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Some path related 4-cordial graphs

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Abstract

In this paper, we discuss 4-cordial labeling of some path related graphs. We prove that middle graph, total graph and splitting graph of the path are 4-cordial. In addition to this we prove that P_n^2 and triangular snakes are 4-cordial.

Keywords: Abelian group, 4-cordial labeling, middle graph, total graph, splitting graph.
AMS Subject Classification(2010): 05C78.

1 Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling(an edge labeling).

A latest survey on various graph labeling problems can be found in Gallian[1].

Definition 1.2. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \to A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$
(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.
where
$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

Here we consider $A = < Z_4, +_4 >$, that is additive group of modulo 4. The concept of A-cordial labeling was introduced by Hovey[3] and proved the following results.

- All the connected graphs are 3-cordial.
- All the trees are 3-cordial and 4-cordial.
- Cycles are k-cordial for all odd k.

In [4, 5] Kanani and Modha proved various results related to 5-cordial and 7-cordial labeling.
In [6] Kanani and Rathod derived some new families of 4-cordial graphs.
Here we consider the following definitions of standard graphs.

- The *middle graph* $M(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.
- The *total graph* $T(G)$ of G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G.
- The *splitting graph* of G is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex which is adjacent to v in G in other words $N(v) = N(v')$. The splitting graph is denoted by $S'(G)$.
- Let G be a simple connected graph. The *square of graph* G denoted by G^2 is defined to be the graph with the same vertex set as G and in which two vertices u and v are joined by an edge $\Leftrightarrow$ in G we have $1 \leq d(u, v) \leq 2$.
- The *triangular snake* TS_n is obtained from the path P_n by replacing every edge of a path by a triangle C_3.

For any undefined term in graph theory we refer to Gross and Yellen[2].

2 Main Results

Theorem 2.1. The middle graph $M(P_n)$ of the path P_n is 4-cordial.

Proof: Let $G = M(P_n)$ be the middle graph of the path P_n. Let $v_1, v_2, ..., v_n$ be vertices of P_n and $v'_1, v'_2, ..., v'_{n-1}$ be the newly added vertices corresponding to the edges $e_1, e_2, ..., e_{n-1}$ to form G. We note that $|V(G)| = 2n - 1$ and $|E(G)| = 3n - 4$.

We define 4-cordial labeling $f : V(G) \to Z_4$ as follows.
$f(v_1) = 3;$
$f(v_i) = 0; \quad i \equiv 1, 2, 5, 6 (\text{mod } 8);$
$f(v_i) = 1; \quad i \equiv 3, 7 (\text{mod } 8);$

$f(v_i) = 2;\quad i \equiv 0, 4 (\text{mod } 8);\; 2 \leq i \leq n$
$f(v_i^{'}) = 1;\quad i \equiv 3, 7 (\text{mod } 8);$
$f(v_i^{'}) = 2;\quad i \equiv 1, 5 (\text{mod } 8);$
$f(v_i^{'}) = 3;\quad i \equiv 0, 2, 4, 6 (\text{mod } 8);\; 1 \leq i \leq n-1.$

Table 1: $n = 8a + b,\ a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1,5	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) + 1 = e_f(3)$
2,6	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3) + 1$
3,7	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) + 1 = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$

Table 1 shows that above defined labeling pattern satisfies the vertex and edge conditions of 4-cordial labeling. Hence the middle graph $M(P_n)$ of path P_n is 4-cordial. ■

Illustration 2.2. The middle graph $M(P_3)$ of path P_3 and its 4-cordial labeling is shown in Figure 1.

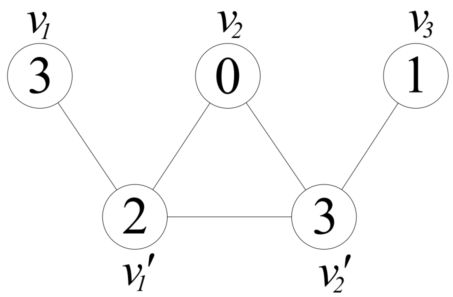

Figure 1: 4-cordial labeling of the middle graph $M(P_3)$ of path P_3.

Theorem 2.3. The total graph $T(P_n)$ of path P_n is 4-cordial.

Proof: Let $G = T(P_n)$ be the total graph of the path P_n. Let $v_1, v_2, ..., v_n$ be the vertices of the path P_n and $e_1, e_2, ..., e_{n-1}$ be the $n-1$ edges. Let $v_1^{'}, v_2^{'}, ..., v_{n-1}^{'}$ be the newly added vertices corresponding to edges $e_1, e_2, ..., e_{n-1}$ to form G. We note that $|V(G)| = 2n - 1$ and $|E(G)| = 4n - 5$.

We define 4-cordial labeling $f : V(G) \to Z_4$ as follows.
$f(v_i) = 1;\quad i \equiv 3, 7 (\text{mod } 8);$
$f(v_i) = 2;\quad i \equiv 1, 5 (\text{mod } 8);$
$f(v_i) = 3;\quad i \equiv 0, 2, 4, 6 (\text{mod } 8);\; 1 \leq i \leq n$
$f(v_i^{'}) = 0;\quad i \equiv 0, 1, 4, 5 (\text{mod } 8);$
$f(v_i^{'}) = 1;\quad i \equiv 2, 6 (\text{mod } 8);$
$f(v_i^{'}) = 2;\quad i \equiv 3, 7 (\text{mod } 8);\; 1 \leq i \leq n-1.$

Table 2: $n = 8a + b,\ a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0)+1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)$
1,5	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)+1$	$e_f(0)+1 = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1)+1 = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1) = e_f(2) = e_f(3)$
3,7	$v_f(0)+1 = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)+1$

Table 2 shows that above defined labeling pattern satisfies the vertex and edge conditions of 4-cordial labeling. Hence The total graph $T(P_n)$ of the path P_n is 4-cordial. ∎

Illustration 2.4. The total graph $T(P_4)$ of path P_4 and its 4-cordial labeling is shown in Figure 2.

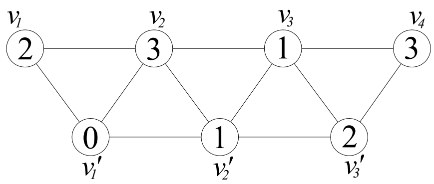

Figure 2: 4-cordial labeling of the total graph $T(P_4)$ of path P_4.

Theorem 2.5. The splitting graph $S'(P_n)$ of the path P_n is 4-cordial.

Proof: Let $G = S'(P_n)$ be the splitting graph of the path P_n. Let $v_1, v_2, ..., v_n$ be vertices of the path P_n and $v_1', v_2', ..., v_n'$ be the newly added vertices corresponding to the vertices $v_1, v_2, ..., v_n$ to form G. We note that $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

We define 4-cordial labeling $f : V(G) \to Z_4$ as follows.

$f(v_i) = 0;\quad i \equiv 2, 3, 6, 7 (\text{mod } 8);$
$f(v_i) = 2;\quad i \equiv 0, 4 (\text{mod } 8);$
$f(v_i) = 3;\quad i \equiv 1, 5 (\text{mod } 8);\ \ 1 \le i \le n$
$f(v_i') = 1;\quad i \equiv 2, 3, 6, 7 (\text{mod } 8);$
$f(v_i') = 2;\quad i \equiv 1, 5 (\text{mod } 8);$
$f(v_i') = 3;\quad i \equiv 0, 4 (\text{mod } 8);\ \ 1 \le i \le n.$

Table 3: $n = 8a + b,\ a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0)+1 = e_f(1)+1 = e_f(2)+1 = e_f(3)$
1,5	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2) = e_f(3)$
3,7	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)+1$

Table 3 shows that above defined labeling pattern satisfies the vertex and edge conditions of 4-cordial labeling. Hence the splitting graph $S'(P_n)$ of the path P_n is 4-cordial. ∎

Illustration 2.6. The splitting graph $S'(P_7)$ of path P_7 and its 4-cordial labeling is shown in Figure 3.

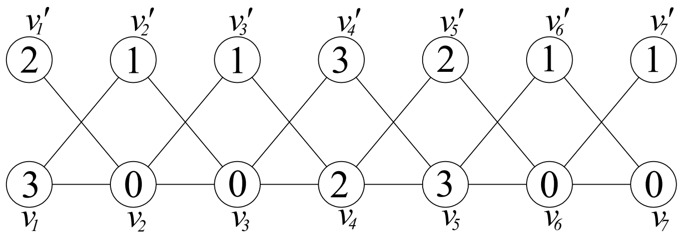

Figure 3: 4-cordial labeling of Splitting graph $S'(P_7)$ of path P_7.

Theorem 2.7. The graph P_n^2 is 4-cordial.

Proof: Let $G = P_n^2$ be the square of the path P_n with vertices $v_1, v_2, ..., v_n$. We note that $|V(G)| = n$ and $|E(G)| = 2n - 3$.

We define 4-cordial labeling $f : V(G) \to Z_4$ as follows.

$f(v_i) = 0; \quad i \equiv 1, 5 (\text{mod } 8);$

$f(v_i) = 1; \quad i \equiv 2, 6 (\text{mod } 8);$

$f(v_i) = 2; \quad i \equiv 3, 7 (\text{mod } 8);$

$f(v_i) = 3; \quad i \equiv 0, 4 (\text{mod } 8); \ 1 \leq i \leq n.$

Table 4: $n = 8a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$
1,5	$v_f(0) = v_f(1) + 1 = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3) + 1$
3,7	$v_f(0) = v_f(1) = v_f(2) = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$

Table 4 shows that above defined labeling pattern satisfies the vertex and edge conditions of 4-cordial labeling. Hence the graph P_n^2 is 4-cordial. ∎

Illustration 2.8. The square graph P_6^2 and its 4-cordial labeling is shown in Figure 4.

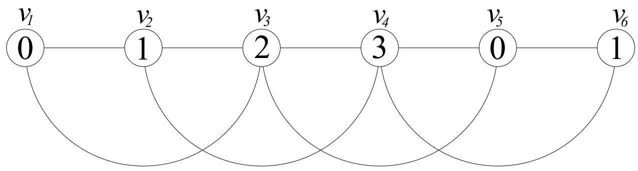

Figure 4: 4-cordial labeling of the square graph P_6^2.

Theorem 2.9. The triangular snake TS_n obtained from the path P_n is 4-cordial.

Proof: Let $G = TS_n$ be the triangular snake obtained from the path P_n. Let $v_1, v_2, ..., v_n$ be the vertices of the path P_n and $v'_1, v'_2, ..., v'_{n-1}$ be the newly added vertices to form G. We note that $|V(G)| = 2n - 1$ and $|E(G)| = 3n - 3$.

Define a vertex labeling $f : V(G) \to Z_4$ as follows:

$f(v_i) = 1; \quad i \equiv 3, 7(\text{mod } 8);$
$f(v_i) = 2; \quad i \equiv 1, 5(\text{mod } 8);$
$f(v_i) = 3; \quad i \equiv 0, 2, 4, 6(\text{mod } 8); \quad 1 \leq i \leq n.$
$f(v'_i) = 0; \quad i \equiv 0, 1, 4, 5(\text{mod } 8);$
$f(v'_i) = 1; \quad i \equiv 2, 6(\text{mod } 8);$
$f(v'_i) = 2; \quad i \equiv 3, 7(\text{mod } 8); \quad 1 \leq i \leq n - 1.$

Table 5: $n = 8a + b,\ a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3) + 1$
1,5	$v_f(0) + 1 = v_f(1) + 1 = v_f(2) = v_f(3) + 1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1) + 1 = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$
3,7	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3) + 1$

Table 5 shows that above defined labeling pattern satisfies the vertex and edge conditions of 4-cordial labeling. Hence the triangular snake TS_n obtained from the path P_n is 4-cordial. ■

Illustration 2.10. The triangular snake TS_5 obtained from the path P_5 and its 4-cordial labeling is shown in *Figure 5*.

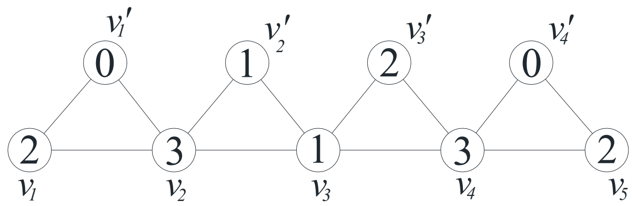

Figure 5: 4-cordial labeling of the triangular snake TS_5 obtained from the path P_5.

Concluding Remarks: We proved five results related to 4-cordial labeling. To investigate similar results for any integer k is an open problem.

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