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On anti fuzzy bi-ideals in near-rings

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Abstract

In this paper we introduce the notion of anti fuzzy bi-ideals in near-rings and give some characterizations of anti fuzzy bi-ideals in near-rings.

Keywords: Near-ring, bi-ideal, fuzzy bi-ideal, anti fuzzy bi-ideal.

AMS Subject Classification (2010): 16Y30, 03E72, 08A72.

1 Introduction

The fundamental concept of fuzzy set was introduced by Zadeh [7]. Kuroki [5 ,6] studied fuzzy ideals, fuzzy bi-ideals in semigroup . In [1], R. Biswas introduced the concept of anti fuzzy subgroups of groups and K.H. Kim and Y.B. Jun[3] studied the notion of anti fuzzy R-subgroups of near-rings. In this paper, we introduced the notion of anti fuzzy bi-ideals of near-rings and investigate some properties.

2 Preliminaries

Definition 2.1. Let N be a near-ring. A fuzzy set μ of N is called a fuzzy subnear-ring of N if for all x, $y\in N$,

(*i*) $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$,

(*ii*) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$.

Definition 2.2. Let N be a near-ring. A fuzzy set μ of N is called a fuzzy bi-ideal of N if for all $x, y, z\in N$,

(*i*) $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$,

(*ii*) $\mu(xyz) \geq min\{\mu(x), \mu(z)\}$.

Definition 2.3. Let N be a near-ring. A fuzzy set μ of N is called an anti fuzzy subnear-ring of N if for all $x, y\in N$,

(*i*) $\mu(x-y) \leq max\{\mu(x), \mu(y)\}$, (*ii*) $\mu(xy) \leq max\{\mu(x), \mu(y)\}$.

Definition 2.4. A family of fuzzy set $\{\mu_i \ / \ i \in \wedge\}$ is a near-ring N , the union $\vee_{i\in\wedge} \mu_i$ of $\{\mu_i / i \in \wedge\}$ is defined by $\left(\vee_{i\in\wedge} \mu_i\right)(x) = \sup\{\mu_i(x) / i \in \wedge\}$ for each $x \in N$.

Definition 2.5. A family of fuzzy set $\{\mu_i / i \in \wedge\}$ is a near-ring N , the intersection $\cap_{i\in\wedge} \mu_i$ of $\{\mu_i / i \in \wedge\}$ is defined by $\left(\cap_{i\in\wedge} \mu_i\right)(x) = \inf\{\mu_i(x) / i \in \wedge\}$ for each $x \in N$.

Definition 2.6. Let N and N' be two near-rings and f a function of N into N'.

(i) If λ is a fuzzy set in N' , then the pre image of λ under f is the fuzzy set in N defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for each $x \in N$.

(ii) If μ is a fuzzy set of N, then the image of μ under f is the fuzzy set in N' defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases} \quad \text{for each } y \in N'.$$

Definition 2.7. Let N and N' be two near-rings and 'f' a function of N into N'. If μ is a fuzzy set of N, then the anti image of μ under f is the fuzzy set $f_-(\mu)$ in N' defined by

$$f_-(\mu)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases} \quad \text{for each } y \in N'.$$

Definition 2.8. A fuzzy bi-ideal μ of a near-ring N is said to be normal if $\mu(0) = 1$.

Definition 2.9. An anti fuzzy bi-ideal μ of a near-ring N is said to be complete if it is normal and there exists $z \in N$ such that $\mu(z) = 0$.

3 Anti fuzzy bi-ideals

Definition 3.1. Let N be a near-ring. A fuzzy set μ of N is called an anti fuzzy bi-ideal of N if for all $x, y, z \in N$,

(*i*) $\mu(x-y) \leq max\{\mu(x), \mu(y)\}$,

(*ii*) $\mu(xyz) \leq max\{\mu(x), \mu(z)\}$.

Example 3.2. Let $N = \{0, a, b, c\}$ be the Klein's four group. Define addition and multiplication in N as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Then $(N,+,.)$ is a near-ring. Define a fuzzy set $\mu: N\rightarrow[0,1]$ by $\mu(0)=0.6$, $\mu(a)=0.7$, $\mu(b) = \mu(c) = 0.8$. It is easy to verify that μ is an anti fuzzy bi-ideal of N. But, μ is not a fuzzy bi-ideal of N since $\mu(0)= \mu(b-b) \ngeq \min\{\mu(b), \mu(b)\}$.

Theorem 3.3. Let $f: N\rightarrow N'$ be an onto homomorphism of near-rings.

(i) If λ is a fuzzy bi-ideal in N', then $f^{-1}(\lambda)$ is a fuzzy bi-ideal in N

(ii) If μ is a fuzzy bi-ideal in N, then $f(\mu)$ is a fuzzy bi-ideal in N'.

Proof: (i) Let λ be a fuzzy bi-ideal of N'.

For any $x,y,z\in N$,

$$\begin{aligned} f^{-1}(\lambda)(x-y) &= \lambda(f(x-y)) \\ &= \lambda(f(x)-f(y)) \\ &\geq \min\{\lambda(f(x)), \lambda(f(y))\} \\ &= \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\} \end{aligned}$$

Therefore, $f^{-1}(\lambda)(x-y) \geq \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\}$ and

$$\begin{aligned} f^{-1}(\lambda)(xyz) &= \lambda(f(xyz)) \\ &= \lambda(f(x)f(y)f(z)) \\ &\geq \min\{\lambda(f(x)), \lambda(f(z))\} \\ &= \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z)\} \end{aligned}$$

Thus, $f^{-1}(\lambda)(xyz) \geq \min\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(z)\}$. Hence, $f^{1}(\lambda)$ is a fuzzy bi-ideal in N.

(ii) Let μ be a fuzzy bi-ideal in N.

Let $y_1, y_2, y_3\in N'$. Then we have $\{x/ x\in f^{-1}(y_1-y_2)\}\supseteq\{x_1-x_2 / x_1\in f^{-1}(y_1)$ and $x_2\in f^{-1}(y_2)\}$, and hence

$$\begin{aligned} f(\mu)(y_1-y_2) &= \sup\{\mu(x) / x\in f^{-1}(y_1-y_2)\} \\ &\geq \sup\{\mu(x_1-x_2) / x_1\in f^{-1}(y_1) \text{ and } x_2\in f^{-1}(y_2)\} \\ &\geq \sup\{\min\{\mu(x_1), \mu(x_2)\} / x_1\in f^{-1}(y_1) \text{ and } x_2\in f^{-1}(y_2)\} \\ &= \min\{\sup\{\mu(x_1) / x_1\in f^{-1}(y_1)\} \text{ and } \sup\{\mu(x_2) / x_2\in f^{-1}(y_2)\}\} \\ &= \min\{f(\mu)(y_1), f(\mu)(y_2)\} \end{aligned}$$

Thus, $f(\mu)(y_1-y_2) \geq \min\{f(\mu)(y_1), f(\mu)(y_2)\}$.

Let $y_1, y_2, y_3\in N'$. Then we have,

$$\begin{aligned} f(\mu)(y_1y_2y_3) &= \sup\{\mu(x)/ x\in f^{-1}(y_1y_2y_3)\} \\ &\geq \sup\{\mu(x_1x_2x_3) / x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2), x_3\in f^{-1}(y_3)\} \\ &\geq \sup\{\min\{\mu(x_1), \mu(x_3)\} / x_1\in f^{-1}(y_1) \text{ and } x_3\in f^{-1}(y_3)\} \\ &= \min\{\sup\{\mu(x_1) / x_1\in f^{-1}(y_1)\} \text{ and } \sup\{\mu(x_3) / x_3\in f^{-1}(y_3)\}\} \\ &= \min\{f(\mu)(y_1), f(\mu)(y_3)\} \end{aligned}$$

Thus, $f(\mu)(y_1y_2y_3) \geq \min\{f(\mu)(y_1), f(\mu)(y_3)\}$. Hence, $f(\mu)$ is a fuzzy bi-ideal of N'. ■

Proposition 3.4. Let N be a near-ring and μ be a fuzzy set in N. Then μ is an anti fuzzy bi-ideal in N if and only if μ^c is a fuzzy bi-ideal in N.

Proof: Let N be a near-ring and μ be an anti fuzzy bi-ideal in N.

For $x, y \in N$,

$$\begin{aligned} \mu^c(x-y) &= 1-\mu(x-y) \\ &\geq 1-max\{\mu(x), \mu(y)\} \\ &= min\{1-\mu(x), 1-\mu(y)\} \\ &= min\{\mu^c(x), \mu^c(y)\} \end{aligned}$$

Therefore, $\mu^c(x-y) \geq min\{\mu^c(x), \mu^c(y)\}$.

For any $x,y,z \in N$,

$$\begin{aligned} \mu^c(xyz) &= 1-\mu(xyz) \\ &\geq 1-max\{\mu(x), \mu(z)\} \\ &= min\{1-\mu(x), 1-\mu(z)\} \\ &= min\{\mu^c(x), \mu^c(z)\} \end{aligned}$$

Therefore, $\mu^c(xyz) \geq min\{\mu^c(x), \mu^c(z)\}$.Hence, μ^c is a bi-ideal in N.

Conversely, Suppose that μ^c is a bi-ideal in N.

For any $x, y \in N$,

$$\begin{aligned} \mu(x-y) &= 1-\mu^c(x-y) \\ &\leq 1- min\{\mu^c(x), \mu^c(y)\} \\ &= max\{1-\mu^c(x), 1-\mu^c(y)\} \\ &= max\{\mu(x), \mu(y)\} \end{aligned}$$

Therefore, $\mu(x-y) \leq max\{\mu(x), \mu(y)\}$.

For any $x, y, z \in N$,

$$\begin{aligned} \mu(xyz) &= 1-\mu^c(xyz) \\ &\leq 1- min\{\mu^c(x), \mu^c(z)\} \\ &= max\{1-\mu^c(x), 1-\mu^c(z)\} \\ &= max\{\mu(x), \mu(z)\}. \end{aligned}$$

Therefore, $\mu(xyz) \leq max\{\mu(x), \mu(z)\}$.

Hence μ is an anti fuzzy bi-ideal in N. ■

Proposition 3.5. Let μ be a fuzzy set in a near-ring N. Then μ is an anti fuzzy bi-ideal of N if and only if the lower level cut $L(\mu ; t)$ of N is a bi-ideal of N for each $t \in [\mu(0), 1]$.

Proof: Let μ be an anti fuzzy bi-ideal of N. Let $x, y \in L(\mu ; t)$.Then $\mu(x) \leq t$ and $\mu(y) \leq t$.Now, $\mu(x-y) \leq max\{\mu(x), \mu(y)\} = t$ which implies that $\mu(x-y) \leq t$ and so $x-y \in L(\mu ; t)$. Hence $L(\mu ; t)$ is a subgroup of N. Let $x, z \in L(\mu ; t)$ and $y \in N$ .Then $\mu(x) \leq t$ and $\mu(z) \leq t$. Now, $\mu(xyz) \leq max\{\mu(x), \mu(z)\} \leq t$ which implies that $\mu(xyz) \leq t$ and hence $xyz \in L(\mu;t)$. Hence, $L(\mu ; t)$ is a bi-ideal of N.

Conversely, suppose that $L(\mu ; t)$ is a bi-ideal of N. Suppose that $x, y \in N$ and $\mu(x-y) > max\{\mu(x), \mu(y)\}$. Choose t such that $\mu(x-y) > t > max\{\mu(x), \mu(y)\}$. Then we get $x , y \in L(\mu ; t)$. But $x-y \notin L(\mu ; t)$, a contradiction. Hence $\mu(x-y) \leq max\{\mu(x), \mu(y)\}$. Similarly we can prove that $\mu(xyz) \leq max\{\mu(x), \mu(z)\}$. Hence, μ is an anti fuzzy bi-ideal of N. ∎

Proposition 3.6. If $\{\mu_i / i \in \wedge\}$ is a family of anti fuzzy bi-ideals of a near-ring N, then $\underset{i\in\wedge}{\vee}\mu_i$ is an anti fuzzy bi-ideal.

Proof: Let $\{\mu_i / i \in \wedge\}$ be a family of anti fuzzy bi-ideals of N and $x, y, z \in N$. Then we have,

$$\begin{aligned}\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x-y) &= \sup\{\mu_i(x-y)/i\in\wedge\}\\ &\leq \sup\{\max\{\mu_i(x),\mu_i(y)\}/i\in\wedge\}\\ &= \max\{\sup\{\mu_i(x)/i\in\wedge\},\sup\{\mu_i(y)/i\in\wedge\}\}\\ &= \max\left\{\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x),\left(\underset{i\in\wedge}{\vee}\mu_i\right)(y)\right\}\end{aligned}$$

Therefore, $\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x-y) \leq \max\left\{\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x),\left(\underset{i\in\wedge}{\vee}\mu_i\right)(y)\right\}$.

$$\begin{aligned}\left(\underset{i\in\wedge}{\vee}\mu_i\right)(xyz) &= \sup\{\mu_i(xyz)/i\in\wedge\}\\ &\leq \sup\{\max\{\mu_i(x),\mu_i(z)\}/i\in\wedge\}\\ &= \max\{\sup\{\mu_i(x)/i\in\wedge\},\sup\{\mu_i(z)/i\in\wedge\}\}\\ &= \max\left\{\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x),\left(\underset{i\in\wedge}{\vee}\mu_i\right)(z)\right\}\end{aligned}$$

Therefore, $\left(\underset{i\in\wedge}{\vee}\mu_i\right)(xyz) \leq \max\left\{\left(\underset{i\in\wedge}{\vee}\mu_i\right)(x),\left(\underset{i\in\wedge}{\vee}\mu_i\right)(y)\right\}$.

Hence, $\underset{i\in\wedge}{\vee}\mu_i$ is an anti fuzzy bi-ideal of N. ∎

Proposition 3.7. If $\{\mu_i / i \in \wedge\}$ is a family of anti fuzzy bi-ideals of a near-ring N, then $\underset{i\in\wedge}{\cap}\mu_i$ is an anti fuzzy bi-ideal.

Proof: The proof is similar to Proposition 3.6. ∎

Theorem.3.8. Let $f: N \to N'$ be an onto homomorphism of near-rings. Then we have that

(i) If λ is an anti fuzzy bi-ideal of N', then $f^{-1}(\lambda)$ is an anti fuzzy bi-ideal in N.

(ii) If μ is an anti fuzzy bi-ideal of N, then $f_{-}(\mu)$ is an anti fuzzy bi-ideal of N'.

Proof: Let λ be an anti fuzzy bi-ideal of N'.

Let $x, y, z \in N$,

$$\begin{aligned}f^{-1}(\lambda)(x-y) &= \lambda(f(x-y))\\ &= \lambda(f(x)-f(y))\\ &\leq max\{\lambda(f(x)), \lambda(f(y))\}\\ &= max\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\}\end{aligned}$$

Therefore, $f^{-1}(\lambda)(x-y) \leq max\{f^{-1}(\lambda)(x), f^{-1}(\lambda)(y)\}$ and

$$\begin{aligned}f^{-1}(\lambda)(xyz) &= \lambda(f(xyz))\\ &= \lambda(f(x)f(y)f(z))\end{aligned}$$

$\leq max\{\lambda(f(x)\,,\ \lambda(f(z))$

$= max\{f^{-1}(\lambda)(x)\,,\, f^{-1}(\lambda)(z)\}$

Therefore, $f^{-1}(\lambda)(xyz) \leq max\{f^{-1}(\lambda)(x)\,,\, f^{-1}(\lambda)(z)\}$.Hence, $f^{-1}(\lambda)$ is an anti fuzzy bi-ideal in N.

(ii) Let μ be an anti fuzzy bi-ideal in N

Let $y_1, y_2, y_3 \in N'$. Then we have $\{x \,/\, x \in f^{1}(y_1\text{-}y_2)\} \supseteq \{\, x_1\text{-}x_2 \,/ x_1 \in f^{1}(y_1) \text{ and } x_2 \in f^{1}(y_2)\}$ and hence $f_{-}(\mu)(y_1\text{-}y_2) = inf\{\mu(x) \,/\, x \in f^{1}(y_1\text{-}y_2)\}$

$\leq inf\{\mu(x_1\text{-}x_2) \,/\, x_1 \in f^{1}(y_1) \text{ and } x_2 \in f^{1}(y_2)\}$

$\leq inf\{max\{\, \mu(x_1)\,,\, \mu(x_2)\} \,/\, x_1 \in f^{1}(y_1) \text{ and } x_2 \in f^{1}(y_2)\}$

$= max\{inf\{\mu(x_1) \,/\, x_1 \in f^{1}(y_1)\,\} \text{ and } inf\{\mu(x_2) \,/\, x_2 \in f^{1}(y_2)\}\}$

$= max\{f_{-}(\mu)(y_1)\ ,\ f_{-}(\mu)(y_2)\}$

Therefore, $f_{-}(\mu)(y_1\text{-}y_2) \leq max\{f_{-}(\mu)(y_1)\,,\ f_{-}(\mu)(y_2)\}$.

Let $y_1, y_2, y_3 \in N'$. Then we have

$f_{-}(\mu)(y_1y_2y_3) = inf\,\{\mu(x)/\, x \in f^{1}(y_1y_2y_3)\}$

$\leq inf\{\mu(x_1x_2x_3) \,/\, x_1 \in f^{1}(y_1)\,,\ x_2 \in f^{1}(y_2)\,,\ x_3 \in f^{1}(y_3)\}$

$\leq inf\{max\{\mu(x_1)\ ,\mu(x_3)\}\ \,/\, x_1 \in f^{1}(y_1) \text{ and } x_3 \in f^{1}(y_3)\}$

$= miax\{inf\{\mu(\, x_1) \,/\, x_1 \in f^{1}(y_1)\} \text{ and } inf\{\mu(x_3) \,/\, x_3 \in f^{1}(y_3)\}\}$

$= max\{f_{-}(\mu)(y_1)\,,\ f_{-}(\mu)(y_3)\}$

Therefore, $f_{-}(\mu)(y_1y_2y_3) \leq max\{f_{-}(\mu)(y_1)\,,\ f_{-}(\mu)(y_3)\}$.

Hence, $f_{-}(\mu)$ is an anti fuzzy bi-ideal of N′. ■

Theorem.3.9. Let μ be an anti fuzzy bi-ideal of a near-ring N and μ^* be a fuzzy set in N defined by $\mu^*(x) = \mu(x) + 1 - \mu(0)\ \forall\ x \in N$. Then μ^* is a normal anti fuzzy bi-ideal of N containing μ.

Proof: Let μ be an anti fuzzy bi-ideal of a near-ring N.

For any $x, y \in N$,

$\mu^*(x\text{-}y) = \mu(x\text{-}y) + 1 - \mu(0)$

$\leq max\{\mu(x)\,,\, \mu(y)\} + 1 - \mu(0)$

$= max\{\mu(x)+1-\mu(0)\,,\, \mu(y)+1-\mu(0)\}$

$= max\{\mu^*(x)\,,\, \mu^*(y)\}$

Therefore, $\mu^*(x\text{-}y) \leq max\{\mu^*(x)\,,\, \mu^*(y)\}$.

For any $x, y, z \in N$,

$\mu^*(xyz) = \mu(xyz) + 1 - \mu(0)$

$\leq max\{\mu(x)\,,\, \mu(z)\} + 1 - \mu(0)$

$= max\{\mu(x)+1-\mu(0)\,,\, \mu(z)+1-\mu(0)\}$

$= max\{\mu^*(x)\,,\, \mu^*(z)\}$

Therefore, $\mu^*(xyz) \leq max\{\mu^*(x)\,,\, \mu^*(z)\}$. Clearly $\mu^*(0) = \mu(0) + 1 - \mu(0) = 1$ and hence μ^* is normal. Hence, μ^* is a normal anti fuzzy bi-ideal of N, and obviously $\mu \subseteq \mu^*$. ■

Theorem.3.10. If μ is an anti fuzzy bi-ideal of a near-ring N, then $(\mu^*)^* = \mu^*$.

Proof: For any $x \in N$, we have

$$\begin{aligned}(\mu^*)^*(x) &= \mu^*(x) + 1 - \mu^*(0)\\ &= [\mu(x) + 1 - \mu(0)] + 1 - [\mu(0) + 1 - \mu(0)]\\ &= \mu(x) + 1 - \mu(0) + 1 - \mu(0) - 1 + \mu(0)\\ &= \mu(x) + 1 - \mu(0)\\ &= \mu^*(x)\end{aligned}$$

Therefore, $(\mu^*)^* = \mu^*$. ■

Theorem 3.11. If μ is normal anti fuzzy bi-ideal of a near-ring N if and only if $\mu^* = \mu$.

Proof: The sufficient part is obvious. To prove the necessary part, let us suppose that μ is normal anti fuzzy bi-ideal of a near-ring N. Let $x \in N$. Since μ is normal, $\mu^*(x) = \mu(x) + 1 - \mu(0) = \mu(x) + 1 - 1 = \mu(x)$.Hence $\mu^* = \mu$. ■

Theorem 3.12. Let μ be an anti fuzzy bi-ideal of a near-ring N, and t be fixed element of N such that $\mu(0) \neq \mu(t)$. Define a fuzzy set μ^* in N by $\mu^*(x) = \frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}$ for all $x \in N$. Then μ^* is a normal anti fuzzy bi-ideal of the near-ring N.

Proof: Let μ be an anti fuzzy bi-ideal of a near-ring N.
For any $x, y \in N$,

$$\begin{aligned}\mu^*(x-y) &= \frac{\mu(x-y) - \mu(t)}{\mu(0) - \mu(t)}\\ &\leq \frac{max\{\mu(x), \mu(y)\} - \mu(t)}{\mu(0) - \mu(t)}\\ &= max\left\{\frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}, \frac{\mu(y) - \mu(t)}{\mu(0) - \mu(t)}\right\}\\ &= max\{\mu^*(x), \mu^*(y)\}.\end{aligned}$$

Therefore, $\mu^*(x-y) \leq max\{\mu^*(x), \mu^*(y)\}$.

For any $x, y, z \in N$,

$$\begin{aligned}\mu^*(xyz) &= \frac{\mu(xyz) - \mu(t)}{\mu(0) - \mu(t)}\\ &\leq \frac{max\{\mu(x), \mu(z)\} - \mu(t)}{\mu(0) - \mu(t)}\\ &= max\left\{\frac{\mu(x) - \mu(t)}{\mu(0) - \mu(t)}, \frac{\mu(z) - \mu(t)}{\mu(0) - \mu(t)}\right\}\\ &= max\{\mu^*(x), \mu^*(z)\}.\end{aligned}$$

Therefore, $\mu^*(xyz) \leq max\{\mu^*(x), \mu^*(z)\}$.Hence μ^* is an anti fuzzy bi-ideal of N.

Also $\mu^*(0) = \frac{\mu(0) - \mu(t)}{\mu(0) - \mu(t)} = 1$, μ^* is normal.

Since $t \in N$ and $\mu^{*}(t) = \frac{\mu(t)-\mu(t)}{\mu(0)-\mu(t)} = 0$ we have μ^{*} is a complete anti fuzzy bi-ideals on N. ■

Theorem.3.13. Let μ be an anti fuzzy bi-ideal of a near-ring N and let $f: [0, \mu(0)] \to [0,1]$ be an increasing function. Then the fuzzy set $\mu_f : N \to [0,1]$ defined by $\mu_f(x) = f(\mu(x))$ is an anti fuzzy bi-ideal of N. In particular, if $f[\mu(0)] = 1$ then μ_f is normal and if $f(t) \geq t$ for all $t \in [0, \mu(0)]$, then $\mu \subseteq \mu_f$.

Proof: For any $x, y \in N$,

$$\begin{aligned} \mu_f(x-y) &= f(\mu(x-y)) \\ &\leq f(max\{\mu(x), \mu(y)\}) \\ &= max\{f(\mu(x), f(\mu(y))\} \\ &= max\{\mu_f(x), \mu_f(y)\} \end{aligned}$$

Therefore, $\mu_f(x-y) \leq max\{\mu_f(x), \mu_f(y)\}$.

For any $x, y, z \in N$,

$$\begin{aligned} \mu_f(xyz) &= f(\mu(xyz)) \\ &\leq f(max\{\mu(x), \mu(z)\}) \\ &= max\{f(\mu(x)), f(\mu(z))\} \\ &= max\{\mu_f(x), \mu_f(z)\} \end{aligned}$$

Therefore, $\mu_f(xyz) \leq max\{\mu_f(x), \mu_f(z)\}$. Hence μ_f is an anti fuzzy bi-ideal of N. If $f[\mu(0)] = 1$, then $\mu_f(0) = 1$.Thus μ_f is normal. Assume that $f(t) = f[\mu(x)] \geq \mu(x)$, for any $x \in N$ which implies $\mu \subseteq \mu_f$. ■

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