Crick automata.

Watson-Crick automata, introduced in [2] represent one instance of mathematical model abstracting biological properties for computational purposes. They are finite automata with two reading heads, working on double stranded sequences. One of the main features of these automata is that characters on corresponding positions from the two strands of the input are related by a complementarity relation similar with the Watson-Crick complementarity of DNA nucleotides. The two strands of the input are separately scanned from left to right by read only heads controlled by a common state.

Local languages are described by the factors of length two or more. Special classes of automata called scanners are considered as a model for computations that require only "local" information. Informally, a scanner [1] is an automaton equipped with a finite memory and a sliding window of a fixed length. In a typical computation, the sliding window is moved from left to right on the input, so that the scanner can remember the factors of length two or more.

We use this concept of local on languages and define Watson-Crick local languages. We also define Watson-Crick tiling system and also recognizability of Watson-Crick local languages. We prove some closure properties of Watson-Crick tiling system languages.

We extend this concept to finite arrays and define Watson-Crick two-dimensional tiling system and prove some closure properties.

2 Preliminaries

Let Γ be a finite alphabet. Γ^* denotes the set of all fiinite words over Γ, λ is the empty word and Γ^+ is the set of all non empty finite words over Γ. i.e., $\Gamma^+ = \Gamma^* \setminus \{\lambda\}$.

Definition 2.1. [4] We now define a "complementarity" relation on the alphabet Γ (like the Watson-Crick complementarity relation among the four DNA nucleotides), $\rho \subseteq \Gamma \times \Gamma$ which is symmetric.

Denote $\{\begin{bmatrix} a \\ b \end{bmatrix} / a, b \in \Gamma, (a, b) \in \rho\}^*$ by $WK_\rho(\Gamma)$.

The set $WK_\rho(\Gamma)$ is called the Watson-Crick domain associated to Γ and ρ. The elements $\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}\cdots\begin{bmatrix} a_n \\ b_n \end{bmatrix} \in WK_\rho(\Gamma)$ are also written in the form $\begin{bmatrix} w_1 \\ w_1 \end{bmatrix}$ for $w_1 = a_1\ a_2 \ldots a_n$ and $w_2 = b_1\ b_2 \ldots b_n$. We call such elements $\begin{bmatrix} w_1 \\ w_1 \end{bmatrix} \in WK_\rho(\Gamma)$ molecules. According to the usual way of representing DNA molecules as double-stranded sequences, we also write the product monoid (Γ^*, Γ^*) in the form $\begin{bmatrix} \Gamma^* \\ \Gamma^* \end{bmatrix}$ and its elements in the form $\begin{bmatrix} x \\ y \end{bmatrix}$.

Definition 2.2. [3] A two-dimensional string (or a picture p) over Γ is a two-dimensional rectangular array of elements of Γ. The set of all two-dimensional strings over Γ is denoted as Γ^{**}. A two-dimensional language over Γ is a subset of Γ^{**}.

The boundary of a picture p is identified by a special symbol $\# \notin \Gamma$ which surrounds p. Watson-Crick domain is also extended to two-dimensional arrays. We define

$$WK_\rho\text{-}2D(\Gamma) = \{(p_1, p_2) / p_1, p_2 \text{ are } m \times n \text{ arrays and } (p_1^{ij}, p_2^{ij}) \in \Gamma\}.$$

We now define some concatenation operations between pictures and two-dimensional languages.

Definition 2.3. [3] Let p and q be two pictures over an alphabet Σ of size (m, n) and (m′, n′), m, n, m′, n′ > 0 respectively.

$$p = \boxed{\begin{matrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{matrix}} \qquad q = \boxed{\begin{matrix} q_{11} & \cdots & q_{1n'} \\ \vdots & \ddots & \vdots \\ q_{m'1} & \cdots & p_{m'n'} \end{matrix}}$$

The column catenation of p and q denoted by p ⦶ q is a partial operation, defined only if m = m′ given by

$$p \oplus q = \begin{array}{|ccc|ccc|} p_{11} & \cdots & p_{1n} & q_{11} & \cdots & q_{1n'} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} & q_{m'1} & \cdots & p_{m'n'} \end{array}$$

Similarly the row concatenation of p and q denoted by $p \ominus q$ is defined only if $n = n'$.

Definition 2.4. [3] Let L_1 and L_2 be two two-dimensional languages over an alphabet Σ, the column concatenation of L_1 and L_2 denoted by $L_1 \oplus L_2$ is defined by

$$L_1 \oplus L_2 = \{p \oplus q \,/\, p \in L_1 \text{ and } q \in L_2$$

Similarly the row concatenation of L_1 and L_2 is defined.

Definition 2.5. [3] Let L be a picture language. The column closure of L denoted as $L^{*\oplus}$ is defined as $L^{*\oplus} = \cup_{i\geq 0} L^{i\oplus}$ where $L^{0\oplus} = \lambda$, $L^{1\oplus} = L$, $L^{n\oplus} = L \oplus L^{(n-1)\oplus}$. Similarly the row closure of L is defined.

3 Watson-Crick Local Languages

Definition 3.1. Let Γ be a finite alphabet. If $w \in WK_\rho(\Gamma)$ then $L \subset WK_\rho(\Gamma)$ is local if there exist finite set of tiles $\theta \in \Gamma^{2\times 2}$ over the alphabet $\Gamma \cup \{\#\}$ such that $L = \{w \in WK_\rho(\Gamma) \,/\, B_{2,2}(w) \subseteq \theta\}$ where $B_{2,2}(w)$ is the set of all blocks or tiles of size (2, 2). We write it as $L = L(\theta)$. The family of Watson-Crick local languages is denoted as $WK_\rho(LOC)$.

Example 3.1. Let $\Gamma = \{a, b\}$ be an alphabet and θ be the following set of tiles over Γ. $\rho = \{(a, a), (b, b)\}$

$$\theta = \left\{ \begin{pmatrix} \# & a \\ \# & a \end{pmatrix}, \begin{pmatrix} a & a \\ a & a \end{pmatrix}, \begin{pmatrix} a & b \\ a & b \end{pmatrix}, \begin{pmatrix} b & b \\ b & b \end{pmatrix}, \begin{pmatrix} b & \# \\ b & \# \end{pmatrix} \right\}$$

The Watson-Crick local language is $L = L(\theta) = \left\{ \begin{pmatrix} a^n & b^m \\ a^n & b^m \end{pmatrix} / n, m \geq 1 \right\}$.

4 Tiling Recongizable Watson-Crick Languages

Definition 4.1. A Watson-Crick tiling system is a 4-tuple $T = (\Sigma, \Gamma, \theta, \pi)$ where Σ and Γ are two finite alphabets, θ is a finite set of tiles over the alphabet $\Gamma \cup \{\#\}$ and $\pi : \Gamma \to \Sigma$ is a projection.

The tiling system defines (recognizes) a language L over the alphabet Σ as $L = \pi(L')$ where $L' = L(\theta)$, is the local language over Γ corresponding to the set of tiles θ written as $L = L(T)$.

We say that L is the language recognized by T. We say that a language $L \subseteq WK_\rho(\Gamma)$ is recognizable by tiling systems if there exists a tiling system $T = (\Sigma, \Gamma, \theta, \pi)$ such that $L = L(T)$.

We denote by $\mathcal{L}$ (WK_ρ-TS) the family of all Watson-Crick recognizable tiling systems.

Example 4.1. Considering Example 3.1 if we apply the projection $\pi : \Gamma \to \Sigma$ such that $\pi(a) = \pi(b) = a$ then we see that $L = \pi(L')$.

We now prove some closure properties.

Theorem 4.1. The family $\mathcal{L}(WK_\rho\text{-TS})$ is closed under projection.

Proof: Let Σ_1, Σ_2 be two finite alphabets and let $\phi : \Sigma_1 \to \Sigma_2$ be a projection.

We have to prove, if $L_1 \subseteq \Sigma_1^*$ is recognizable by Watson-Crick tiling systems then $L_2 = \phi(L_1)$ is recognizable by tiling system.

Let $T_1 = (\Sigma_1, \Gamma, \theta, \pi_1)$ be a Watson-Crick tiling system for L_1. i.e., $L_1 = L(T_1)$. Then $L_1 = \pi_1(L')$ where L' is the Watson-Crick local language represented by θ. We see that L' is a Watson-Crick local language. Also $L_2 = \pi_2(L')$ where $\pi_2 = \phi \, . \, \pi_1 : \Gamma \rightarrow \Sigma_2$. Hence $T_2 = (\Sigma_2, \Gamma, \theta, \pi_2)$, is a Watson-Crick tiling system for L_2. ■

Theorem 4.2. The family $\mathcal{L}(WK_\rho\text{-}TS)$ is closed under catenation.

Proof: Let L_1 and L_2 be languages over the alphabet Σ and let $L = L_1 \, . \, L_2$ be the language obtained by catenating L_1 with L_2. By definition of catenation, a word $w \in L$ is composed of a pair of words $w_1 \in L_1$ and $w_2 \in L_2$ such that the rightmost letter of w_1 is glued to the leftmost letter of w_2.

Let $(\Sigma, \Gamma_1, \theta_1, \pi_1)$ and $(\Sigma, \Gamma_2, \theta_2, \pi_2)$ be two Watson-Crick tiling systems for L_1 and L_2 respectively. Without loss of generality we assume $\Gamma_1 \cap \Gamma_2 = \phi$. We define a Watson-Crick tiling system for L as $(\Sigma, \Gamma, \theta, \pi)$ where $\Gamma = \Gamma_1 \cup \Gamma_2$ and θ has to contain all the elements from set θ_1 except those corresponding to the right borders and all elements from set θ_2 except those corresponding to the left borders. Hence we define θ as follows:

$$\theta = \theta_1 \cup \theta_2 \cup \left\{ \begin{pmatrix} a & c \\ b & d \end{pmatrix} \text{ if } \begin{pmatrix} a & \# \\ b & \# \end{pmatrix} \in \theta_1 \text{ and } \begin{pmatrix} \# & c \\ \# & d \end{pmatrix} \in \theta_2 \right\}$$

The projection $\pi : \Gamma \rightarrow \Sigma$ is given as $\forall$ a $\in \Gamma$, $\pi(a) = \begin{cases} \pi_1(a) & \text{if } a_1 \in \Gamma_1 \\ \pi_2(a) & \text{if } a_2 \in \Gamma_2 \end{cases}$. ■

Theorem 4.3. The family $\mathcal{L}(WK_\rho\text{-}TS)$ is closed under $\cup$.

Proof: Let L and L_2 be two languages over an alphabet Σ and let $(\Sigma, \Gamma_1, \theta_1, \pi_1)$ and $(\Sigma, \Gamma_2, \theta_2, \pi_2)$ be two WK_ρ-TS to recognize L_1 and L_2 respectively.

We assume $\Gamma_1 \cap \Gamma_2 = \phi$.

A WK_ρ-TS $(\Sigma, \Gamma, \theta, \pi)$ for $L_1 \cup L_2 = L$ is defined where $\Gamma = \Gamma_1 \cup \Gamma_2$ is the local alphabet and projection π is defined so that its restrictions to alphabets Γ_1 and Γ_2 coincide with π_1 and π_2 as in Theorem 4.2. Then $\theta = \theta_1 \cup \theta_2$. ■

Watson-Crick local languages can also be extended to two dimensional finite arrays. Instead of the double strands in the Watson-Crick automaton here we have two finite planes. The 1^{st} plane corresponds to the upper level strand and the 2^{nd} plane corresponds to the lower level strand of a double strand sequence.

5 Watson-Crick Two-Dimensional Local Languages

Definition 5.1. Let Γ be a finite alphabet. If $p \in WK_\rho\text{-}2D(\Gamma)$ then $p = (p_1, p_2)$ and $L \subset WK_\rho\text{-}2D(\Gamma)$ is local if there exist finite set of tiles $\theta_1, \theta_2 \subset \Gamma^{2\times 2}$ such that $\theta = (\theta_1, \theta_2)$ and

$L = \{p \in WK_\rho\text{-}2D(\Gamma) \,/\, B_{2,2}(p_1) \subset \theta_1 \text{ and } B_{2,2}(p_2) \subset \theta_2\}$ where $B_{2,2}$ - represent tiles of size 2×2.

We write it as $L = L(\theta)$.

The family of Watson-Crick two-dimensional local languages is denoted as WK_{ρ}-2D(LOC).

Example 5.1. Let $\Gamma = \{0, 1\}$ be an alphabet and $\theta = (\theta_1, \theta_2)$ be the following set of tiles over Γ.

$$\theta_1 = \left\{ \begin{array}{|c|c|}\hline \# & \# \\ \hline \# & 1 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 1 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 0 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 0 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 1 \\ \hline \# & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}, \right.$$

$$\left. \begin{array}{|c|c|}\hline 0 & \# \\ \hline 0 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 0 \\ \hline \# & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 1 & \# \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 0 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & \# \\ \hline 1 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 1 \\ \hline \# & \# \\ \hline \end{array} \right\}$$

$$\begin{array}{cccccccc} \# & \# & \# & \# & \# & \# & \# & \# \\ \# & 1 & 0 & 0 & 0 & 0 & 0 & \# \\ \# & 0 & 1 & 0 & 0 & 0 & 0 & \# \\ \# & 0 & 0 & 1 & 0 & 0 & 0 & \# \\ \# & 0 & 0 & 0 & 1 & 0 & 0 & \# \\ \# & 0 & 0 & 0 & 0 & 1 & 0 & \# \\ \# & 0 & 0 & 0 & 0 & 0 & 1 & \# \\ \# & \# & \# & \# & \# & \# & \# & \# \end{array}$$

$$\theta_2 = \left\{ \begin{array}{|c|c|}\hline \# & \# \\ \hline \# & 1 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 1 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 0 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline 0 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 1 \\ \hline \# & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array}, \right.$$

$$\left. \begin{array}{|c|c|}\hline 0 & \# \\ \hline 0 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 0 \\ \hline \# & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & 0 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 0 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & \# \\ \hline 1 & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 0 & 1 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline 1 & \# \\ \hline \# & \# \\ \hline \end{array} \right\}$$

$$\begin{array}{cccccccc} \# & \# & \# & \# & \# & \# & \# & \# \\ \# & 1 & 0 & 0 & 0 & 0 & 0 & \# \\ \# & 0 & 1 & 0 & 0 & 0 & 0 & \# \\ \# & 0 & 0 & 1 & 0 & 0 & 0 & \# \\ \# & 0 & 0 & 0 & 1 & 0 & 0 & \# \\ \# & 0 & 0 & 0 & 0 & 1 & 0 & \# \\ \# & 0 & 0 & 0 & 0 & 0 & 1 & \# \\ \# & \# & \# & \# & \# & \# & \# & \# \end{array}$$

The language $L = L(\theta)$, $\theta = (\theta_1, \theta_2)$ is the language of squares of pictures in which all main diagonal position are 1 the remaining positions are 0.

6 Tiling Recognizable WKρ-2D(LOC) Languages

Definition 6.1. A Watson-Crick two-dimensional tiling system (WK_{ρ}-2D-TS) is a 4-tuple $T = (\Sigma, \Gamma, \theta, \pi)$ where Σ and Γ are two finite alphabets, $\theta = (\theta_1, \theta_2)$ is finite set of tiles over the alphabet $\Gamma \cup \{\#\}$ and $\pi : \Gamma \to \Sigma$ is a projection.

The tiling system T defines (recognizes) a language L over the alphabet Σ as $L = \pi(L')$ where $L' = L(\theta)$, $\theta = (\theta_1, \theta_2)$ is the local language over Γ corresponding to the set of tiles $\theta = (\theta_1, \theta_2)$ written as $L = L(T)$. We say that L is the language recognized by T. The WK_{ρ}-2D local language $L' \subseteq \Gamma^{**}$ is called as WK_{ρ}-2D local language for L.

We say that a language $L \subseteq \Sigma^{**}$ is recognizable by tiling system if there exists a tiling system $T = (\Sigma, \Gamma, \theta, \pi)$ where $\theta = (\theta_1, \theta_2)$ such that $L = L(T)$.

Example 6.1. The language of squares over the alphabets $\Gamma = \{0, 1\}$ with 1's in the main diagonal and 0's in the other positions and if we apply the projection $\pi : \Gamma \to \Sigma$ such that $\pi(0) = \pi(1) = a$ then we see that $L = \pi(L')$.

We now prove some closure properties.

Theorem 6.1. The family L(WK$_\rho$-2D-TS) is closed under projection.

Proof: Let Σ_1, Σ_2 be two finite alphabets and let $\phi : \Sigma_1 \to \Sigma_2$ be a projection.

To prove: If $L_1 \subseteq \Sigma_1^{**}$ is recognizable by tiling systems, then $L_2 = \phi(L_1)$ is recognizable by tiling systems.

Let $T_1 = (\Sigma_1, \Gamma, \theta, \pi_1)$, $\theta = (\theta_1, \theta_2)$ be a tiling system for L_1 i.e., $L_1 = L(T_1)$. Then $L_1 = \pi_1(L')$ where L' is WK$_\rho$-2D local languages represented by $\theta = (\theta_1, \theta_2)$. We see that L' is the WK$_\rho$-2D local language. Also $L_2 = \pi_2(L')$ where $\pi_2 = \phi \circ \phi_1 : \Gamma \to \Sigma_2$. Hence $T_2 = (\Sigma_2, \Gamma, \theta, \pi_2)$, $\theta = (\theta_1, \theta_2)$ is a tiling system for L_2. ■

Theorem 6.2. The family $\mathcal{L}$(WK$_\rho$-2D-TS) is closed under row and column concatenation operations.

Proof: Let L_1 and L_2 be picture languages over an alphabet Σ and let $L = L_1 ⦶ L_2$ be the language corresponding to the column concatenation of L_1 and L_2. By definition of column concatenation, a picture $p \in L$ is composed by a pair of pictures $p_1 \in L_1$ and $p_2 \in L_2$ with the same number of rows such that the rightmost column of p_1 is glued to the leftmost column of p_2.

Let $(\Sigma, \Gamma_1, \theta_1, \pi_1)$ where $\theta_1 = (\theta_{1_1}, \theta_{1_2})$ and $(\Sigma, \Gamma_2, \theta_2, \pi_2)$ where $\theta_2 = (\theta_{2_1}, \theta_{2_2})$ be two tiling systems for L_1 and L_2. We assume that the local alphabets Γ_1 and Γ_2 are disjoint. We define a WK$_\rho$-2D tiling system for L as $(\Sigma, \Gamma, \theta, \pi)$. Here $\Gamma = \Gamma_1 \cup \Gamma_2$.

θ has to contain all the elements from set θ_1 where $\theta_1 = (\theta_{1_1}, \theta_{1_2})$ except those corresponding to the right borders and all elements from set θ_2 where $\theta_2 = (\theta_{2_1}, \theta_{2_2})$ except those corresponding to the left borders. Some middle tiles corresponding to the two columns must be added where the gluing is done. Such tiles contain pieces of the right border of pictures in L_1 in the left side and pieces of the left border of pictures in L_2 in the right side. i.e., the following three set of tiles are defined.

$$\theta'_{1_i} = \left\{ \begin{array}{|c|c|}\hline a_1 & b_1 \\ \hline c_1 & d_1 \\ \hline \end{array} \;\middle|\; \begin{array}{|c|c|}\hline a_1 & b_1 \\ \hline c_1 & d_1 \\ \hline \end{array} \in \theta_1 \quad \text{and} \quad b_1, d_1, \neq \# \right\}$$

$$\theta'_{2_i} = \left\{ \begin{array}{|c|c|}\hline a_2 & b_2 \\ \hline c_2 & d_2 \\ \hline \end{array} \;\middle|\; \begin{array}{|c|c|}\hline a_2 & b_2 \\ \hline c_2 & d_2 \\ \hline \end{array} \in \theta_2 \quad \text{and} \quad a_2, c_2, \neq \# \right\}$$

$$\theta_{1_i}\theta_{2_i} = \left\{ \begin{array}{|c|c|}\hline a_1 & a_2 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline b_1 & b_2 \\ \hline \end{array}, , \begin{array}{|c|c|}\hline c_1 & c_2 \\ \hline d_1 & d_2 \\ \hline \end{array} \;\middle|\; \begin{array}{l} \begin{array}{|c|c|}\hline a_1 & \# \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline b_1 & \# \\ \hline \end{array}, , \begin{array}{|c|c|}\hline c_1 & \# \\ \hline d_1 & \# \\ \hline \end{array} \in \theta_1 \\ \\ \begin{array}{|c|c|}\hline \# & a_2 \\ \hline \# & \# \\ \hline \end{array}, \begin{array}{|c|c|}\hline \# & \# \\ \hline \# & b_2 \\ \hline \end{array}, , \begin{array}{|c|c|}\hline \# & c_2 \\ \hline \# & d_1 \\ \hline \end{array} \in \theta_2 \end{array} \right\}$$

Then $\theta = \theta'_{1_i} \cup \theta'_{2_i} \cup \theta_{1_i}\theta_{2_i}$, $i = 1,2$.

The projection $\pi : \Gamma \to \Sigma$ is $\forall\, a \in \Gamma$, $\pi(a) = \begin{cases} \pi_1(a) & \text{if } a_1 \in \Gamma_1, \\ \pi_2(a) & \text{if } a_2 \in \Gamma_2. \end{cases}$

Similarly we can obtain a WK_ρ-2D tiling system for the row concatenation. ■

Theorem 6.3. The family $\mathcal{L}(WK_\rho\text{-2D-TS})$ is closed under row and column closure operations.

Proof: Let L be a picture language over an alphabet Σ which is recognizable by tiling systems. By definition of the column closure $L^{*\oplus}$ of L is given by successions of column concatenation operations between pictures in L. Then we can find a WK_ρ-2D tiling system $(\Sigma, \Gamma, \theta, \pi)$ for $L^{*\oplus}$ using the same procedure as in Theorem 6.2.

Consider two different WK_ρ-2D tiling systems for L as $(\Sigma, \Gamma_1, \theta_1, \pi_1)$ where $\theta_1 = (\theta_{1_1}, \theta_{1_2})$ and $(\Sigma, \Gamma_2, \theta_2, \pi_2)$ where $\theta_2 = (\theta_{2_1}, \theta_{2_2})$ where the local alphabets Γ_1 and Γ_2 are disjoint.

Let $\Gamma = \Gamma_1 \cup \Gamma_2$ and construct $\theta_{1_i}\theta_{2_i}$ as in Theorem 6.2. Then $\theta = \theta_1 \cup \theta_{1_i}\theta_{2_i}$. The projection $\pi : \Gamma \to \Sigma$ is same as in Theorem 6.2.

Similarly the row closure $L^{*\ominus}$ can be done. ■

Theorem 6.4. The family $\mathcal{L}(WK_\rho\text{-2D-TS})$ is closed under $\cup$ and $\cap$.

Proof: Let L_1 and L_2 be two picture language over an alphabet Σ and let $(\Sigma, \Gamma_1, \theta_1, \pi_1)$ where $\theta_1 = (\theta_{1_1}, \theta_{1_2})$ and $(\Sigma, \Gamma_2, \theta_2, \pi_2)$ where $\theta_2 = (\theta_{2_1}, \theta_{2_2})$ be two WK_ρ-2D tiling systems to recognize L_1 and L_2 respectively. Assume $\Gamma_1 \cap \Gamma_2 = \phi$. A WK_ρ-2D tiling system $(\Sigma, \Gamma, \theta, \pi)$ for $L_1 \cup L_2 = L$ is defined. Let $\Gamma = \Gamma_1 \cup \Gamma_2$ be the local alphabet and define the projection π so that its restrictions to alphabets Γ_1 and Γ_2 coincide with π_1 and π_2 respectively (same as in Theorem 6.2). Then $\theta = \theta_1 \cup \theta_2$ where $\theta_1 = (\theta_{1_1}, \theta_{1_2})$ and $\theta_2 = (\theta_{2_1}, \theta_{2_2})$.

For $L = L_1 \cap L_2$ we construct a WK_ρ-2D tiling system $(\Sigma, \Gamma, \theta, \pi)$ we find a local language L whose pictures belong to local languages L_1 and L_2. For this, let $\Gamma \subseteq \Gamma_1 \times \Gamma_2$ such that
$(a_1, a_2) \in \Gamma \Leftrightarrow \pi_1(a_1) = \pi_2(a_2)$.

Then θ which represents $\theta_1 \cap \theta_2$ is defined as

$$\theta = \left\{ \begin{array}{|c|c|} \hline (a_1,a_2) & (b_1,b_2) \\ \hline (c_1,c_2) & (d_1,d_2) \\ \hline \end{array} \;\middle|\; \begin{array}{|c|c|} \hline a_1 & b_1 \\ \hline c_1 & d_1 \\ \hline \end{array} \in \theta_1 \quad \text{and} \quad \begin{array}{|c|c|} \hline a_2 & b_2 \\ \hline c_2 & d_2 \\ \hline \end{array} \in \theta_2 \right\}$$

The projection $\pi : \Gamma \to \Sigma$ is well defined as $\pi((a_1, a_2)) = \pi_1(a_1) = \pi_2(a_2)\ \forall\ (a_1, a_2) \in \Gamma_1 \times \Gamma_2$. ■

Theorem 6.5. The family $\mathcal{L}(WK_\rho\text{-2D-TS})$ is closed under rotation.

Proof: Let $L \subseteq \Sigma^{**}$ and let $T = (\Sigma, \Gamma, \theta, \pi)$, $\theta = (\theta_1, \theta_2)$ be WK_ρ-2D tiling system to recognize L. Rotation of L is recognized by WK_ρ-2D tiling system $T^R = (\Sigma, \Gamma, \theta^R, \pi)$ where θ^R is the rotation of set $\theta = (\theta_1, \theta_2)$. ■

Conclusion: We used the concept of local languages on Watson-Crick domain and proved some closure results. We have extended these concepts to arrays and some results have been established.

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