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Option pricing formulas for Modified Log-payoff Function

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Abstract

Paul Wilmott derived BSM option pricing formula for the payoff function $\max\{\ln(\frac{S_T}{K}), 0\}$. In this article, we derive the BSM option pricing formula for the modified payoff function $\max\{S_T \ln(\frac{S_T}{K}), 0\}$. It turns out that the present BSM formula is quite close to the plain vanilla option pricing formula.

Keywords: BSM Formulas, Log and modified log payoff functions, comparison of BSM formulas.
AMS Subject Classification(2010): 91B24, 91B28.

1 Introduction

One of the best product among various derivatives in financial market is undoubtedly the 'option'. Millions of people in financial market use Black-Schole-Merton option pricing formulas. After the success of BSM formulas for plain vanilla, several other types of option pricing formulas were derived with different payoff functions [1]. In Section-2, we derive the BSM (European) option pricing formula for the payoff function $\max\{S_T \ln(\frac{S_T}{K}), 0\}$. This is a modification of Paul Wilmott's log payoff function $\max\{\ln(\frac{S_T}{K}), 0\}$ [4, p 149]. Our search for a payoff function closely related with Wilmott's log payoff function, eliminating some of its deficiencies, and that is closer to the celebrated plain vanilla payoff function leads to the modified log payoff function stated above. Notice that it is closely related with the entropy function $x \log x$ in Information Theory. In Section-3, for the modified log payoff function, we write down the expressions for the financial parameters such as put option formula and greek letters Δ and Γ. We also compute numerical call/put option values and payoff values for the three payoff functions; namely, plain vanilla, log payoff, and modified log payoff. It turns out that the option values for the modified log payoff function are close to the corresponding values for the plain vanilla. The present note supplements Paul Wilmott's analysis on log payoff function.

2 BSM formula for the modified log payoff

Theorem 2.1. The European call option pricing formula for the modified log payoff function $\max\{S_T \ln(\frac{S_T}{K}), 0\}$ is

$$C(S,t) = S[\ln(\frac{S}{K})N(d) + \sigma\sqrt{T-t}\,\eta(d) + (r + \frac{1}{2}\sigma^2)(T-t)N(d)]$$

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where

$$d = \frac{\ln(\frac{S}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad \textit{and} \quad \eta(d) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{d^2}{2}}.$$

Proof: The Black-Scholes-Merton partial differential equation along with the boundary conditions for a Europian call option $C(S,t)$ is as follow [5, page-76]:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{2.1}$$

with $C(0,t) = 0$, $C(S,t) \to S$ when $S \to \infty$ and $C(S,T) = \max\{S_T \ln(\frac{S_T}{K}), 0\}$, where K is the striking price. Taking $S = Ke^x$, $t = T - \frac{\tau}{\frac{1}{2}\sigma^2}$, $C(S,\, t) = Kv(x,\tau)$, the equation (2.1) reduces to

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv. \tag{2.2}$$

where $k = \frac{r}{\frac{1}{2}\sigma^2}$. Again, by taking

$$v(x,\tau) = e^{-\frac{1}{2}(k-1)x - \frac{1}{4}(k+1)^2\tau}u(x,\tau), \tag{2.3}$$

the equation (2.2) reduces to the heat equation

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} \qquad (\tau > 0,\ -\infty < x < \infty). \tag{2.4}$$

Now $C(S,T) = \max\{S_T \ln(\frac{S_T}{K}), 0\}$ gives

$$u_0(x) = u(x,0) = \max\{xe^{\frac{1}{2}(k+1)x},\ 0\}. \tag{2.5}$$

The solution of the heat equation (2.4) is

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} u_0(s) e^{-\frac{(s-x)^2}{4\tau}} ds, \tag{2.6}$$

where $u_0(x)$ is given by equation (2.5). Substituting the value of $u_0(x)$ from equation (2.5) into the equation (2.6), we get

$$\begin{aligned} u(x,\tau) &= e^{\frac{1}{2}(k+1)x + \frac{1}{4}(k+1)^2\tau}[xN(d) \\ &\quad + \sqrt{\frac{\tau}{\pi}} e^{-\frac{1}{2}(\frac{x}{\sqrt{2\tau}} + \frac{1}{2}(k+1)\sqrt{2\tau})^2} + \sqrt{\frac{\tau}{2}} N(d)\sqrt{2}\tau] \end{aligned} \tag{2.7}$$

where

$$d = \frac{x}{\sqrt{2\tau}} + \frac{1}{2}(k+1)\sqrt{2\tau} \quad \text{and} \quad N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx.$$

From equation (2.3) , we get

$$v(x,\tau) = e^x[xN(d) + \sqrt{\frac{\tau}{\pi}} e^{-\frac{1}{2}(\frac{x}{\sqrt{2\tau}} + \frac{1}{2}(k+1)\sqrt{2\tau})^2} + (k+1)\tau N(d)]. \tag{2.8}$$

Thus

$$C(S,t) = S[\ln(\frac{S}{K})N(d) + \sigma\sqrt{T-t}\,\eta(d) + (r+\frac{1}{2}\sigma^2)(T-t)N(d)], \quad (2.9)$$

where

$$d = \frac{\ln(\frac{S}{K}) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \text{ and } \eta(d) = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{d^2}{2}}.$$

The expression (2.9) is the BSM formula for the European call option with payoff function $C(S,T) = \max\{S_T \ln(\frac{S_T}{K}), 0\}$. ■

For the sake of completeness, we list the following notations and formulas for our main three payoff functions which will be required later.

$$\begin{aligned}
PO_{C_1} &= C_1(S,T) = \max\{S_T - K, 0\} \\
PO_{C_2} &= C_2(S,T) = \max\{\ln(\frac{S_T}{K}), 0\} \\
PO_{C_3} &= C_3(S,T) = \max\{S_T\ln(\frac{S_T}{K}), 0\} \\
PO_{P_1} &= P_1(S,T) = \max\{K - S_T, 0\} \\
PO_{P_2} &= P_2(S,T) = \max\{\ln(\frac{K}{S_T}), 0\} \\
PO_{P_3} &= P_3(S,T) = \max\{S_T\ln(\frac{K}{S_T}), 0\} \\
C_1 &= SN(d_1) - Ke^{-r(T-t)}N(d_2) \\
C_2 &= e^{-r(T-t)}\eta(d_2)\sigma\sqrt{T-t} + e^{-r(T-t)}[\,\ln(\frac{S}{K}) + (r-\frac{1}{2}\sigma^2)(T-t)]N(d_2) \\
C_3 &= S[\,\ln(\frac{S}{K})N(d_1) + \sigma\sqrt{T-t}\,\eta(d_1) + (r+\frac{1}{2}\sigma^2)(T-t)N(d_1)] \\
P_1 &= C_1 - S + Ke^{-r(T-t)} \\
P_2 &= C_2 - e^{-r(T-t)}[\,\ln(\frac{S}{K}) + (r-\frac{1}{2}\sigma^2)(T-t)] \\
P_3 &= C_3 - S[\,\ln(\frac{S}{K}) + (r-\frac{1}{2}\sigma^2)(T-t)]
\end{aligned}$$

where

$$\begin{aligned}
d_1 &= \frac{\ln(\frac{S}{K}) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln(\frac{S}{K}) + (r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\
\eta(x) &= \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}, \quad N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{x^2}{2}}\,dx.
\end{aligned}$$

The delta Δ of a call option is of fundamental importance in both theory and practice. It indicates the rate of change of the value of option with respect to the asset price S. Equally important is the gamma Γ. It is the rate of change of Δ with respect to the asset price S. Here we write down expressions for Δ and Γ for these three payoff functions.

$$\begin{aligned}
\Delta_{C_1} &= N(d_1) \\
\Delta_{C_2} &= \frac{e^{-r(T-t)}}{S\sigma\sqrt{T-t}}[\ln(\frac{S}{K}) + (r-\frac{\sigma^2}{2})(T-t) - d_2\,\sigma\sqrt{T-t}]\,\eta(d_2)
\end{aligned}$$

$$
\begin{aligned}
& + \frac{e^{-r(T-t)} N(d_2)}{S} \\
\Delta_{C_3} &= \left[\frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t) + \sigma^2(T-t) - d_1\, \sigma\sqrt{T-t}}{\sigma\sqrt{T-t}} \right] \eta(d_1) \\
& + [\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t) + 1] N(d_1) \\
\Delta_{P_1} &= \Delta_{C_1} - 1 \\
\Delta_{P_2} &= \Delta_{C_2} - \frac{1}{S}\, e^{-r(T-t)} \\
\Delta_{P_3} &= \Delta_{C_3} - [\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)] - 1 \\
\Gamma_{C_1} &= \frac{\eta(d_1)}{S\sigma\sqrt{T-t}} \\
\Gamma_{C_2} &= -\frac{d_2\, \eta(d_2) e^{-r(T-t)}}{S^2\sigma^2(T-t)} \left[\ln(\frac{S}{K}) + (r - \frac{\sigma^2}{2})(T-t) - d_2\, \sigma\sqrt{T-t} \right] \\
& + \frac{\eta(d_2) e^{-r(T-t)}}{S^2\sigma\sqrt{T-t}} - \frac{1}{S}\Delta_{C_2} \\
\Gamma_{C_3} &= \frac{\eta(d_1)}{S\sigma\sqrt{T-t}} \left[\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t) + 1 \right] + \frac{N(d_1)}{S} \\
& - \frac{\eta(d_1)\, d_1}{S} \left[\frac{\ln(\frac{S}{K})}{\sigma^2(T-t)} - \frac{d_1}{\sigma\sqrt{T-t}} + \frac{(r + \frac{\sigma^2}{2})}{\sigma^2} + 1 \right] \\
\Gamma_{P_1} &= \Gamma_{C_1} \\
\Gamma_{P_2} &= \Gamma_{C_2} + \frac{1}{S^2}\, e^{-r(T-t)} \\
\Gamma_{P_3} &= \Gamma_{C_3} - \frac{1}{S}
\end{aligned}
$$

3 Related Numerical Computations

Throughout this section, we fix the current asset price $S_0 = 100$, the maturity time $T = 0.5$ and the risk free interest rate $r = 0.08$; using these, we compute the call/put option values as well as payoff for different striking prices K and volatilities σ.

Table - 01 : $K = 80, \sigma = 0.30$

C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
24.0929	0.2443	29.4384	80	0	0	0	0	0	0
			90	10	0.1178	10.6020	41.51	48.22	36.01
			100	20	0.2231	22.3100	83.01	91.32	75.78
			110	30	0.3185	35.0350	124.52	130.37	119.01
			120	40	0.4055	48.6600	166.02	165.98	165.29

Table - 02 : $K = 90, \sigma = 0.30$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
16.4094	0.1536	19.3754	100	10	0.1054	10.5400	60.94	68.62	54.40
			110	20	0.2007	22.0770	121.88	130.66	113.94
			120	30	0.2877	34.5240	182.82	187.30	178.18

Table - 03 : $K = 100, \sigma = 0.30$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
10.3881	0.0900	11.9525	100	0	0	0	0	0	0
			110	10	0.0953	10.4841	96.26	105.89	87.71
			120	20	0.1823	21.8786	192.53	202.56	183.05

Table - 04 : $K = 80, \sigma = 0.35$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	10	0.1178	10.6020	40.44	47.96	34.42
24.7292	0.2456	30.8027	100	20	0.2231	22.3100	80.88	90.84	72.43
			110	30	0.3185	35.0350	121.31	129.68	113.74
			120	40	0.4055	48.6600	161.75	165.11	157.97

Table - 05 : $K = 90, \sigma = 0.35$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
17.4750	0.1600	21.0687	100	10	0.1054	10.5400	57.22	65.87	50.03
			110	20	0.2007	22.0770	114.45	125.44	104.79
			120	30	0.2877	34.5240	171.67	179.81	163.86

Table - 06 : $K = 100, \sigma = 0.35$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
11.7408	0.0994	13.8039	100	0	0	0	0	0	0
			110	10	0.0953	10.4841	85.17	95.87	75.95
			120	20	0.1823	21.8786	170.35	183.40	158.50

Table - 07 : $K = 80, \sigma = 0.40$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	10	0.1178	10.6020	39.25	47.56	32.76
25.4772	0.2477	32.3600	100	20	0.2231	22.3100	78.50	90.07	68.94
			110	30	0.3185	35.0350	117.75	128.58	108.27
			120	40	0.4055	48.6600	157.00	163.71	150.37

Table - 08 : $K = 90, \sigma = 0.40$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
18.5931	0.1665	22.8918	100	10	0.1054	10.5400	53.78	63.30	46.04
			110	20	0.2007	22.0770	107.57	120.54	96.44
			120	30	0.2877	34.5240	161.35	172.79	150.81

Table - 09 : $K = 100, \sigma = 0.40$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
13.0957	0.1084	15.7322	100	0	0	0	0	0	0
			110	10	0.0953	10.4841	76.36	87.91	66.64
			120	20	0.1823	21.8786	152.72	168.17	139.07

Table - 10 : $K = 80, \sigma = 0.45$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	10	0.1178	10.6020	38.01	47.06	31.09
26.3097	0.2503	34.0990	100	20	0.2231	22.3100	76.02	89.13	65.42
			110	30	0.3185	35.0350	114.03	127.25	102.75
			120	40	0.4055	48.6600	152.03	162.01	142.70

Table - 11 : $K = 90, \sigma = 0.45$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
19.7463	0.1728	24.8281	100	10	0.1054	10.5400	50.64	60.99	42.45
			110	20	0.2007	22.0770	101.28	116.15	88.92
			120	30	0.2877	34.5240	151.93	166.49	139.05

Table - 12 : $K = 100, \sigma = 0.45$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
14.4506	0.1170	17.7369	100	0	0	0	0	0	0
			110	10	0.0953	10.4841	69.20	81.45	59.11
			120	20	0.1823	21.8786	138.40	155..81	123.35

Table - 13 : $K = 80, \sigma = 0.50$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	10	0.1178	10.6020	36.76	46.52	29.47
27.2061	0.2532	35.9802	100	20	0.2231	22.3100	73.51	88.11	62.01
			110	30	0.3185	35.0350	110.27	125.79	97.37
			120	40	0.4055	48.6600	147.03	160..15	135.24

Table - 14 : $K = 90, \sigma = 0.50$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
20.9236	0.1790	26.8675	100	10	0.1054	10.5400	47.79	58.88	39.23
			110	20	0.2007	22.0770	95.59	112.12	82.17
			120	30	0.2877	34.5240	143.38	160.73	128.50

Table - 15 : $K = 100, \sigma = 0.50$									
C_1	C_2	C_3	S_T	PO_{C_1}	PO_{C_2}	PO_{C_3}	$PO_{C_1}\%$	$PO_{C_2}\%$	$PO_{C_3}\%$
			80	0	0	0	0	0	0
			90	0	0	0	0	0	0
15.8041	0.1250	19.8184	100	0	0	0	0	0	0
			110	10	0.0953	10.4841	63.27	76.24	52.90
			120	20	0.1823	21.8786	126.55	145.84	110.39

Table - 16 : $K = 100, \sigma = 0.30$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	20	0.2231	17.8515	309.26	304.78	313.05
			90	10	0.1054	9.4824	154.63	143.99	166.28
6.4671	0.0732	5.7025	100	0	0	0	0	0	0
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 17 : $K = 110, \sigma = 0.30$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	30	0.3185	25.4763	253.74	256.65	249.66
			90	20	0.2007	18.0604	169.16	161.72	176.99
11.8230	0.1241	10.2044	100	10	0.0953	9.5310	84.58	76.79	93.39
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 18 : $K = 120, \sigma = 0.30$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	40	0.4055	32.4372	213.88	220.50	205.70
			90	30	0.2877	25.8914	160.41	156.44	164.19
18.7019	0.1839	15.7692	100	20	0.1823	18.2322	106.94	99.13	115.60
			110	10	0.0870	9.5713	53.47	47.31	60.70
			120	0	0	0	0	0	0

Table - 19 : $K = 100, \sigma = 0.35$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	20	0.2231	17.8515	255.76	246.79	264.80
			90	10	0.1054	9.4824	127.88	116.59	140.66
7.8197	0.0904	6.7414	100	0	0	0	0	0	0
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 20 : $K = 110, \sigma = 0.35$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	30	0.3185	25.4763	226.89	224.61	228.35
			90	20	0.2007	18.0604	151.26	141.54	161.88
13.2221	0.1418	11.1566	100	10	0.0953	9.5310	75.63	67.21	85.42
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 21 : $K = 120, \sigma = 0.35$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	40	0.4055	32.4372	200.58	202.55	197.22
			90	30	0.2877	25.8914	150.43	143.71	157.42
19.9426	0.2002	16.4470	100	20	0.1823	18.2322	100.29	91.06	110.84
			110	10	0.0870	9.5713	50.14	43.46	58.19
			120	0	0	0	0	0	0

Table - 22 : $K = 100, \sigma = 0.40$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
9.1746	0.1084	7.7322	80	20	0.2231	17.8515	217.99	205.81	230.87
			90	10	0.1054	9.4824	109.00	97.23	122.63
			100	0	0	0	0	0	0
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 23 : $K = 110, \sigma = 0.40$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
14.6284	0.1603	12.0658	80	30	0.3185	25.4763	205.08	198.69	211.14
			90	20	0.2007	18.0604	136.72	149.68	149.68
			100	10	0.0953	9.5310	68.36	59.45	78.98
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 24 : $K = 120, \sigma = 0.40$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
21.2417	0.2178	17.1303	80	40	0.4055	32.4372	188.31	186.18	189.35
			90	30	0.2877	25.8914	141.23	132.09	151.14
			100	20	0.1823	18.2322	94.15	83.70	106.42
			110	10	0.0870	9.5713	47.08	39.94	55.87
			120	0	0	0	0	0	0

Table - 25 : $K = 100, \sigma = 0.45$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
10.5295	0.1271	8.6744	80	20	0.2231	17.8515	189.94	175.53	205.79
			90	10	0.1054	9.4824	94.97	82.93	109.31
			100	0	0	0	0	0	0
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 26 : $K = 110, \sigma = 0.45$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	30	0.3185	25.4763	187.05	177.44	197.03
			90	20	0.2007	18.0604	124.70	111.81	139.68
16.0380	0.1795	12.9299	100	10	0.0953	9.5310	62.35	53.09	73.70
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 27 : $K = 120, \sigma = 0.45$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	40	0.4055	32.4372	177.15	171.53	182.20
			90	30	0.2877	25.8914	132.86	121.70	145.44
22.5798	0.2364	17.8026	100	20	0.1823	18.2322	88.57	77.11	102.40
			110	10	0.0870	9.5713	44.29	36.80	53.76
			120	0	0	0	0	0	0

Table - 28 : $K = 100, \sigma = 0.50$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	20	0.2231	17.8515	168.31	152.18	186.57
			90	10	0.1054	9.4824	84.15	71.90	99.10
11.883	0.1466	9.5684	100	0	0	0	0	0	0
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 29 : $K = 110, \sigma = 0.50$									
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
			80	30	0.3185	25.4763	171.93	159.57	185.31
			90	20	0.2007	18.0604	114.62	100.55	131.36
17.4484	0.1996	13.7482	100	10	0.0953	9.5310	57.31	47.74	69.32
			110	0	0	0	0	0	0
			120	0	0	0	0	0	0

Table - 30 : $K = 120, \sigma = 0.50$										
P_1	P_2	P_3	S_T	PO_{P_1}	PO_{P_2}	PO_{P_3}	$PO_{P_1}\%$	$PO_{P_2}\%$	$PO_{P_3}\%$
23.9445	0.2560	18.4538	80	40	0.4055	32.4372	167.05	158.40	175.77
			90	30	0.2877	25.8914	125.29	112.38	140.30
			100	20	0.1823	18.2322	83.53	71.21	98.79
			110	10	0.0870	9.5713	41.76	33.98	51.87
			120	0	0	0	0	0	0

4 Conclusion

The plain vanilla option is the most used one in financial market. The very first BSM formula was derived for this option. Then there are many exotic options also. All of the other options are compared directly or indirectly with plain vanilla. We also compare our modified log option with the plain vanilla option.

Our first conclusion is the following. From all the thirty tables above, we can see that the option values and payoff values for the log option are strictly less than unity. Sometimes, they are even less than the transaction costs; see also Table 4.4 in [1, p 121]. So this BSM formula does not seem to be practically useful in the financial market. On the other hand, our modified log option contract is very close to the plain vanilla (see Tables).

Our second important conclusion is that as compared to the plain vanilla, generally the *writer* is more beneficial to enter into a call option using the modified log payoff (Tables 1 to 15) whereas the *holder* is more beneficial to enter into a put option using the same (Tables 16 to 30).

References

[1] E. G. Haug, *The Complete Guide to Option Pricing Formulas*, McGraw-Hill, second edition, 2007.

[2] J. C. Hull, *Options Futures and other Derivatives*, Prentice Hall, seventh edition, 2008.

[3] A. Neuberger, *The Log Contract: A New Instrument to Hedge Volatility*, Journal of Portfolio Management, Winter. 1994, 74-80.

[4] P. Wilmott, *Paul Wilmott on Quantitative Finance*, John Wiley & Sons, Ltd., second edition, 2006.

[5] P. Wilmott, S. Howison and J. Dewynne, *Mathematics of Financial Derivatives*, Cambridge University Press, 2002.