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Some edge product cordial graphs

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Abstract

For a graph $G = (V(G), E(G))$, a function $f : E(G) \to \{0, 1\}$ is called an edge product cordial labeling of G if the induced vertex labeling function defined by the product of incident edge labels be such that the edges with label 1 and label 0 differ by at most 1 and the vertices with label 1 and label 0 also differ by at most 1. In this paper we investigate some new families of edge product cordial graph.

Keywords: Cordial graph, product cordial graph, edge product cordial graph.
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1 Introduction

In this section we give some definitions and other information which are useful for the present work. The terms that are not defined here are used in the sense of Chartrand and Lesniak [2].

Definition 1.1. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

For an extensive survey on graph labeling and bibliographic references we refer to Gallian [3].

Most of the graph labeling techniques trace their origin to graceful labeling introduced by Rosa [6] and Golomb [4]. The famous Ringel-Kotzig graceful tree conjecture and the illustrious work by Kotzig [5] brought a tide of labeling problems having graceful theme.

In 1987, Cahit [1] introduced the concept of cordial labeling as a weaker version of graceful and harmonious labelings. Some labeling schemes are also introduced with minor variations in cordial theme. In 2004, Sundaram et al. [7] introduced the notion of product cordial labeling in which the absolute difference in cordial labeling is replaced by product of the vertex labels.

Vaidya and Barasara [8] introduced the edge analogue of product cordial labeling and named it as edge product cordial labeling which is defined as follows.

Definition 1.2. For a graph G, the edge labeling function is defined as $f : E(G) \to \{0, 1\}$ and the induced vertex labeling function $f^* : V(G) \to \{0, 1\}$ is given by $f^*(v) = f(e_1)f(e_2)\ldots f(e_n)$ if $e_1, e_2, \ldots, e_n$ are the edges incident with the vertex v.

We denote the number of vertices of G having label i under f^* by $v_f(i)$ and the number of edges of G having label i under f by $e_f(i)$ for $i = 0, 1$.

f is called an edge product cordial labeling of a graph G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called edge product cordial if it admits an edge product cordial labeling.

Vaidya and Barasara studied edge product cordial labeling for some new families of graphs in [9] .

Definition 1.3. The graph $C_n^{(t)}$ is a one point union of t cycles C_n with a vertex in common.

In this paper we investigate some families of graphs that admit and that do not admit edge cordial labeling.

2 Main Results

Theorem 2.1. The graph $K_n (n \geq 4)$ is not an edge product cordial graph.

Proof: In order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to at least $\left\lfloor \frac{n(n-1)}{4} \right\rfloor$ edges out of $\frac{n(n-1)}{2}$ edges. The edges with label 0 will give rise at least $n - \left\lfloor \frac{n}{4} \right\rfloor$ vertices with label 0 and at most $\left\lfloor \frac{n}{4} \right\rfloor$ vertices with label 1 out of total n vertices. Therefore $|v_f(0) - v_f(1)| \geq n - 2\left\lfloor \frac{n}{4} \right\rfloor \geq 2$. Thus the vertex condition for edge product cordial graph is violated. Consequently, $K_n (n \geq 4)$ is not an edge product cordial graph. ■

Theorem 2.2. The graph $K_{m,n} (m, n \geq 2)$ is not an edge product cordial graph.

Proof: Without loss of generality we assume that $n \geq m$.

Case 1: n is even.

In order to satisfy the edge condition for an edge product cordial graph it is essential to assign label 0 to $\frac{mn}{2}$ edges out of mn edges. The edges with label 0 will give rise at least $m + \frac{n}{2}$ vertices with label 0 and at most $\frac{n}{2}$ vertices with label 1 out of the total $m + n$ vertices. Therefore, $|v_f(0) - v_f(1)| \geq m \geq 2$. Thus the vertex condition for an edge product cordial graph is violated.

Case 2: m is even.

In order to satisfy the edge condition for an edge product cordial graph it is essential to assign label 0 to $\frac{mn}{2}$ edges out of mn edges. The edges with label 0 will give rise at least $n + \frac{m}{2}$ vertices with label 0 and at most $\frac{m}{2}$ vertices with label 1 out of the total $m + n$ vertices. Therefore $|v_f(0) - v_f(1)| \geq n \geq 2$. Thus the vertex condition for an edge product cordial graph is violated.

Case 3: Both m and n are odd.

In order to satisfy the edge condition for an edge product cordial graph it is essential to assign label 0 to at least $\left\lfloor \frac{mn}{2} \right\rfloor$ edges out of mn edges. The edges with label 0 will give rise at least $m + \left\lceil \frac{n}{2} \right\rceil$ vertices with label 0 and at most $\left\lfloor \frac{n}{2} \right\rfloor$ vertices with label 1 out of the total $m + n$ vertices. Therefore, $|v_f(0) - v_f(1)| \geq m + 1 \geq 2$. Thus the vertex condition for edge product cordial graph is violated.

Therefore, $K_{m,n} (m, n \geq 2)$ is not an edge product cordial graph. ■

Remark 2.3. Vaidya and Barasara [8] proved that all tress are edge product cordial. Hence, the graph $K_{1,n}$ is edge product cordial

Theorem 2.4. The graph $C_n^{(t)}$ is an edge product cordial graph for t even or t and n are both odd while is not an edge product cordial graph for t odd and n even.

Proof: Let $e_{k,1}, e_{k,2}, \ldots, e_{k,n}$ be the edges of k^{th} copy of cycle C_n and v be a common vertex of $C_n^{(t)}$. The edges $e_{k,1}$ and $e_{k,n}$ of k^{th} copy of cycle C_n are incident to v. We consider the following three cases.
Case 1: t is even.

$$\begin{array}{lll} f(e_{i,j}) = 0; & 1 \leq i \leq \frac{t}{2} & \text{and} \quad 1 \leq j \leq n \\ f(e_{i,j}) = 1; & \frac{t}{2} + 1 \leq i \leq t & \text{and} \quad 1 \leq j \leq n. \end{array}$$

In view of the above defined labeling pattern we have,

$$v_f(0) - 1 = v_f(1) = \frac{t(n-1)}{2}$$
$$e_f(0) = e_f(1) = \frac{tn}{2}$$

Case 2: Both t and n are odd.

$$\begin{array}{lll} f(e_{i,j}) = 0; & 1 \leq i \leq \frac{t-1}{2} & \text{and} \quad 1 \leq j \leq n \\ f(e_{i,j}) = 1; & \frac{t+1}{2} \leq i \leq t-1 & \text{and} \quad 1 \leq j \leq n \\ f(e_{t,j}) = 0; & 1 \leq j \leq \frac{n-1}{2} & \\ f(e_{t,j}) = 1; & \frac{n+1}{2} \leq i \leq n. & \end{array}$$

In view of the above defined labeling pattern we have,

$$v_f(0) - 1 = v_f(1) = \frac{t(n-1)}{2}$$
$$e_f(0) = e_f(1) - 1 = \frac{tn-1}{2}$$

Case 3: t is odd and n is even.

In order to satisfy the edge condition for an edge product cordial graph it is essential to assign label 0 to $\frac{tn}{2}$ edges out of tn edges. The edges with label 0 will give rise at least $\frac{t(n-1)+3}{2}$ vertices with label 0 and at most $\frac{t(n-1)-1}{2}$ vertices with label 1 out of total $t(n-1)+1$ vertices. Therefore $|v_f(0) - v_f(1)| \geq 2$. Thus the vertex condition for edge product cordial graph is violated.

Therefore, the graph $C_n^{(t)}$ is an edge product cordial graph for t even or t and n are both odd while it is not an edge product cordial graph for t odd and n even. ■

Illustration 2.5. The graph $C_5^{(3)}$ and its edge product cordial labeling is shown in Figure 1.

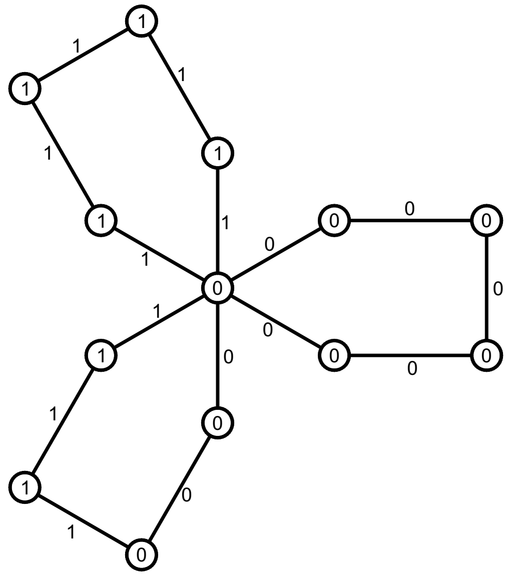

Figure 1: *An edge product cordial labeling of* $C_5^{(3)}$.

3 Concluding Remarks

Cordial and edge product cordial labeling of a graph are two independent concepts and a graph may possess one or both of these labelings or neither as exhibited below.

1. Every tree is cordial as well as edge product cordial.
2. The friendship graph $C_3^{(t)}$ for $t \equiv 2 (\mathrm{mod}\ 4)$ is not cordial but it is edge product cordial.
3. The complete bipartite graph $K_{m,n}$ for $m, n \geq 2$ is cordial but not edge product cordial.
4. The complete graph K_n for $n \geq 4$ is neither cordial nor edge product cordial.

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