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Some new families of 5-cordial graphs

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Abstract

In this paper, we discuss here 5-cordial labeling of some new graph families. We prove that prisms are 5-cordial. We also prove that web graph, flower graph and closed helm admit 5-cordial labeling.

Keywords: Abelian Group; 5-cordial labeling.
AMS Subject Classification(2010): 05C78.

1 Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s) If the domain of the mapping is the set of vertices(edges) then the labeling is called a *vertex labeling(an edge labeling.)* The latest survey on various graph labeling techniques can be found in Gallian[1].

Definition 1.1. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \to A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$
(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.
where
$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling. Here we consider $A =< Z_5, +_5 >$, that is additive group of modulo 5.

The concept of A-cordial labeling was introduced by Hovey[3]. He proved that all connected graphs and trees are 3-cordial, all trees are also 4-cordial and cycles are k-cordial for all odd k. Youssef[5] proved that the complete graph K_n is 4-cordial $\Longleftrightarrow n \leq 6$, the complete bipartite graph $K_{m,n}$ is 4-cordial $\Longleftrightarrow m$ or $n \not\equiv 2(mod4)$. Also he proved that the graph C_n^2 is 4-cordial $\Longleftrightarrow n \not\equiv 2(mod4)$.

We consider the following definitions of standard graphs.

- The *prism* $P_m \times C_n$ is obtained taking the cartesian product of path P_m with cycle C_n.
- The *web graph* $W(2, n)$ is obtained by joining pendant vertices of helm H_n to form a cycle and then adding a pendant edge to each vertex of outer cycle.
- The *flower graph* Fl_n is obtained from helm H_n by joining each pendant vertex to the central vertex of the helm.
- The *closed helm* CH_n is obtained from helm H_n by joining each pendant vertex to form a cycle.

For any undefined term in graph theory we rely upon Gross and Yellen[2].

2 Main Results

Theorem 2.1. All the prisms $P_m \times C_n$ are $5-$cordial.

Proof: Let $G = P_m \times C_n$ be the prism. Let $v_1, v_2, ..., v_{mn}$ be vertices of the prism arranged in a clockwise direction consecutively. Let $v_1, v_2, v_3, ..., v_n$ be vertices of outer most cycle. We start the labeling pattern from $v_1, v_2, v_3, ..., v_n$. Let v_{n+1} be the vertex of first inner cycle which is adjacent to v_n. Again label the vertices $v_{n+1}, v_{n+2}, v_{n+3}, ..., v_{2n}$ in the clockwise direction. Continue this pattern up to last inner cycle. We note that $|V(G)| = mn$ and $|E(G)| = 2mn - n$.

To define 5- cordial labeling $f : V(G) \to Z_5$ we consider the following cases.

Case 1: $n \equiv 0, 1, 3(mod5)$ and $m \equiv 0, 1, 2, 3, 4(mod5)$.
For $1 \leq i \leq mn$,
$f(v_i) = 0;\quad i \equiv 3(\text{mod } 5),\qquad f(v_i) = 1;\quad i \equiv 1(\text{mod } 5)$
$f(v_i) = 2;\quad i \equiv 4(\text{mod } 5),\qquad f(v_i) = 3;\quad i \equiv 2(\text{mod } 5)$
$f(v_i) = 4;\quad i \equiv 0(\text{mod } 5).$

Case 2: $n \equiv 2(mod5)$.
Subcase (i): $m \equiv 0(mod5)$.
For $1 \leq i \leq mn - 3$,
$f(v_i) = 0;\quad i \equiv 3(\text{mod } 5),\qquad f(v_i) = 1;\quad i \equiv 1(\text{mod } 5)$
$f(v_i) = 2;\quad i \equiv 4(\text{mod } 5),\qquad f(v_i) = 3;\quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4;\quad i \equiv 0(\text{mod } 5).$
$f(v_{mn-2}) = 2$, $f(v_{mn-1}) = 4$, $f(v_{mn}) = 0$.

Subcase (ii): $m \equiv 1, 4 (mod 5)$.
For $1 \le i \le mn - 2$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$. $f(v_{mn-1}) = 2$, $f(v_{mn}) = 3$.

Subcase (iii): $m \equiv 2(mod 5)$.
For $1 \le i \le mn - 3$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$. $f(v_{mn-2}) = 2$, $f(v_{mn-1}) = 3$, $f(v_{mn}) = 0$.

Subcase (iv): $m \equiv 3(mod 5)$.
For $1 \le i \le mn$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$.

Case 3: $n \equiv 4(mod 5)$.
Subcase (i): $m \equiv 0(mod 5)$.
For $1 \le i \le mn - 5$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$.
$f(v_{mn-4}) = 4$, $f(v_{mn-3}) = 1$, $f(v_{mn-2}) = 0$, $f(v_{mn-1}) = 2$, $f(v_{mn}) = 3$.

Subcase (ii): $m \equiv 1(mod 5)$.
For $1 \le i \le mn - 1$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$.
$f(v_{mn}) = 4$.

Subcase (iii): $m \equiv 2, 3, 4(mod 5)$.
For $1 \le i \le mn$,
$f(v_i) = 0$; $i \equiv 3(\text{mod } 5)$, $f(v_i) = 1$; $i \equiv 1(\text{mod } 5)$,
$f(v_i) = 2$; $i \equiv 4(\text{mod } 5)$, $f(v_i) = 3$; $i \equiv 2(\text{mod } 5)$,
$f(v_i) = 4$; $i \equiv 0(\text{mod } 5)$.

Table 1 shows that above defined labeling pattern satisfies the vertex conditions and edge conditions for 5-cordial labeling. Hence the prism $P_m \times C_n$ is 5-cordial. ∎

Table 1: Vertex and edge conditions for the prism $P_m \times C_n$, where $n = 5a + b$, $m = 5c + d$, $a, b, c, d \in N \cup \{0\}$.

b	d	Vertex conditions	Edge conditions
0	0,1,2,3,4	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
1	0	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
	1	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4)$
	2	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4)$
	3	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
	4	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4)$
2	0	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$
	1	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) + 1 = e_f(4)$
	2	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) = e_f(4) + 1$
	3	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
	4	$v_f(0) + 1 = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
3	0	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$
	1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$
	2	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$
	3	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
	4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$
4	0	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$

b	d	Vertex conditions	Edge conditions
4	1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) = e_f(4)$
	2	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) = e_f(4)$
	3	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
	4	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4) + 1$

Illustration 2.2. The prism $P_3 \times C_7$ and its $5-$ cordial labeling is shown in Figure 2.

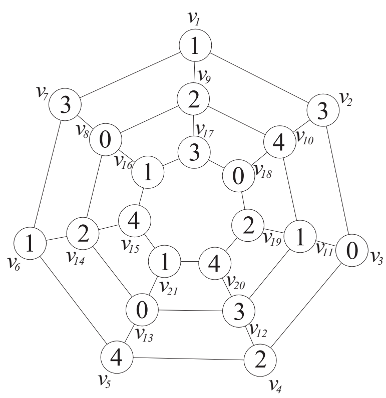

Figure 1: 5-Cordial labeling of prism $P_3 \times C_7$.

Theorem 2.3. All web graphs $W(2, n)$ are $5-$cordial.

Proof: Let $G = W(2, n)$ be the web graph. Let v_0 be the central vertex. Let $v_1, v_2, ..., v_n$ be the rim vertices and $v_1^{'}, v_2^{'}, ...v_n^{'}$ be the pendent vertices of the helm H_n which are to be joined to form an outer cycle. Let $v_1^{''}, v_2^{''}, ..., v_n^{''}$ be the pendent vertices to obtain the pendent edges $v_i^{'}v_i^{''}$ of the web graph $W(2, n)$. We note that $|V(G)| = 3n + 1$ and $|E(G)| = 5n$.

To define $5-$ cordial labeling $f : V(G) \to Z_5$ we consider the following cases.

Case 1: $n \equiv 0(mod5)$.

$f(v_0) = 0$,

For $1 \leq i \leq n$,

$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5), \qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5), \qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5), \qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5), \qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5).$
$f(v_i^{''}) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{''}) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{''}) = 4; \quad i \equiv 0(\text{mod } 5).$

Case 2: $n \equiv 1(mod 5)$.
$f(v_0) = 3,$
For $1 \leq i \leq n$,
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$
For $1 \leq i \leq n-1$,
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $f(v_n^{'}) = 4,$
$f(v_i^{''}) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{''}) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{''}) = 4; \quad i \equiv 0(\text{mod } 5),$ $f(v_n^{''}) = 2.$

Case 3: $n \equiv 2(mod 5)$.
$f(v_0) = 0;$
For $1 \leq i \leq n-2$,
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $f(v_{n-1}) = 2$, $f(v_n) = 3;$
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $f(v_{n-1}^{'}) = 4$, $f(v_n^{'}) = 1,$
For $1 \leq i \leq n-1$,
$f(v_i^{''}) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{''}) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{''}) = 4; \quad i \equiv 0(\text{mod } 5),$ $f(v_n^{''}) = 4.$

Case 4: $n \equiv 3(mod 5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n$,
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

For $1 \leq i \leq n-1$,
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n^{'}) = 2.$
For $1 \leq i \leq n-3$,
$f(v_i^{''}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{''}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{''}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-2}^{''}) = 4, f(v_{n-1}^{''}) = 4, f(v_n^{''}) = 2.$

Case 5: $n \equiv 4(mod 5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-2$,
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}) = 2, f(v_n) = 4.$
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}^{'}) = 2, f(v_n^{'}) = 4.$
For $1 \leq i \leq n-3$
$f(v_i^{''}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{''}) = 1; \quad i \equiv 1(\text{mod } 5);$
$f(v_i^{''}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{''}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{''}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-2}^{''}) = 0, f(v_{n-1}^{''}) = 0, f(v_n^{''}) = 4.$

Table 2 shows that above defined labeling pattern satisfies the vertex conditions and edge conditions for 5-cordial labeling. Hence, the web graph $W(2, n)$ is$5-$cordial. ■

Table 2: Vertex conditions and edge conditions for $W(2, n)$ where $n = 5a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
1	$v_f(0) + 1 = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
2	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
3	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
4	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$

Illustration 2.4. The web graph $W(2,6)$ and its $5-$ cordial labeling is shown in Figure 2.

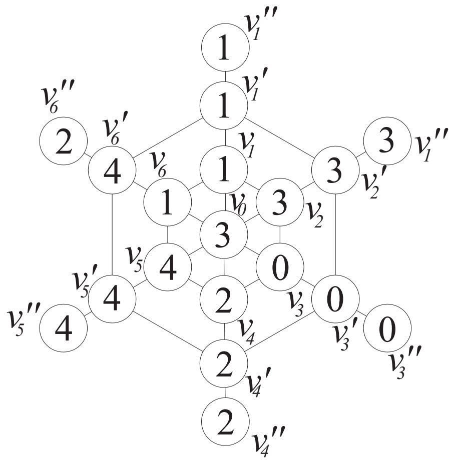

Figure 2: 5-Cordial labeling of web graph $W(2,6)$.

Theorem 2.5. All the flower graphs Fl_n are $5-$cordial.

Proof: Let $G = Fl_n$ be the flower graph. Let v_0 be the central vertex. Let $v_1, v_2, ..., v_n$ be the rim vertices and $v_1^{'}, v_2^{'}, ...v_n^{'}$ be the pendent vertices of the helm H_n. By joining $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ with the central vertex, we obtain the flower graph Fl_n. We note that $|V(G)| = 2n+1$ and $|E(G)| = 4n$.

To define $5-$ cordial labeling $f : V(G) \to Z_5$ we consider the following cases.

Case 1: $n \equiv 0(mod 5)$.

$f(v_0) = 0$,

For $1 \leq i \leq n$,

$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5).$

Case 2: $n \equiv 1(mod 5)$.

$f(v_0) = 0$,

For $1 \leq i \leq n$,

$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

For $1 \leq i \leq n-1$,

$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n^{'}) = 4.$

Case 3: $n \equiv 2(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-2,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}) = 4,$ $\qquad f(v_n) = 3.$

For $1 \leq i \leq n-1,$
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n^{'}) = 2.$

Case 4: $n \equiv 3(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

For $1 \leq i \leq n-2,$
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}^{'}) = 2, f(v_n^{'}) = 4.$

Case 5: $n \equiv 4(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-2,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}) = 2,$ $\qquad f(v_n) = 4.$

For $1 \leq i \leq n-3,$
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5),$
$f(v_{n-2}^{'}) = 2,$ $\qquad f(v_{n-1}^{'}) = 3,$ $\qquad f(v_n^{'}) = 4.$

Table 3 shows that above defined labeling pattern satisfies the vertex conditions and edge conditions for 5-cordial labeling. Hence, the flower graph Fl_n is $5-$cordial. ■

Table 3: Vertex and edge conditions for Fl_n where $n = 5a + b$, $a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4)$
2	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$ $= e_f(3) = e_f(4)$
3	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$
4	$v_f(0) + 1 = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$

Illustration 2.6. The flower graph Fl_6 and its $5-$ cordial labeling is shown in Figure 3.

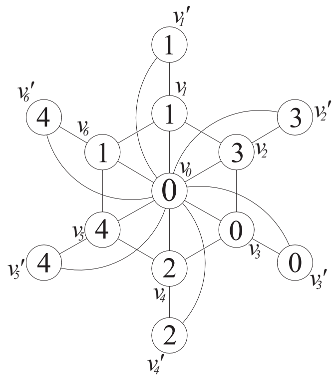

Figure 3: 5-Cordial labeling of flower graph Fl_6.

Theorem 2.7. All the closed Helms CH_n are $5-$cordial.

Proof: Let $G = CH_n$ be the closed helm. Let v_0 be the central vertex. Let $v_1, v_2, ..., v_n$ be the rim vertices and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the pendent vertices of H_n. Join the pendent vertices to obtain CH_n. We note that $|V(G)| = 2n + 1$ and $|E(G)| = 4n$.

To define $5-$ cordial labeling $f : V(G) \to Z_5$ we consider the following cases.

Case 1: $n \equiv 0(mod 5)$.

$f(v_0) = 0$,

For $1 \leq i \leq n$,

$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$

$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$

$f(v_i') = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i') = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i') = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i') = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i') = 4; \quad i \equiv 0(\text{mod } 5).$

Case 2: $n \equiv 1(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5).$
For $1 \leq i \leq n-1,$
$f(v_i') = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i') = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i') = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i') = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i') = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n') = 4.$

Case 3: $n \equiv 2(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-2,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_{n-1}) = 3, f(v_n) = 2.$
For $1 \leq i \leq n-1,$
$f(v_i') = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i') = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i') = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i') = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i') = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n') = 4.$

Case 4: $n \equiv 3(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-1,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i) = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n) = 2,$
$f(v_i') = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i') = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i') = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i') = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i') = 4; \quad i \equiv 0(\text{mod } 5),$ $\qquad f(v_n') = 4.$

Case 5: $n \equiv 4(mod5)$.
$f(v_0) = 0,$
For $1 \leq i \leq n-3,$
$f(v_i) = 0; \quad i \equiv 3(\text{mod } 5),$ $\qquad f(v_i) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i) = 2; \quad i \equiv 4(\text{mod } 5),$ $\qquad f(v_i) = 3; \quad i \equiv 2(\text{mod } 5),$

$f(v_i) = 4; \quad i \equiv 0 (\text{mod } 5).$
$f(v_{n-2}) = 2, \qquad f(v_{n-1}) = 3,\ f(v_n) = 4.$
For $1 \le i \le n-4$,
$f(v_i^{'}) = 0; \quad i \equiv 3(\text{mod } 5), \qquad f(v_i^{'}) = 1; \quad i \equiv 1(\text{mod } 5),$
$f(v_i^{'}) = 2; \quad i \equiv 4(\text{mod } 5), \qquad f(v_i^{'}) = 3; \quad i \equiv 2(\text{mod } 5),$
$f(v_i^{'}) = 4; \quad i \equiv 0(\text{mod } 5).$
$f(v_{n-3}^{'}) = 4, \qquad f(v_{n-2}^{'}) = 2,$
$f(v_{n-1}^{'}) = 3, \qquad f(v_n^{'}) = 1.$

Table 4 shows that above defined labeling pattern satisfies the vertex conditions and edge conditions for 5-cordial labeling. Hence, the closed Helm CH_n is $5-$cordial. ∎

Table 4: Vertex and edge conditions for CH_n where $n = 5a + b,\ a, b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4) + 1$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4)$
1	$v_f(0) = v_f(1) = v_f(2) + 1$ $= v_f(3) + 1 = v_f(4)$	$e_f(0) = e_f(1) = e_f(2)$ $= e_f(3) = e_f(4) + 1$
2	$v_f(0) = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4)$
3	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$ $= v_f(3) = v_f(4) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)$ $= e_f(3) + 1 = e_f(4) + 1$
4	$v_f(0) + 1 = v_f(1) = v_f(2)$ $= v_f(3) = v_f(4)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$ $= e_f(3) + 1 = e_f(4) + 1$

Illustration 2.8. The closed helm CH_8 and its $5-$ cordial labeling is shown in Figure 4.

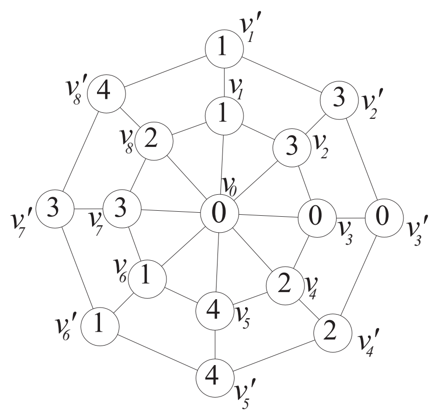

Figure 4: 5-Cordial labeling of closed helm CH_8.

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