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Some new results on strongly prime graphs

S. Meena[1], P. Kavitha[2]

[1] Department of Mathematics, Government Arts College
C-Mutlur, Chidambaram– 608 102, Tamil Nadu, India.
meenasaravanan14@gmail.com

[2] Department of Mathematics, S.R.M University
Chennai– 603 203, Tamil Nadu, India.
kavithavps@gmail.com

Abstract

A graph $G=(V,E)$ with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph and a graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$. In this paper we prove that the graphs $P_n \cup C_{bk}, C_{bk} \cup C_n^*, C_k \cup P_n, G\odot K_1$ where G is a $C_n(C_n)$ graph and splitting graph of $K_{1,n}$ are strongly prime graphs.

Keywords: Prime labeling, prime graph, strongly prime graph.

AMS Subject Classification (2010): 05C78.

1 Introduction

In this paper, we consider only simple, finite, undirected and non-trivial graph $G=(V(G),E(G))$ with vertex set V(G) and edge set E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1]. Two integers a and b are said to be relatively prime if their greatest common divisor is 1. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [9]. H. Fu [4] have proved that path P_n on n vertices is a prime graph. Deresky. T [3] proved that the cycle C_n with n vertices is a prime graph. Lee.S [6] have proved that wheel W_n is a prime graph iff n is even. Around 1980, Roger Etringer conjectured that all trees having prime labeling which is not settled till today. S. K. Vaidya and K.K. Kanani [10] investigated the Prime labeling for some cycle related graph. In [7] and [8] S. Meena and K. Vaithiligam have investigated the prime labeling for some helm related graphs and prime labeling for some crown related graphs.

S.K.Vaidya and Udayan M.Prajapati[11] introduced strongly prime graph and proved C_n, P_n and $K_{1,n}$ are strongly prime graphs and W_n is a strongly prime graph for every even integer $n \geq 4$, in Some new results on prime graph. C. David Raj and C. Jayasekaran [2] have proved some results on super harmonic mean graphs. For latest Dynamic survey on graph labeling we refer to Gallian [4]. We use the following definitions.

Definition 1.1. If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling.

Definition 1.2. Let $G=(V(G),E(G))$ be a graph with p vertices. A bijection $f:\mathrm{V}(G)\to\{1,2,\ldots\ldots p\}$ is called a prime labeling if for each edge $e=uv, \gcd\{f(u),f(v)\}=1$. A graph which admits prime labeling is called a prime graph.

Definition 1.3. A graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$.

Definition 1.4. The union of two graphs G_1 and G_2 is a graph $G_1\cup G_2$ with $V(G_1\cup G_2)=V(G_1)\cup V(G_2)$ and $E(G_1\cup G_2)=E(G_1)\cup E(G_2)$.

Definition 1.5. The corona of a graph G on p vertices $v_1,v_2,\ldots v_p$ is the graph obtained from G by adding p new vertices $u_1,u_2,\ldots u_p$ and new edges u_iv_i for $1\le i\le p$ and is denoted by $G\odot K_1$.

Definition 1.6. The graph $P_n\odot K_1$ is called a comb C_{bn}.

Definition 1.7. The crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at each vertex of the n-cycle.

Definition 1.8. If every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of graph G .

Definition 1.9. The barycentric subdivision of a cycle C_n in which each pair of newly inserted vertices of adjacent edges are joined by an edge is denoted by $C_n(C_n)$, as it looks like C_n inscribed in C_n.

Definition 1.10. For a graph G, the splitting graph is obtained by adding to each vertex v, a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G.

Bertrand's Postulate 1.11. For every positive integer $n>1$ there is a prime p such that $n<p<2n.$

In this paper we investigate strongly prime labeling for some graphs related to cycle, path, comb, crown and star graph.

2 Main Results

Theorem 2.1. The graph $G=P_n\cup C_{bk}$ is a strongly prime graph for all integers $n,k\ge 2$.

Proof: Let G be the graph $P_n\cup C_{bk}$ with vertex set $V(G)=\{w_1,w_2,\ldots w_n,v_1,v_2,\ldots v_k u_1,u_2,\ldots u_k\}$ and edge set $E(G)=\{v_iu_i\,/\,1\le i\le k\}\cup\{v_iv_{i+1}\,/\,1\le i\le k-1\}\cup\{w_iw_{i+1}\,/\,1\le i\le n-1\}$. Here $|V(G)|=2k+n$.

Let a be the vertex for which we assign label 1. We have the following two cases:

Case (i): When a is any vertex of C_{bk}.

Subcase (i): If $a = v_j$ for some $j \in \{1,2,3,...k\}$ then the function $f : V(G) \to \{1,2,3,...2k+n\}$ defined by

$$f(v_i) = \begin{cases} 2k+2i-2j+1 & if\ i = 1,2,...j-1; \\ 2i-2j+1 & if\ i = j, j+1, j+2,...k, \end{cases}$$

$$f(u_i) = \begin{cases} 2k+2i-2j+2 & if\ i = 1,2,...j-1; \\ 2i-2j+2 & if\ i = j, j+1, j+2,...k, \end{cases}$$

$$f(w_i) = 2k+i \qquad if\ i = 1,2,...n,$$

is a prime labeling for G with $f(a) = f(v_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of v_j in C_{bk}.

Subcase (ii): If $a = u_j$ for some $j \in \{1,2,3,...k\}$, then define a labeling f_2 using the labeling f defined in subcase(i) as follows: $f_2(v_j) = f(u_j)$, $f_2(u_j) = f(v_j)$ for $j = 1,2,3,...k$ and $f_2(v) = f(v)$ for all the remaining vertices. Then f_2 is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of u_j in C_{bk}.

Case (ii): When a is any arbitrary vertex of P_n.

Subcase (i): When n is even.

If $a = w_j$ for some $j \in \{1,2,3,...n\}$ then the function $f : V(G) \to \{1,2,3,...2k+n\}$ defined by

$$f(w_i) = \begin{cases} n+i-j+1 & if\ i = 1,2,...j-1; \\ i-j+1 & if\ i = j, j+1, j+2,...n, \end{cases}$$

$$f(v_i) = n+2i-1 \qquad if\ i = 1,2,...k,$$

$$f(u_i) = n+2i \qquad if\ i = 1,2,...k,$$

is a prime labeling for G with $f(a) = f(w_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of w_j in P_n.

Subcase (ii): When n is odd.

If $a = w_j$ for some $j \in \{1,2,3,...n\}$, then define a labeling f_3 using the labeling f defined in subcase(i) of case (ii) as follows: $f_3(u_i) = n+2i-1$, $f_3(v_i) = n+2i$ for $i = 1,2,...k$, and $f_3(v) = f(v)$ for all the remaining vertices. Thus f_3 is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of P_n and C_{bk}.

Thus $G = P_n \cup C_{bk}$ is a strongly prime graph for all integers $n,k \geq 2$. ■

Illustration 2.2. Prime labelings of $P_6 \cup C_{b7}$ and $P_6 \cup C_{b5}$ are given in Figure 1and 2.

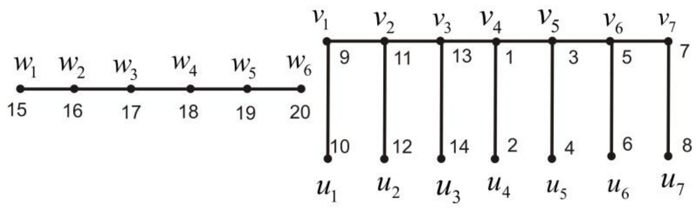

Figure 1: A prime labeling of $P_6 \cup C_{b7}$ having v_4 as label 1.

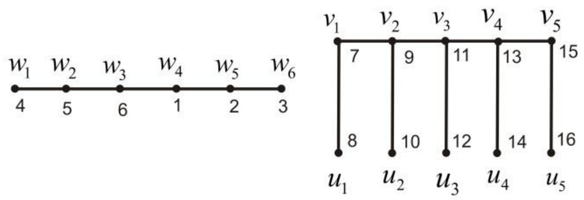

Figure 2: A prime labeling of $P_6 \cup C_{b5}$ having w_4 as label 1.

Theorem 2.3. The graph $G = C_{bk} \cup C_n^*$ is a strongly prime graph for all integers $n \geq 3$ and $k \geq 2$.

Proof: Let G be the graph $C_{bk} \cup C_n^*$ with vertex set $V(G) = \{v_1, v_2, ... v_k, v_1', v_2', ... v_k', u_1, u_2, ... u_n, u_1', u_2', ... u_n'\}$. and edge set $E(G) = \{v_i v_i' / 1 \leq i \leq k\} \cup \{v_i v_{i+1} / 1 \leq i \leq k-1\} \cup \{u_i u_i' / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_1 u_n\}$.

Here $|V(G)| = 2k + 2n$. Let a be the vertex for which we assign label 1 in our labeling method. Then we have the following two cases:

Case (i): When a is any arbitrary vertex of C_{bk}.

Subcase (i): If $a = v_j$ for some $j \in \{1, 2, 3, ... k\}$ then the function $f : V(G) \to \{1, 2, 3, ... 2k + 2n\}$ defined by

$$f(v_i) = \begin{cases} 2k + 2i - 2j + 1 & \text{if } i = 1, 2, ... j-1; \\ 2i - 2j + 1 & \text{if } i = j, j+1, j+2, ... k, \end{cases}$$

$$f(v_i') = \begin{cases} 2k + 2i - 2j + 2 & \text{if } i = 1, 2, ... j-1; \\ 2i - 2j + 2 & \text{if } i = j, j+1, j+2, ... k, \end{cases}$$

$$f(u_i) = 2k + 2i - 1 \qquad \text{if } i = 1, 2, ... n,$$

$$f(u_i') = 2k + 2i \qquad \text{if } i = 1, 2, ... n,$$

If $f(u_1)$ is a multiple of $f(u_n)$ then interchange $f(u_n)$ and $f(u_n')$.

Now this mapping is a prime labeling for G with $f(a) = f(v_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of v_j in C_{bk}.

Subcase (ii): If $a = v_j'$ for some $j \in \{1, 2, 3, ... k\}$, then define a labeling f_2 using the labeling f defined in subcase (i) as follows: $f_2(v_j) = f(v_j')$, $f_2(v_j') = f(v_j)$ for $j = 1, 2, 3, ... k$ and $f_2(v) = f(v)$ for all the remaining vertices. Then the resulting labeling f_2 is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of $v_j^{'}$ in C_{bk} ..

Case (ii): When a is any arbitrary vertex of C_n^*

Subcase (i): If $a = u_j$ for some $j \in \{1,2,3,...n\}$ then the function $f : V(G) \to \{1,2,3,...2k+2n\}$ defined by

$$f(u_i) = \begin{cases} 2(n+i-j)+1 & if\ i = 1,2,...j-1; \\ 2(i-j)+1 & if\ i = j, j+1, j+2,...n, \end{cases}$$

$$f(u_i^{'}) = \begin{cases} 2(n+i-j)+2 & if\ i = 1,2,...j-1; \\ 2(i-j)+2 & if\ i = j, j+1, j+2,...n, \end{cases}$$

$$f(v_i) = 2n+2i-1 \qquad if\ i = 1,2,...k,$$

$$f(v_i^{'}) = 2n+2i \qquad if\ i = 1,2,...k,$$

is a prime labeling for G with $f(a) = f(u_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of u_j in C_n^*.

Subcase (ii): If $a = u_j^{'}$ for some $j \in \{1,2,3,...n\}$, then define a labeling f_3 using the labeling f defined in subcase (i) of case (ii) as follows: $f_3(u_j) = f(u_j^{'})$, $f_3(u_j^{'}) = f(u_j)$ for $j = 1,2,3,...k$ and $f_3(v) = f(v)$ for all the remaining vertices. Then the resulting graph G is a prime graph and also it is possible to assign label 1 to any arbitrary vertex of $u_j^{'}$ .Hence, G is a strongly prime graph. ■

Illustration 2.4. A prime labeling of $C_{b6} \cup C_6^*$ is given in Figure 3.

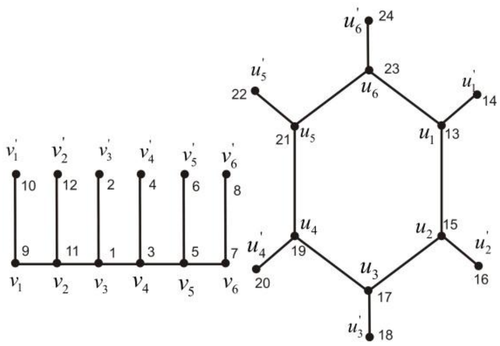

Figure 3: A prime labeling of $C_{b6} \cup C_6^*$ having v_3 as label 1.

Theorem 2.5. The graph $G = C_k \cup P_n$ is a strongly prime graph $n \geq 2$ and even $k \geq 4$.

Proof: Let G be the graph $C_k \cup P_n$ with vertex set $V(G) = \{v_1, v_2,...v_k, u_1, u_2,...u_n\}$ and edge set $E(G) = \{v_i v_{i+1} / 1 \leq i \leq k-1\} \cup \{v_1 v_k\} \cup \{u_i u_{i+1} / 1 \leq i \leq n\}$. Here, $|V(G)| = k+n$.

Let a be the vertex for which we assign label 1 in our labeling method. Then we have the following two cases:

Case (i): When a is any vertex of C_k.

If $a = v_j$ for some $j \in \{1,2,3,...k\}$ then the function $f : V(G) \to \{1,2,3,...k+n\}$ defined by

$$f(v_i) = \begin{cases} k+i-j+1 & \text{if } i = 1,2,...j-1; \\ i-j+1 & \text{if } i = j, j+1, j+2,...k, \end{cases}$$

$$f(u_i) = k+i \qquad \text{if } i = 1,2,...n,$$

is a prime labeling for G with $f(a) = f(v_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of v_j in C_k.

Case (ii): When a is any arbitrary vertex of P_n.

If $a = u_j$ for some $j \in \{1,2,3,...n\}$ then the function $f : V(G) \to \{1,2,3,...k+n\}$ defined by

$$f(u_j) = 1,$$

$$f(u_i) = \begin{cases} k+n+i-j+1 & \text{if } i = 1,2,...j-1; \\ k+i-j+1 & \text{if } i = j+1, j+2,...n, \end{cases}$$

$$f(v_i) = i+1 \qquad \text{if } i = 1,2,...k,$$

is a prime labeling for G with $f(a) = f(u_j) = 1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of u_j in P_n, when k is even. Thus from all the cases described above G is a strongly prime graph if k is even. ■

Illustration 2.6. A prime labeling of $C_8 \cup P_7$ is given in Figure 4.

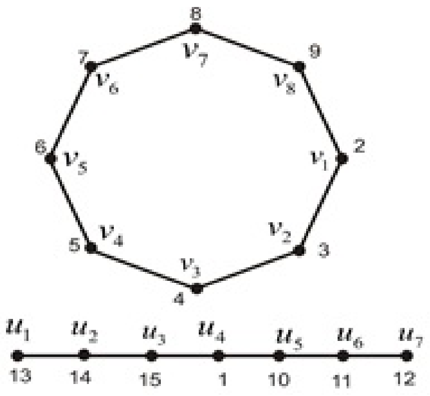

Figure 4: A prime labeling of $C_8 \cup P_7$ having u_4 as label 1.

Theorem 2.7. The graph $G \odot K_1$ is a strongly prime graph where $G = C_n(C_n)$ for all integers $n \geq 3$.

Proof: Let G be the graph with vertices $\{u_1, u_2, ...u_n, v_1, v_2, ...v_n\}$. Let $\{u_1', u_2', ...u_n'\}$ and $\{v_1', v_2', ...v_n'\}$ be the corresponding new vertices, join $u_i u_i'$ and $v_i v_i'$ in G. We get the graph $G_1 = G \odot K_1$ where $G = C_n(C_n)$.

Now the vertex set and edge set of G_1 are given by $V(G_1)=\{u_1,u_2,...u_n,v_1,v_2,...v_n,u_1^{'},u_2^{'},...u_n^{'},v_1^{'},v_2^{'},...v_n^{'}\}$ and $E(G_1)=\{v_iv_{i+1}/1\le i\le n-1\}\cup\{u_iv_i,\mathrm{u}_i\,u_i^{'},v_iv_i^{'}/1\le i\le n\}\cup\{v_iu_{i+1}/1\le i\le n-1\}$. Then, $|V(G_1)|=4n$.

Let a be the vertex for which we assign label 1. Then we have the following four cases:

Case (i): When a is of degree 3.

Let $a=u_j$ for some $j\in\{1,2,3,...n\}$ then the function $f:V(G_1)\to\{1,2,3,...4\mathrm{n}\}$ defined by

$$f(u_i)=\begin{cases}4n+4i-4j+1 & \text{if } i=1,2,...j-1;\\ 4i-4j+1 & \text{if } i=j,j+1,j+2,...\mathrm{n},\end{cases}$$

$$f(\mathrm{v}_i)=\begin{cases}4n+4i-4j+3 & \text{if } i=1,2,...j-2;\\ 4i-4j+3 & \text{if } i=j,j+1,j+2,...\mathrm{n},\end{cases}$$

$$f(\mathrm{u}_i^{'})=\begin{cases}4n+4i-4j+2 & \text{if } i=1,2,3,...j-1;\\ 4i-4j+2 & \text{if } i=j,j+1,j+2,...n,\end{cases}$$

$$f(\mathrm{v}_i^{'})=\begin{cases}4n+4i-4j+4 & \text{if } i=1,2,3,...j-2;\\ 4i-4j+4 & \text{if } i=j,j+1,j+2,...n,\end{cases}$$

$$f(v_{j-1})=\begin{cases}4n, & \text{if } 4n-1 \text{ is a multipleof } 3\\ 4\mathrm{n}-1, & \text{if } 4n-1 \text{is notamultipleof } 3\end{cases}$$

$$f(v_{j-1}^{'})=\begin{cases}4n-1, & \text{if } 4n-1 \text{ is a multipleof } 3\\ 4\mathrm{n}, & \text{if } 4n-1 \text{is notamultipleof } 3\end{cases}$$

is a prime labeling for G_1 with $f(a)=\mathrm{f}(\mathrm{u}_j)=1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of degree 3 in G_1.

Case (ii): Let $a=u_j^{'}$ for some $j\in\{1,2,3,...n\}$, then define a labeling f_2 using the labeling f defined in Case (i) as follows: $f_2(u_j)=f(u_j^{'})$, $f_2(u_j^{'})=f(u_j)$ for $j=1,2,3,...k$ and $f_2(v)=f(v)$ for all the remaining vertices. Then the resulting labeling f_2 is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex of $a=u_j^{'}$ in G_1.

Case (iii): Let $a=v_j$ for some $j\in\{1,2,3,...n\}$ then the function $f:V(G_1)\to\{1,2,3,...4\mathrm{n}\}$ defined by

$$f(u_i)=\begin{cases}4n+4i-4j-1 & \text{if } i=1,2,...j;\\ 4i-4j-1 & \text{if } i=j+1,j+2,...\mathrm{n},\end{cases}$$

$$f(\mathrm{u}_i^{'})=\begin{cases}4n+4i-4j & \text{if } i=1,2,3,...j;\\ 4i-4j & \text{if } i=j+1,j+2,...n,\end{cases}$$

$$f(\mathrm{v}_i)=\begin{cases}4n+4i-4j+1 & \text{if } i=1,2,...j-1;\\ 4i-4j+1 & \text{if } i=j,j+1,j+2,...\mathrm{n},\end{cases}$$

$$f(\mathrm{v}_i^{'})=\begin{cases}4n+4i-4j+2 & \text{if } i=1,2,3,...j-1;\\ 4i-4j+2 & \text{if } i=j,j+1,j+2,...n,\end{cases}$$

is a prime labeling for G_1 with $f(a)=\mathrm{f}(\mathrm{v}_j)=1$. Thus f is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex $a=v_j$ for $j\in\{1,2,3,...n\}$ in G_1.

Case (iv): Let $a=v_j^{'}$ for some $j\in\{1,2,3,...n\}$, then define a labeling f_3 using the labeling f defined in case (iii) as follows: $f_3(v_j)=f(v_j^{'})$, $f_3(v_j^{'})=f(v_j)$, for $j=1,2,3,...k$ and $f_3(\mathrm{v})=f(v)$ for all remaining vertices. Then f_3 is a prime labeling and also it is possible to assign label 1 to any arbitrary vertex $a=v_j^{'}$ in G_1. Hence, G_1 is a strongly prime graph. ■

Illustration 2.8. A prime labeling of $G\odot K_1$ where $G=C_7(C_7)$ is given in Figure 5.

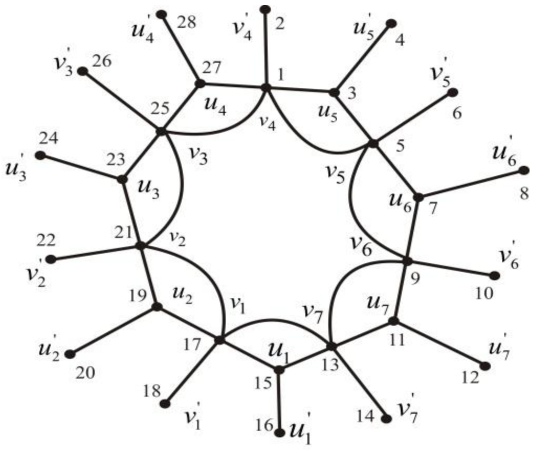

Figure 5: A prime labeling of $G\odot K_1$ where $G=C_7(C_7)$ having v_4 as label 1.

Theorem 2.9. The splitting graph of a star is a strongly prime graph.

Proof: Let $v_0,v_1,v_2,...v_n$ be the vertices of star graph $K_{1,n}$ with v_0 be the apex vertex. Let G be the splitting graph of $K_{1,n}$ and $v_0^{'},v_1^{'},v_2^{'},...v_n^{'}$ be the newly added vertices to $K_{1,n}$ to form G. Let x be the vertex for which we assign label 1. Then we have the following four cases:

Case (i): If x is the first apex vertex $x=v_0$, then the function $f:V(G)\to\{1,2,3,...2(n+1)\}$ defined by

$f(v_i)=i+1$ for $i=0,1,2,...n$,

$f(v_0^{'})=p$,

where p is the largest prime less than or equal to $2(n+1)$. According to Bertrand's postulate such a prime exists with $n+1<p<2n+2$.

$f(v_i^{'})=i$ for $i=\{1,2,\ldots n\}-\{p\}$,

Then clearly f is an injection. For an arbitrary edge $e=ab$ of G we claim that $\gcd\left(f(a),f(b)\right)=1.$

Subcase (i): If $e=v_0v_i$ for some $i\in\{1,2,...n\}$ then $\gcd\left(f(v_0),f(v_i)\right)=\gcd\left(1,f(v_i)\right)=1.$

Subcase (ii): If $e=v_0v_i^{'}$ for some $i\in\{1,2,...n\}$ then $\gcd\left(f(v_0),f(v_i^{'})\right)=\gcd\left(1,f(v_i^{'})\right)=1.$

Subcase (iii): If $e = v_0^{'} v_i$ for some $i \in \{1, 2, ... n\}$ then $\gcd\left(f(v_0^{'}), f(v_i)\right) = \gcd\left(p, f(v_i)\right) = 1$ as p is Co-prime to every integer from $\{1, 2, ... n+1\}$. Hence this function f is a prime labeling on G with $f(x) = f(v_0) = 1$.

Case (ii): If x is the second apex vertex $x = v_0^{'}$, then define a labeling f_2 using the labeling f defined in Case (i) as follows: $f_2(v_0) = f(v_0^{'})$, $f_2(v_0^{'}) = f(v_0)$ for $j = 1, 2, 3, ... k$ and $f_2(v) = f(v)$ for all the remaining vertices. Then the resulting labeling f_2 is a prime labeling.

Case (iii): When v is of degree 2 ($x = v_j$).

If $x = v_j$ for some $j \in \{1, 2, 3, ... n\}$, then define the function $f : V(G) \to \{1, 2, 3, ... 2(n+1)\}$ as $f(v_0^{'}) = p_1$, where p_1 is the largest prime less than or equal to $n+1$, $f(v_0) = p_2$, where p_2 is the largest prime less than or equal to $2(n+1)$. According to Bertrand's postulate,

$$f(v_i) = \begin{cases} i & \text{for } i = 1, 2, 3, ... p_1 - 1; \\ i+1 & \text{for } i = p_1, p_1 + 1, ... n, \end{cases}$$

$$f(v_i^{'}) = \begin{cases} n+i+1 & \text{for } i = 1, 2, 3, ... \left(\frac{p_2 - 1}{2} - 1\right); \\ n+i+2 & \text{for } i = \frac{p_2 - 1}{2}, \frac{p_2 - 1}{2} + 1, ... n, \end{cases}$$

Then clearly f is an injection. For an arbitrary edge $e = ab$ of G we claim that $\gcd\left(f(a), f(b)\right) = 1$. To prove our claim the following cases are to be considered.

Subcase (i): If $e = v_0 v_i$ for some $i \in \{1, 2, ... n\}$ then $\gcd\left(f(v_0), f(v_i)\right) = \gcd\left(p_2, f(v_i)\right) = 1$ as p_2 is Co-prime to every integer from $\{1, 2, ... n+1\} - \{p_1\}$.

Subcase (ii): If $e = v_0 v_i^{'}$ for some $i \in \{1, 2, ... n\}$ then $\gcd\left(f(v_0), f(v_i^{'})\right) = \gcd\left(p_2, f(v_i^{'})\right) = 1$ as p_2 is Co-prime to every integer from $\{n+2, n+3 ... 2n+2\} - \{p_2\}$.

Subcase (iii): If $e = v_0^{'} v_i$ for some $i \in \{1, 2, ... n\}$ then $\gcd\left(f(v_0^{'}), f(v_i)\right) = \gcd\left(p_1, f(v_i)\right) = 1$ as p_1 is Co-prime to every integer from $\{1, 2, ... n+1\} - \{p_1\}$. Then f is a prime labeling for G with $f(x) = f(v_j) = 1$.

Case (iv): When v is of degree 2 ($x = v_j^{'}$).

If $x = v_j^{'}$ for some $j \in \{1, 2, 3, ... n\}$, then define a labeling f_3 using the labeling f defined in case (iii) as follows: $f_3(v_1) = f(v_1^{'})$, $f_3(v_1^{'}) = f(v_1)$ and for $j = 1, 2, 3, ... k$ and $f_3(v) = f(v)$ for all remaining vertices. Then f_3 is a prime labeling for G. Hence, G is a strongly prime graph. ■

Illustration 2.10. Prime labelings of the splitting graph of $K_{1,5}$ is given in Figure 6 and 7.

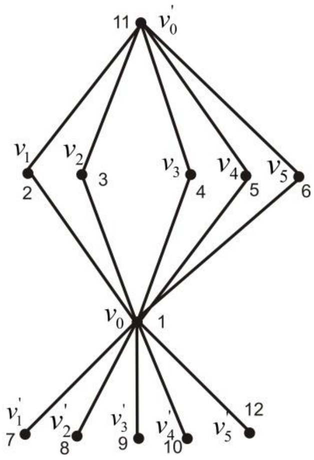

Figure 6: A prime labeling of the splitting graph of $K_{1,5}$ having the first apex vertex(v_0) as label 1.

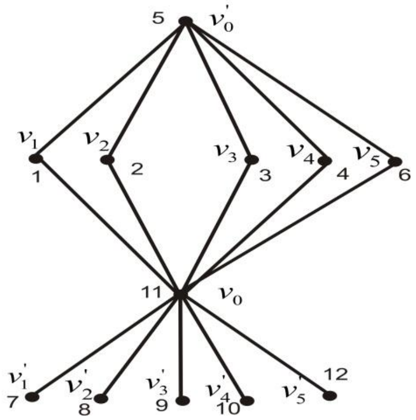

Figure 7: A prime labeling of the splitting graph of $K_{1,5}$ having v is of degree 2 ($x = v_j$) as label 1.

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