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On soft πgb-closed sets in soft topological spaces

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Abstract

This paper focuses on soft πgb-closed sets and soft πgb-open sets in soft topological spaces and to investigate its properties. Further soft πgb-$T_{1/2}$ space is introduced and its basic properties are discussed.

Keywords: Soft topological spaces, soft closed, soft generalized closed, soft πgb-closed, soft πgb-$T_{1/2}$ space.

AMS Subject Classification (2010): 54A05.

1 Introduction

In 1999, D. Molodtsov [11] introduced the notion of soft set and applied the soft theory in several fields such as smoothness of functions, game theory, probability, Perron integration, Riemann integration and theory of measurement. The concept of soft set is used to solve complicated problems in other sciences such as, engineering, economics and the like. Maji et al. [9] described an application of soft set theory in decision-making problem. Pei and Miao [13] investigated the relationships between soft sets and information systems. Lashin et al. [7] introduced generalized rough set theory in the framework of topological spaces. Recently, Shabir and Naz [14] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They have also studied some of the basic concepts of soft topological spaces. J. Mahanta, P. K. Das[12] studied soft topological space via semiopen and semiclosed soft sets. Maji, Biswas and Roy [9] made a theoretical study of soft set theory. Tridev Jyoti Naog[15] studied a new approach to the theory of soft sets. Soft semi open sets, Soft g closed sets, soft πgr closed sets, soft gβ and soft gsβ closed sets, soft regular generalized closed sets have been introduced and studied by the researchers [1,2,5,6,16].

In this paper, we define a new class of sets called soft πgb-closed sets and study the relationships with other soft sets. Also we introduce soft πgb-$T_{1/2}$ and study its basic properties.

2 Preliminaries

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let A ⊆ E.

Definition 2.1. [10] A pair (F,A) is called a soft set over U, where F is a mapping given by F : A → P(U).In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F,A).

Definition 2.2. [3] For two soft sets (F,A) and (G,B) over a common universe U, we say that (F,A) is a soft subset of (G,B) if
(i) A ⊆B and
(ii)∀e ∈ A, F(e) $\tilde{\subseteq}$G(e).
We write (F,A) $\tilde{\subseteq}$ (G,B). (F,A) is said to be a soft super set of (G,B), if (G,B) is a soft subset of (F,A) and is denoted by (F,A) $\tilde{\supseteq}$ (G,B).

Definition 2.3. [9] A soft set (F,A) over U is said to be
(i) null soft set denoted by ϕ if ∀e ∈ A, F(e) = ϕ.
(ii) absolute soft set denoted by A, if ∀e ∈ A, F(e) = U.

Definition 2.4. [9] For two soft sets (F,A) and (G,B) over a common universe U, union of two soft sets of (F,A) and (G,B) is the soft set (H,C),where C = A ∪ B and ∀e ∈ C,

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

We write (F,A) ∪(G,B)=(H,C).

Definition 2.5. [3] The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe U denoted by (F,A) ∩ (G,B) is defined as C= A∩ B and H(e) = F(e) ∩ G(e) for all e∈C.

Definition 2.6. [14] Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y,E) over X for which Y (e) = Y, for all e ∈ E. In particular, (X,E) is denoted by $\tilde{X}$.

Definition 2.7. [14] For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by (F,A)′ and is defined by (F,A)′=(F′,A), where F′: A →P(U) is a mapping defined by F′(e)= U-F(e) for all e∈A.

Definition 2.8. [14] Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

1) ϕ, $\tilde{X}$ belong to τ
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. For simplicity, throughout the work we denote the soft topological space (X, τ, E) as X.

Definition 2.9. [14] Let (X, τ, E) be soft space over X. A soft set (F,E) over X is said to be soft closed in X, if its relative complement (F,E)′ belongs to τ. The relative complement is a mapping F′:E→P(X) defined by F′(e)= X-F(e) for all e ∈A.

Definition 2.10. [6] Let X be an initial universe set , E be the set of parameters and τ={ ϕ, $\tilde{X}$ }. Then τ is called the soft indiscrete topology on X and (X, τ,E) is said to be a soft indiscrete space over X. If τ is the collection of all soft sets which can be defined over X, then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Definition 2.11. [6] Let (X, τ, E) be a soft topological space over X and the soft interior of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly, (F,E) is the largest soft open set over X which is contained in (F,E).The soft closure of (F,E) denoted by Cl(F,E) is the intersection of all closed sets containing (F,E). Clearly, (F,E) is the smallest soft closed set containing (F,E).

Int (F,E) = ∪ { (O,E): (O,E) is soft open and (O,E) $\tilde{\subseteq}$ (F,E)}.

Cl(F,E) = ∩{ (O,E): (O,E) is soft closed and (F,E) $\tilde{\subseteq}$ (O,E)}.

Definition 2.12. [6] Let U be the common universe set and E be the set of all parameters. Let (F,A) and (G,B) be soft sets over the common universe set U and A,B $\tilde{\subseteq}$E. Then (F, A) is a subset of (G,B), denoted by (F,A) $\tilde{\subseteq}$ (G,B). (F,A) equals (G,B) , denoted by (F,A)=(G,B) if (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) $\tilde{\subseteq}$ (F,A).

Definition 2.13. A soft subset (A,E) of X is called

(i) a soft generalized closed (soft g-closed)[6] if Cl(A,E) $\tilde{\subseteq}$ U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

(ii) a soft semi open [2]if (A,E) $\tilde{\subseteq}$Int(Cl(A,E)).

(iii) a soft regular open[5] if (A,E)= Int(Cl(A,E)).

(iv) a soft α-open[5] if (A,E) $\tilde{\subseteq}$Int(Cl(Int(A,E))).

(v) a soft b-open[5] if (A,E) $\tilde{\subseteq}$Cl(Int(A,E)) Int(Cl(A,E)).

(vi) a soft pre-open[5] set if (A,E) $\tilde{\subseteq}$Int (Cl(A,E)).

(vii) a soft clopen[5] is (A,E) is both soft open and soft closed.

(viii) a soft β-open set[16] if (A,E) $\tilde{\subseteq}$ Cl(Int (Cl(A,E))).

(ix) a soft generalized β closed (Soft gβ-closed)[1] in a soft topological space (X, τ,E) if βCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

(x) a soft gsβ closed[1] if βCl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi open in X.

The complement of the soft semi open , soft regular open , soft α-open, soft b-open , soft pre-open sets are their respective soft semi closed , soft regular closed , soft α-closed , soft b-closed and soft pre -closed sets.

The finite union of soft regular open sets is called soft π-open set and its complement is soft π-closed set. The soft regular open set of X is denoted by SRO(X) or SRO(X, τ, E).

Definition 2.14. [6] A soft topological space X is called a soft $T_{1/2}$-space if every soft g-closed set is soft closed in X.

Definition 2.15. [5] The soft regular closure of (A,E) is the intersection of all soft regular closed sets containing (A,E). That is the smallest soft regular closed set containing (A,E) and is denoted by srcl(A,E).

The soft regular interior of (A,E) is the union of all soft regular open sets contained in (A,E) and is denoted by srint(A,E).

Similarly , we define soft α-closure, soft pre-closure, soft semi closure and soft b-closure of the soft set (A,E) of a topological space X and are denoted by sαcl(A,E), spcl(A,E), sscl(A,E) and sbcl(A,E) respectively.

Proposition 2.16. [4] Let (X, τ ,E) be a soft topological space over X and (F,E) and (G,E) be a soft set over X. Then

(1) int (int (F,E)) = int (F,E)

(2) (F,E) $\tilde{\subseteq}$ (G,E) implies int (F,E) $\tilde{\subseteq}$int(G,E)

(3) cl (cl (F,E)) = cl (F,E)

(4) (F,E) $\tilde{\subseteq}$ (G,E) implies cl (F,E) $\tilde{\subseteq}$cl(G,E).

Definition 2.16. [8] A subset A in a topological space is defined to be a Q-set if int(cl(A)=cl(int(A)).

3 Soft πgb -Closed sets

In this section, we define a new class of sets called soft πgb-closed sets and establish its relationship with other soft sets is discussed.

Definition 3.1. A soft subset (A,E) of a soft topological space X is called soft πgb-closed in X if sbcl(A,E) $\tilde{\subseteq}$ (U,E) whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open in X.
By SπGBC(X), we mean the family of all soft πgb-closed subsets of the space X.

Theorem 3.2.

1. Every soft closed set is soft πgb-closed.

2. Every soft g-closed is soft πgb-closed.

3. Every soft α-closed set is soft πgb-closed.

4. Every soft pre-closed set is soft πgb-closed.

5. Every soft πgr-closed set is soft πgb-closed.

6. Every soft πg -closed set is soft πgb-closed.

7. Every soft πgp -closed set is soft πgb-closed.

8. Every soft πgα-closed set is soft πgb-closed.

9. Every soft πgs-closed set is soft πgb-closed.

Proof: 1.Let (A,E) $\tilde{\subseteq}$(U,E) and (U,E) be soft π-open. Since (A,E) is soft closed we have cl(A,E) $\tilde{\subseteq}$(U,E) . Since every closed set is soft b-closed , we have sbcl (A,E)$\tilde{\subseteq}$cl (A,E) $\tilde{\subseteq}$ U. Hence A is soft πgb-closed.

2. Let (A,E) be soft g-closed in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft π- open. Since every soft π-open set is soft open and A is soft g-closed, we have cl(A) $\tilde{\subseteq}$ U. Hence sbcl(A) $\tilde{\subseteq}$ cl(A) $\tilde{\subseteq}$ U. Then A is soft πgb-closed.

3. Let (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π-open. Since A is soft α-closed, we have sαcl(A) $\tilde{\subseteq}$ (A,E) $\tilde{\subseteq}$ (U,E). Since every soft α- closed is soft b-closed, we have sbcl (A,E)$\tilde{\subseteq}$sαcl (A,E) $\tilde{\subseteq}$(U,E). Then A is soft πgb- closed.

4. Let (A, E) $\tilde{\subseteq}$ U and U is soft π-open. Since (A, E) is soft pre-closed, we have spcl (A,E) $\tilde{\subseteq}$ U. Since every soft pre-closed set is soft b-closed, we have sbcl (A, E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft πgb-closed.

5. Let (A,E) $\tilde{\subseteq}$ (U,E) and U be soft π-open. By assumption, srcl (A,E) $\tilde{\subseteq}$ (U,E). Then every soft πgr-closed set is soft πgb- closed. Then (A,E) is soft πgb-closed.

6. Let (A,E) be soft πg-closed in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft π-open. By assumption cl(A) $\tilde{\subseteq}$ U . Hence sbcl (A,E) $\tilde{\subseteq}$ cl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft πgb-closed.

7. Let (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft π- open. Since every soft pre-closed set is soft b-closed, we have sbcl (A,E) $\tilde{\subseteq}$ spcl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft πgb- closed.

8. Let (A,E) be soft πgα-closed in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft π- open. By assumption sαcl (A,E) $\tilde{\subseteq}$ (U,E) .Every soft α-closed is soft b-closed implies sbcl (A,E) $\tilde{\subseteq}$ sαcl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft πgb-closed.

9. Let (A,E) be soft πgs-closed in X and (A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft π- open. By assumption sscl(A) $\tilde{\subseteq}$ (U,E) . Hence sbcl (A,E) $\tilde{\subseteq}$ sscl (A,E) $\tilde{\subseteq}$ (U,E). Then (A,E) is soft πgb-closed. ■

Remark 3.3. Converse of the above need not be true as seen in the following example.

Example 3.4: Let X={a,b,c,d}, E={e_1,e_2}. Let F_1,F_2,….,F_6 are functions from E to P(X) and are defined as follows:

$F_1(e_1)$={c},$F_1(e_2)$={a}, $F_2(e_1)$={d},$F_2(e_2)$={b}, $F_3(e_1)$={c,d},$F_3(e_2)$={a,b}, $F_4(e_1)$={a,d},$F_4(e_2)$={b,d}, $F_5(e_1)$={b,c,d},$F_5(e_2)$={a,b,c} and $F_6(e_1)$={a,c,d},$F_6(e_2)$={a,b,d}.

Then τ_1={Φ,X,(F_1,E),.....(F_6,E)} is a soft topology and elements in τ are soft open sets.

(i) The soft set (A,E)={{a},{a}} is πgb-closed but not soft -closed.

(ii) The soft set (B,E)={{a,c,d},{d}} is πgb-closed but not soft πgα-closed, soft πgs-closed .

(iii) The soft set (C,E)={{a,c},{a,d}} is πgb-closed but not soft πgp-closed.

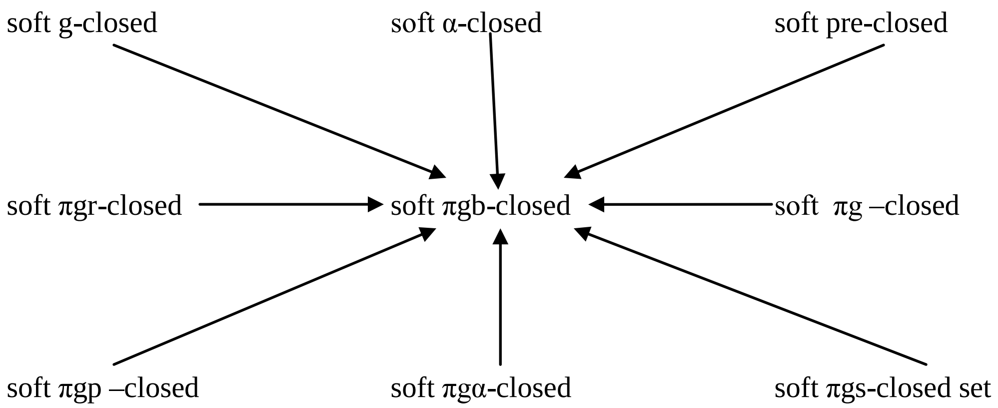
(iv) The soft set (D,E)={{c},{a}} is πgb-closed but not soft πg-closed.

(v) The soft set (H,E)={{b,c,d},{a,b,c}} is πgb-closed but not soft g-closed.

(vi) The soft set (F,E)={{c,d},{a,b}} is πgb-closed but not soft α-closed, soft pre-closed..

(vii) The soft set (G,E)={{a},{d}} is πgb-closed but not soft πgr-closed.

Remark 3.5. We depict the above discussions in the following diagram.



Theorem 3.6. If (A,E) is soft π-open and soft πgb- closed, then (A,E) is soft b-closed.

Proof : Let (A,E) be soft π-open and soft πgb-closed. Let (A,E) $\subset$ (A,E) where (A,E) is soft π-open. Since (A,E) is soft πgb- closed, we have sbcl (A,E) $\tilde{\subseteq}$ (A,E). Then (A,E) = sbcl (A,E) .Hence (A,E) is soft b-closed. ■

Theorem 3.7. Let (A,E) be soft πgb-closed in X .Then sbcl (A,E) - (A,E) does not contain any non-empty soft π-closed set.

Proof: Let (F,E) be a non-empty soft π-closed set such that (F,E) $\tilde{\subseteq}$sbcl(A,E) $\tilde{\subseteq}$(A,E) . Since (A,E) is soft πgb-closed, (A,E) $\tilde{\subseteq}$X-(F,E) where X-(F,E) is soft π-open implies sbcl (A) $\tilde{\subseteq}$ X-(F,E) . Hence (F,E) $\tilde{\subseteq}$ X-sbcl(A).Now, (F,E) $\tilde{\subseteq}$ (sbcl (A,E) -(A,E))$\cap$ (X-sbcl (A,E)) implies (F,E) =ϕ which is a contradiction. Therefore, sbcl(A,E) -(A,E) does not contain any non-empty soft π- closed set. ■

Corollary 3.8. Let (A,E) be soft πgb -closed in X .Then (A,E) is soft b-closed if and only if sbcl(A,E) -(A,E) is soft π-closed.

Proof: Let (A,E) be soft b-closed. Then sbcl(A,E) = (A,E). This implies sbcl(A)-A= ϕ which is soft π- closed. Assume that sbcl(A,E)-(A,E) is soft π- closed. Then sbcl(A,E) -(A,E) = ϕ. Hence, sbcl (A,E) = (A,E). ■

Theorem 3.9. For a soft subset (A, E) of X, the following statements are equivalent:

(1) (A, E) is soft π-open and soft πgb- closed.

(2) (A,E) is soft regular open.

Proof: (1)⇒(2): Let (A,E) be a soft π-open and soft πgb-closed subset of X. Then sbcl(A,E) $\tilde{\subseteq}$ (A,E).Hence int cl(A,E) $\tilde{\subseteq}$(A,E). Since (A,E) is soft open, we have (A,E) is soft pre-open and thus (A,E) $\tilde{\subseteq}$int cl(A,E). Therefore, we have int cl(A,E) =(A,E) , which shows that (A,E) is soft regular open.

(2)⇒(1): Since every soft regular open set is soft π-open then sbcl(A,E) =(A,E) and sbcl(A,E) $\tilde{\subseteq}$ (A,E). Hence, (A,E) is soft πgb-closed. ■

Theorem 3.10. For a soft subset (A,E) of X, the following statements are equivalent:

(1) (A,E) is soft π-clopen.

(2) (A,E) is soft π-open, a Q- set and soft πgb-closed.

Proof: (1) ⇒ (2): Let (A,E) be a soft π-clopen subset of X. Then (A,E) is soft π-closed and soft π-open. Thus (A,E) is soft closed and soft open. Therefore, (A,E) is a Q-set. Since every soft π-closed is soft πgb-closed then (A, E) is soft πgb-closed.

(2) ⇒ (1): By Theorem 3.9, (A,E) is soft regular open. Since (A,E) is a Q-set,we have (A,E) =int(cl(A,E))=cl(int(A,E)).Therefore (A,E) is regular closed. Then (A,E) is soft π-closed. Hence (A, E) is soft π-clopen. ■

Remark 3.11. Finite union of soft soft πgb-closed sets need not be πgb-closed.

Example 3.12. In Example 3.4, let the two soft sets be G(e_1)={a}, G(e_2)={a} and H(e_1)={c,d}, H(e_2)={d}. Then (G, E) and (H, E) are soft πgb-closed sets over X but their union (A,E)={{a,c,d},{a,d}} is not soft πgb-closed.

Remark 3.13. Finite intersection of soft πgb -closed sets need not be soft πgb- closed.

Example 3.14. In Example 3.4, **l**et the two soft sets be G(e_1)={d},G(e_2)={a,b,c,d} and H(e_1)={a,b,c,d}, H(e_2)={a,b}. Then (G,E) and (H,E) are soft πgb-closed sets over X. But their intersection (A,E)={{d},{a,b}} is not soft πgb-closed.

Definition 3.15. A soft topological space X is said to be soft hyperconnected if the closure of every soft open subset is X.

Theorem 3.16. Let X be a soft hyperconnected space. Then every soft πgb-closed subset of X is soft πgs- closed.

Proof: Assume that X is soft hyperconnected. Let (A,E) be soft closed and let (U,E) be an soft π-open set containing (A,E).Then sbcl(A,E)=(A,E)∪[int(cl((A,E))∩cl(int(A,E))]= (A,E)∪ int(cl(A,E)) = sscl((A,E)). Since sbcl((A,E)) = sscl((A,E)), sscl((A,E)) $\tilde{\subseteq}$ U. Hence, (A,E) is soft πgs- closed. ■

Theorem 3.17. Let (A,E) be a soft πgb- closed set and soft dense in X. Then (A,E) is soft πgp- closed.

Proof: Suppose that (A,E) be soft πgb-closed set and soft dense in X. Let (U,E) be a soft π -open set containing (A,E). Since sbcl(A,E) =(A,E) ∪[int(cl(A,E))∩cl(int(A,E))] and soft dense, we obtain bcl((A,E))= (A,E) ∪cl(int((A,E))) =pcl((A,E)) $\tilde{\subseteq}$ (U,E). Therefore (A,E) is πgp-closed. ■

Theorem 3.18. If (A,E) is soft πgb-closed set and (B,E) is any soft subset such that (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ sbcl (A,E), then (B,E) is a soft πgb-closed set.

Proof: Let (B,E) $\tilde{\subseteq}$ (U,E) and (U,E) be soft π-open. Given (A,E) $\tilde{\subseteq}$ (B,E). Then (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is soft πgb- closed, (A,E) $\tilde{\subseteq}$ (U,E) implies sbcl(A,E) $\tilde{\subseteq}$ (U,E). By assumption it follows that sbcl(B,E) $\tilde{\subseteq}$ sbcl (A,E) $\tilde{\subseteq}$ (U,E). Hence (B,E) is a soft πgb-closed set. ■

4 Soft πgb-Open sets

Definition 4.1. A soft subset (A,E) $\tilde{\subseteq}$ X is called soft πgb-open if its relative complement is soft πgb- closed. By SπGBO(X) we mean the family of all soft πgb-open subsets of the space X.

Lemma 4.2. Let (F,A) be a soft subset of a topological space X ,then sbcl (X-(F,A)) = (X-sbint (F,A)).

Proof: Let x∈X- sbint((F, A)).Then x∉ sbint ((F,A)).That is every soft b-open set (G,A) containing x is such that (G,A) $\tilde{\not\subset}$ (F,A).Hence every soft b-open set (G,A) containing x intersects X-(F,A).This implies x∈ sbcl (X-(F,A)).Hence X- sbint((F, A)) $\tilde{\subseteq}$ sbcl (X-(F,A)).

Conversely, let x∈ sbcl (X-(F,A)).Thus every soft b-open set (H,A) containing x intersects (X-(F,A)).That is every b-open set (H,A) containing x is such that (H,A) $\tilde{\not\subset}$ (F,A).This implies x∉sbint((F,A)).Thus sbcl (X-(F,A)) $\tilde{\subseteq}$ X- sbint((F, A)).Hence sbcl (X-(F,A)) = (X-sbint (F,A)). ■

Theorem 4.3. The soft subset (A,E) of X is soft πgb-open if and only if F $\tilde{\subseteq}$ sbint(A,E) whenever (F,E) is soft π-closed and (F,E) $\tilde{\subseteq}$ (A,E).

Proof: Necessity: Let (A,E) be soft πgb-open. Let (F,E) be soft π-closed and (F,E) ⊂(A,E). Then X-(A,E) $\tilde{\subseteq}$ X-(F,E) where X-(F,E) is soft π-open. By assumption, sbcl(X-(A,E)) ⊂ X-(F,E). By Lemma 4.2, X-sbint (A,E) $\tilde{\subseteq}$ X-(F,E). Thus (F,E) $\tilde{\subseteq}$ sbint (A,E).

Sufficiency: Suppose (F,E) is soft π- closed and (F,E) $\tilde{\subseteq}$ (A,E) such that (F,E) $\tilde{\subseteq}$ sbint(A,E). Let X-(A,E) $\tilde{\subseteq}$ (U,E) where (U,E) is soft π-open. Then X-(U,E) $\tilde{\subseteq}$ (A,E) where X-(U,E) is soft π-closed. By hypothesis, X-(U,E) $\tilde{\subseteq}$ sbint (A,E).That is X-sbint(A,E) $\tilde{\subseteq}$ (U,E). Hence sbcl(X-(A,E)) $\tilde{\subseteq}$ (U,E) . Thus X-(A,E) is soft πgb-closed and A is soft πgb-open. ■

Theorem 4.4. If sbint (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ (A,E) and (A,E) is soft πgb-open then (B,E) is soft πgb-open.

Proof: Let sbint(A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ (A,E). Thus X-(A,E) $\tilde{\subseteq}$ X-(B,E) $\tilde{\subseteq}$ X-sbint(A).That is X-(A,E) $\tilde{\subseteq}$ X-(B,E) $\tilde{\subseteq}$ sbcl (X-(A,E)) by Lemma 4.2 .Since X-(A,E) is soft πgb- closed , by Theorem 2.18,(X-(A,E)) $\tilde{\subseteq}$ (X-B) $\tilde{\subseteq}$ sbcl (X-(A,E)) implies (X-(B,E)) is soft πgb- closed. Hence (B,E) is soft πgb-open. ■

Remark 4.5. For any soft subset (A,E) of X, sbint (sbcl(A,E) -(A,E))= ϕ.

Theorem 4.6. If (A,E) $\tilde{\subseteq}$ X is soft πgb- closed, then sbcl(A,E) -(A,E) is soft πgb- open.

Proof: Let (A,E) be soft πgb- closed let (F,E) be soft π-closed set such that (F,E) $\tilde{\subseteq}$ sbcl (A,E) -(A,E).By Theorem 3.7, (F,E) = ϕ.Thus (F,E) $\tilde{\subseteq}$ sbint (sbcl(A,E) -(A,E)).By Theorem 4.3, sbcl (A,E) -(A,E) is soft πgb- open. ■

5 Soft πgb-T1/2 Spaces

Definition 5.1. A soft topological space X is called

(i) soft πgb-$T_{1/2}$ space if every soft πgb- closed set is soft b-closed.

(ii) soft πgb- space if every soft πgb- closed is soft closed.

(iii) soft $T_{\pi gb}$ -space if every soft πgb-closed set is soft πg-closed.

Theorem 5.2. For a soft topological space (X, τ,E) the following are equivalent

(i) X is soft πgb-$T_{1/2}$ space.

(ii) Every singleton set is either soft π-closed or soft b-open.

Proof: To prove (i) ⇒ (ii): Let X be a soft πgb-$T_{1/2}$ space .Let (A,E) be a soft singleton set in X and assume that (A,E) is not soft π- closed. Then clearly X- (A,E) is not soft π- open. Hence X- (A,E) is trivially a soft πgb- closed. Since X is soft πgb- $T_{1/2}$ space, X-(A,E) is soft b-closed. Therefore (A,E) is soft b-open.

(ii⇒(i): Assume that every singleton of X is either soft π-closed or soft b-open. Let (A,E) be a πgb-closed set. Let (A,E) ∈ sbcl (A,E)

Case (i): Let the singleton set (F,E) be soft π-closed. Suppose (F,E) does not belong to (A,E). Then (F,E) ∈ bcl (A,E) - (A,E) . By Theorem 3.7, (F,E) ∈ (A,E). Hence sbcl (A,E) $\tilde{\subseteq}$ (A,E).

Case (ii): Let the singleton set (F,E) be soft b-open . Since (F,E) ∈ sbcl (A,E), we have (F,E) ∩ (A,E) ≠ ϕ implies that (F,E) ∈ (A,E). In both the cases sbcl (A,E) $\tilde{\subseteq}$ (A,E) or equivalently (A,E) is soft b-closed. ■

Theorem 5.3. For a soft topological space (X, τ, E) the following are equivalent

(i) X is soft πgb-$T_{1/2}$ space.

(ii) For every soft subset (A,E) of X, (A,E) is soft πgb- open if and only if (A,E) is soft b-open.

Proof: (i)⇒(ii): Let the soft topological space X be soft πgb-$T_{1/2}$ and let (A,E) be a soft πgb-open soft subset of X. Then X- (A,E) is soft πgb-closed and so X- (A,E) is soft b-closed. Hence (A,E) is soft b-open.

Conversely, let (A,E) be a soft b-open subset of X. Thus X- (A,E) is soft b-closed. Since every soft b-closed set is soft πgb-closed then X- (A,E) is soft πgb-closed. Hence (A,E) is soft πgb-open.

(ii)⇒(i): Let (A,E) be a soft πgb-closed subset of X. Then X-(A,E) is soft πgb-open. By the hypothesis X- (A,E) is soft b-open. Thus (A,E) is soft b-closed. Since every soft πgb-closed is soft b-closed, X is soft πgb-$T_{1/2}$ space. ■

Proposition 5.4. (i) Every soft πgb- space is soft πgb-$T_{1/2}$.

(ii) Every soft πgb- space is soft $T_{\pi gb}$ –space.

Theorem 5.5. SBO(X, τ, E) $\tilde{\subseteq}$ SπGBO(X, τ, E).

Proof: Let (A,E) be soft b-open, then X- (A,E) is soft b-closed so X- (A,E) is soft πgb-closed. Thus (A,E) is soft πgb-open. Hence SBO (τ) $\tilde{\subseteq}$ SπGBO(τ). ■

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