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Technical Note

Probability of failure determination for tunnels in rock by using Monte Carlo simulation

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Abstract

Convergence - confinement method can be used to analyze rock support systems in interaction with rock mass in order to select the appropriate supports system for tunnels in rock. This method enables calculation of the charge applied to the support system and the factor of safety for the tunnel supports. The factor of safety calculated from traditional deterministic analysis methods cannot fully represent tunnel stability. There are many uncertainties in parameters used to calculate the factor of safety, and these uncertainties are not integrated in a deterministic analysis, but can be taken into account using a reliability analysis. Reliability analyses, used to calculate probability of failure for tunnel support system, is a complement of the factor of safety calculated by using deterministic analyses. In this paper, Monte Carlo Simulation is used to calculate probability of failure for tunnel support system. This article shows that significant error may be introduced by accepting a distribution for the performance function, which is obligatory for methods like FOSM, FORM, PEM. It is concluded that in cases when the distribution for the performance factor is not fully known, Monte Carlo analysis may give better results.

1. Introduction

Three types of approaches can be used to calculate radial displacement on tunnel walls.

Firstly, the analytic approach, which uses closed form solutions to calculate stress and strain state in tunnel wall. Secondly, numerical methods can be used to calculate stress and strain state in rock mass, some of which are FEM (Finite Element Method), FDM (Finite Difference Method), BEM (Boundary Element Method), etc. Thirdly, by using monitoring instruments which can be placed in the rock during tunnel construction. The obtained data can be used for back analyses and to have a better understanding of rock material near tunnel walls. These three approaches are often combined together in order to give the needed information for stress and strain state around the tunnel excavation.

Convergence – confinement method [1] can use data from approaches one and two in the design phase of tunnel construction, also can use data obtained from back analyses in the construction phase. Using convergence – confinement method, stress and strain in tunnel walls can be calculated and after choosing the support system, a factor of safety can be calculated. The above description of tunnel design can be stated as deterministic method,

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because input data are deterministic, represented only by using mean value of parameters. In this study a probabilistic method is employed, where the uncertainties of the input data are represented by using their standard deviation values. A case study is done for the diversion tunnel in Fan Hydropower plant, in Rrëshen, Albania. The probability of failure for tunnel support system, in this tunnel design case, is determined by probabilistic methods.

2. Convergence – Confinement Method

Convergence – confinement method [1] is used to calculate the factor of safety for tunnel support system. This method can be applied if the conditions listed below are satisfied:

- Circular and relatively deep tunnel (H>3D);
- Rock mass around tunnel is considered homogeneous and isotropic;
- Isotropic initial stress state;
- Uniform and radial supports reaction to the tunnel contour.

Below is given the procedure of convergence – confinement method by using Carranza Torrez and Fairhurst [2] based on Hoek – Brown Generalized Failure Criterion [3] to calculate stress and displacement around a circular tunnel.

i) Equations (11) and (13) are used, by giving values to p_i from $p_i = p_o$ to $p_i = 0$.

Equation (11) is used for pi > pcr and equation (13) for pi < pcr. From the given equations, Ground Reaction Curve (GRC) can be obtained to be used in the convergence – confinement method (Fig. 1.).

$$m_b = m_i \cdot e^{\left(\frac{GSI-100}{28-14D}\right)} \tag{1}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \tag{2}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \tag{3}$$

$$P_i = \frac{p_i}{m_b \sigma_{ci}} + \frac{s}{m_b^2} \tag{4}$$

$$S_o = \frac{\sigma_o}{m_b \sigma_{ci}} + \frac{s}{m_b^2} \tag{5}$$

$$P_i^{cr} = \frac{1}{16}\left(1 - \sqrt{1 + 16 S_o}\right)^2 \tag{6}$$

$$p_i^{cr} = \left(P_i^{cr} - \frac{s}{m_b^2}\right) m_b \sigma_{ci} \tag{7}$$

For $\sigma_{ci} \le 100MPa$:

$$E_m = 1000\left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)} \tag{8}$$

For $\sigma_{ci} > 100MPa$:

$$E_m = 1000\left(1-\frac{D}{2}\right)\cdot 10^{((GSI-10)/40)} \tag{9}$$

$$G_{rm} = \frac{E_{rm}}{2\left(1+\upsilon\right)} \tag{10}$$

$$u_r^{el} = \left(\frac{\sigma_o - p_i}{2G_{rm}}\right)R \tag{11}$$

$$R_{pl} = R\cdot e^{\left[2\left(\sqrt{P_i^{cr}}-\sqrt{P_i}\right)\right]} \tag{12}$$

$$K_\psi = \frac{1+\sin\psi}{1-\sin\psi} \tag{13}$$

$$\frac{u_r^{pl}}{R}\frac{2G_{rm}}{\sigma_o - p_i^{cr}} = \frac{K_\psi - 1}{K_\psi + 1} + \frac{2}{K_\psi + 1}\left(\frac{R_{pl}}{R}\right)^{K_\psi + 1} +$$

$$+\frac{1-2\nu}{4\left(S_o - P_i^{cr}\right)}\left[ln\left(\frac{R_{pl}}{R}\right)\right]^2 - [\frac{1-2\nu}{K_\psi + 1}\frac{\sqrt{P_i^{cr}}}{S_o - P_i^{cr}} + \tag{14}$$

$$+\frac{1-\nu}{2}\frac{K_\psi - 1}{\left(K_\psi + 1\right)^2}\frac{1}{S_o - P_i^{cr}}][\left(K_\psi + 1\right)ln\left(\frac{R_{pl}}{R}\right) - \left(\frac{R_{pl}}{R}\right)^{K_\psi + 1} + 1]$$

Symbols in equations were defined in Nomenclature.

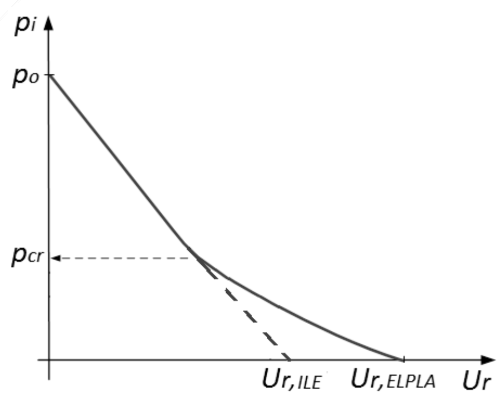

Fig. 1. Ground reaction curve (GRC) for ILE (linear elastic) and ELPLA(elasto – plastic) rock mass. (adapted from M. Barla [4])

ii) Choosing supports system type and plotting Support Reaction Curve (SRC)

Supports system type is chosen based on the elastic stiffness Ks and the maximum limit pressure p_{lim}. Distance x, is the distance from tunnel face at which support system will be installed. This distance can be chosen using 1% of contour displacement that occurs prior to support system installation [5].

According to Vlachopoulos and Diederichs [6], radial displacement can be calculated at distance x from tunnel face.

Radial displacement at tunnel face:

$$u_r(0) = \left(\frac{u_r(\infty)}{3}\right)e^{-0.15\left(\frac{R_{pl}}{a}\right)} \tag{15}$$

Radial displacement in front of tunnel face (x<0):

$$u_r(i) = u_r(0)e^{\left(\frac{x}{a}\right)} \tag{16}$$

Radial displacement behind tunnel face (x>0):

$$u_r(i) = u_r(\infty)\left[1 - \left(1 - \frac{u_r(0)}{u_r(\infty)}\right)e^{\left(\frac{-3x}{a}\right)/\left(\frac{2R_{pl}}{a}\right)}\right] \tag{17}$$

After performing the above calculations, it is possible to construct SRC (Fig. 2):

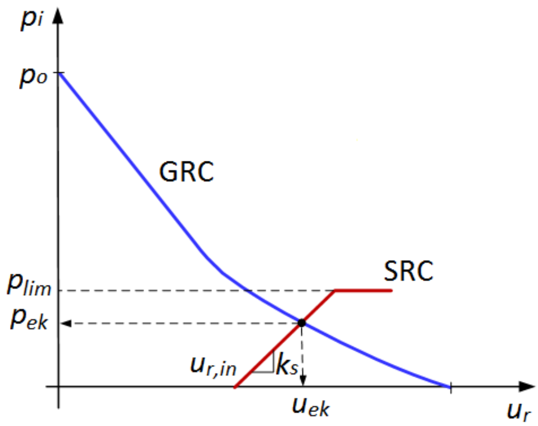

Fig. 2. Ground reaction curve (GRC) and Support Reaction Curve (SRC), (adapted from M. Barla [4])

iii) Factor of safety is calculated by using:

$$Fs = \frac{p_{lim}}{p_{ek}} > 1 \tag{18}$$

3. Probability of Failure of Tunnel Support System Using Monte Carlo Simulation

Monte Carlo Simulation [7] is used to calculate Probability of Failure for tunnel support system. Monte Carlo Simulation uses random generated values to obtain the series of values needed for calculation. In this study, random values are generated by using Normal Distribution. Minimum number of iterations needed for performing Monte Carlo simulation, have been calculated by accepting a confidence level of 95% and an error bound of 1%.

Random values have been generated using Excel software. By performing Monte Carlo simulations, mean value and standard deviation for Factor of Safety have been calculated.

Reliability index is calculated by using:

$$\beta = \frac{\mu_{FS} - 1}{\sigma_{FS}} \tag{19}$$

Where:
μ_{FS} - mean value of Safety Factor
σ_{FS} - standard deviation of Safety Factor
β - Reliability index.
However, probability of Failure can be calculated directly, without using reliability index, based on the cumulative distribution function which is constructed after the data determined by using Monte Carlo Simulation.

4. Case Study. Diversion Tunnel in Rrëshen Hydropower Plant, Albania

Hydropower plant of Rrëshen is built in Fan River, in Rrëshen, Albania. Diversion tunnel has a length of 420m and a diameter of 9.1 m. More details are given in Fig.s 3 and 4.

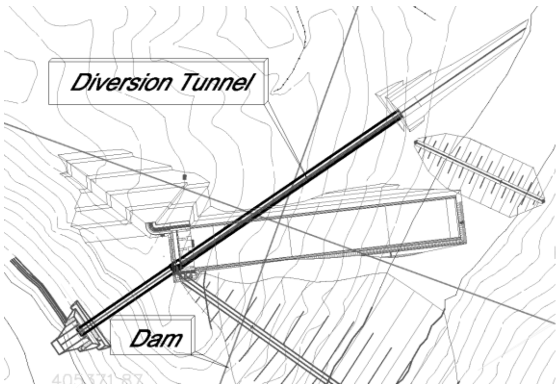

Fig. 3. Rrëshen diversion tunnel. (Fan River Hydro Power Project, published with consent of Aydiner Construction Co, Lezhe, Albania)

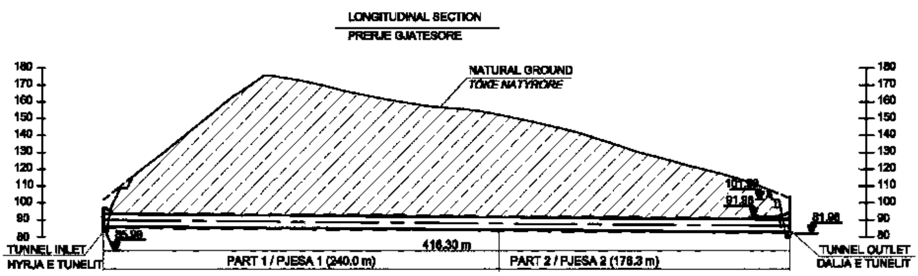

Fig. 4. Longitudinal profile of Rrëshen diversion tunnel. (Fan River Hydropower Project, published with consent of Aydiner Construction Co, Lezhe, Albania)

Tunnel excavation was performed by using explosive charge, and face advance was between 0.5 and 2.5 m, depending on the type of rock. For every face advancement (in total 282 advances), a face sketch has been drawn and data presented for water, type of rock, joint number, joint alteration etc. By using this data, RMR has been calculated (Bieniawski), Q (Barton), GSI (Hoek & Brown). In Fig. 5 one of those tunnel face sketches at 195.6 m is illustrated.

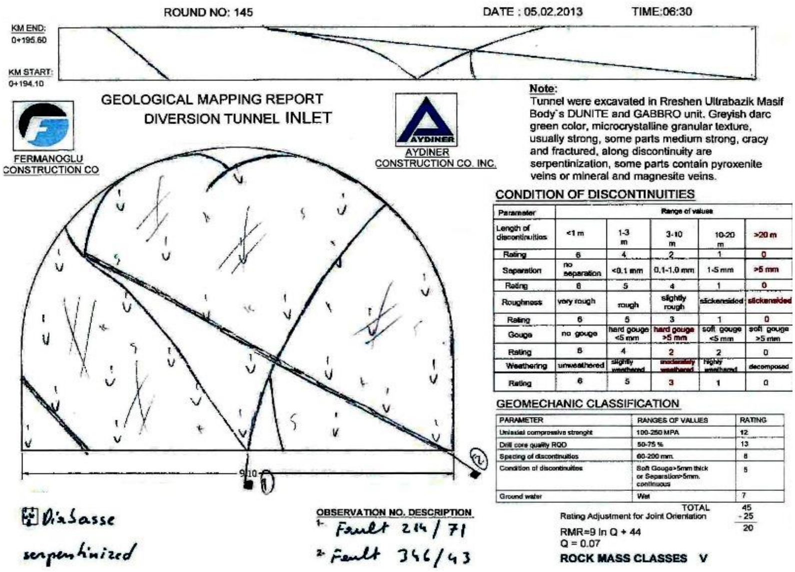

Fig. 5. Tunnel face sketch at 195.60 m. (Fan River Hydro Power Project, published with consent of Aydiner Construction Co, Lezhe, Albania)

By using boreholes near tunnel axis, rock samples have been taken and by laboratory tests, the Uniaxial Compression Strength (UCS) is obtained. Based on the rock description by the geological engineer in site, the value of intact rock mi is approximated to be used in Hoek – Brown failure criterion [3].

For 282 tunnel face advances, 282 values of GSI have been calculated, from which mean value and standard deviation are evaluated. Taking into account that survey is conducted by two independent groups, dividing the tunnel in two parts, only 173 values obtained from the second group are taken into account in this study. By using these 173 values, mean and the standard deviation values have been calculated. The same calculations are performed for UCS. 13 values of UCS were obtained from laboratory tests, from samples collected near tunnel axis from boreholes. Using Marinos & Hoek [8] table, value of mi has been chosen for gabbro and diabase and is 15 with standard deviation of 3. Table 1 gives a summary of the data collected. Blast damage factor D is taken zero, because tunnel blast will be controlled and the rock can be assumed undisturbed.

Table 1. Mean values and standard deviation for UCS, GSI and mi

	Mean Value (MPa)	Standard Deviation
UCS	46.62	10.60
GSI	26.24	7.09
mi	15.00	3.00

A sensitivity analysis is performed for the above parameters in order to simplify the calculations by taking some of the parameters as deterministic. For this, Sensitivity Index (SI) is calculated by using equation (18). Sensitivity index is calculated for Plastic Radius, Radial Displacement and Young Modulus.

Sensitivity index for plastic radius:

$$SI_{Rp} = \frac{Rp_{max} - Rp_{min}}{Rp_{max}} \tag{18}$$

By using the 3σ rule, R_{pmax} is calculated from E[UCS] + 3*σ_{UCS} and R_{pmin} , E[UCS] - 3*σ_{UCS}.

The above calculation is for the sensitivity index of changing UCS.

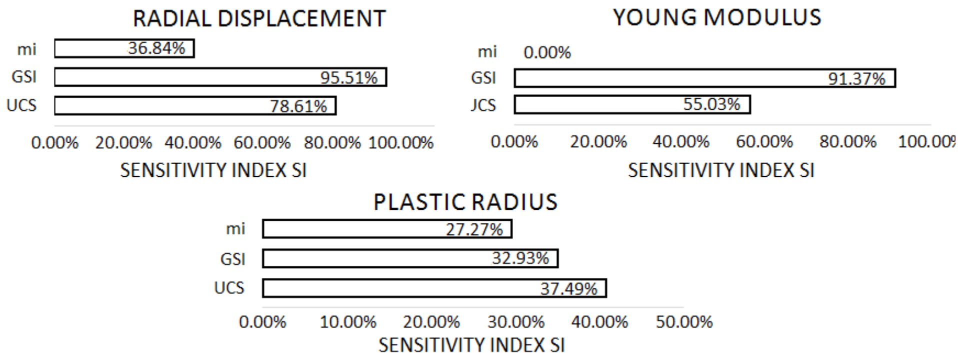

Fig. 6. Results of sensitivity analysis

By using sensitivity analysis, it is obtained that all three parameters influence Radial displacement and plastic radius. Further calculations are continued with all three parameters as uncertain. Tunnel supports are steel ribs and shotcrete. Table 3 gives data for tunnel supports.

Table 3. Data needed to construct Support Reaction Curve (SRC)

Supports install distance from tunnel face	x = 0.5 m
Steel profile IPN 280 with area	A = 0.0061m^2
Distance between steel profiles	i = 0.5 m
Shotcrete thickness	d = 0.15 m
Young modulus for concrete	Ec = 2.5 * 10^7 kPa
Young modulus for steel	Es = 2.1 * 10^8 kPa
Steel yielding stress	fy = 5.4 * 10^5 kPa
Tunnel internal radius	R = 4.55 m

$$\frac{1}{K_c} = \frac{i \cdot R}{E_a \cdot A} + \frac{d}{E_c \cdot R} = 1.7773{*}10\text{-}6 \rightarrow Kc = 562659 \text{ kPa} = 562.659 \text{ MPa}$$

$$p_{lim} = \frac{A \cdot \sigma_a}{i \cdot R} = 1447.91 \text{ kPa} = 1.448 \text{ MPa}$$

Radial displacement at the moment of support installation has been calculated by using Vlachopoulos and Diederichs formulas [6]. Ground Reaction Curve (GRC) has been constructed by using Carranza – Torres and Fairhust method [2] which uses Hoek – Brown Generalized Failure Criterion [3].

Calculations by using mean values (deterministic method), gives the following results:
Ur (x = 0.5 m) = 9.78 mm and FS = 1.072 > 1

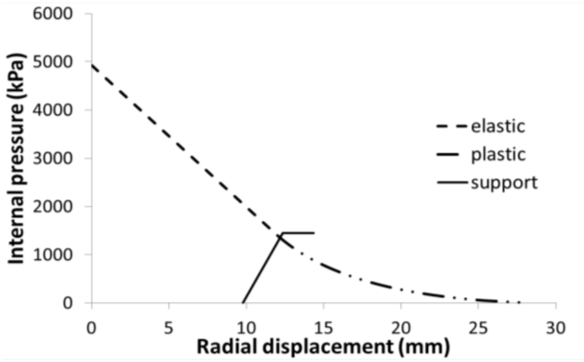

Fig. 7. Ground reaction curve (GRC) and Support Reaction Curve (SRC).

Minimum number of iterations needed in Monte Carlo simulation, has been established for a Confidence Level of 95%, and an error bound of 1% [9],[10] resulting in needed 1055 iterations.

Performing Monte Carlo Simulations with 2000 iterations gives the result below:

$$\mu_{FS} = 1.2136$$
$$\sigma_{FS} = 0.2010$$
$$\beta = 1.0626$$

Cumulative distribution function (CDF) for the Safety Factor is given in Fig.8. Calculated value for probability of failure is pf = 9.45 %

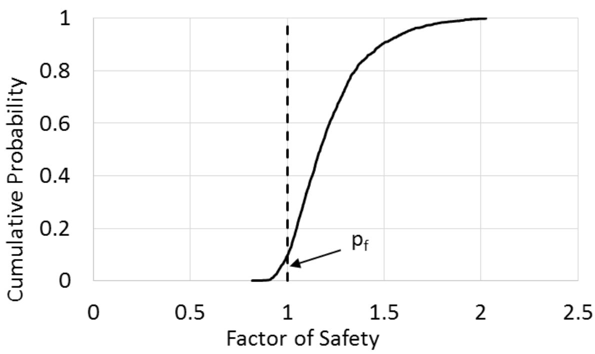

Fig. 8. Cumulative distribution function for factor of safety

By using Monte Carlo Simulation to determine Probability of Failure, there is no need to use Reliability Index, also is not needed to accept a distribution for the performance function for the Factor of Safety. Other reliability methods (FOSM, PEM, FORM), have to accept a distribution type for the performance function, for example Normal Distribution, and from that distribution and the reliability index, probability of failure is calculated.

Calculations below show the difference from the "exact" probability of failure and the probability of failure calculated if a distribution is accepted for the performance function. If distribution for Safety Factor is accepted as Normal, probability of failure calculated is pf=14.40%, a value 52.4% bigger than the "exact" solution. Fig. 9 gives a comparison between PDF and CDF constructed from data collected from Monte Carlo Simulation, with PDF and CDF for Normal Distribution.

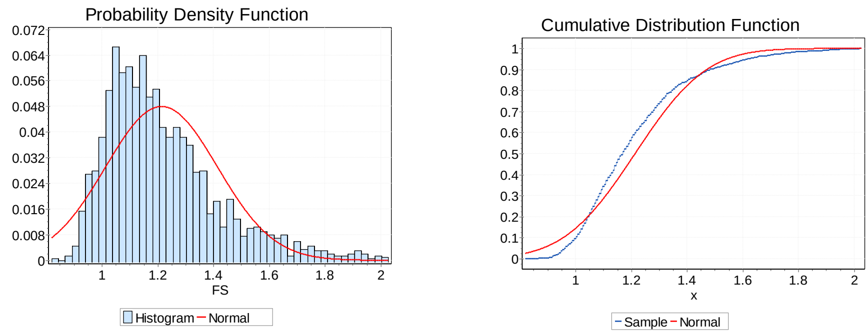

Fig. 9. PDF and CDF obtained from data and by using Normal distribution for Factor of Safety

The distribution that best approximates data collected by using Monte Carlo Simulation is Wakeby Distribution (Fig. 10)

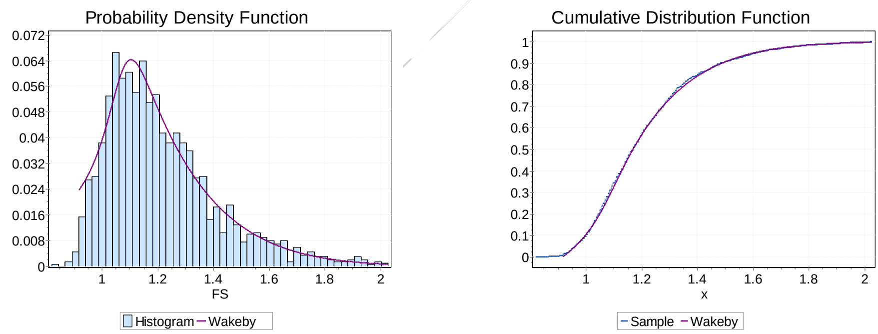

Fig. 10. PDF and CDF obtained from data and by using Wake by distribution for Factor of Safety

Probability of Failure calculated by using Wakeby Distribution is pf = 10.35%, a value 9.5% larger that the "exact" solution. Fig. 11 gives a comparison between PDF a CDF of data obtained from Monte Carlo Simulation by using LogNormal Distribution. Probability of failure obtained by using LogNormal Distribution is pf = 12.24%, a value 29.5% bigger than the "exact" solution.

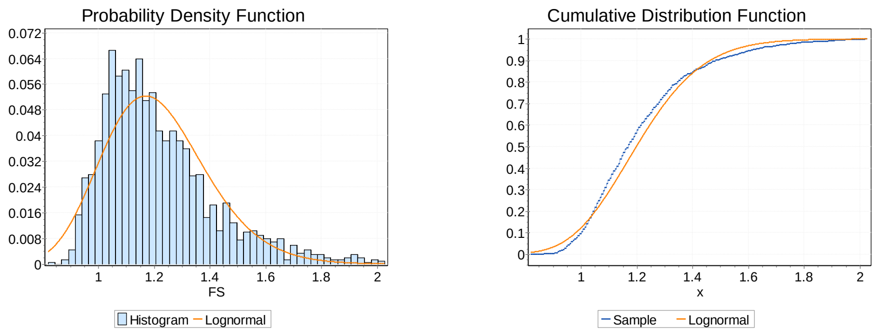

Fig. 11. PDF and CDF obtained from data and by using LogNormal distribution for Factor of Safety

4. Summary and Conclusions

The deterministic factor of safety is generally insufficient for characterizing the stability of underground support structures. Probabilistic calculations offer an advantage especially if sufficient data on the quality of the rock mass are available. This paper gives a procedure to calculate the probability of failure using Monte Carlo Simulation.

A sensitivity analysis was performed in order to identify the main parameters influencing the support design. In this case, three parameters were taken into account in the sensitivity analysis (UCS, GSI, mi). It was concluded that all three parameters influence Radial displacement and plastic radius. Further calculations were continued with all three parameters as uncertain.

The proposed approach was illustrated with the example of diversion tunnel in Rrëshen dam site. The calculations were performed by using steel ribs and shotcrete as primary support. The distribution law of input variables was expressed by their mean values and standard deviation, both calculated from data collected in field works in Rrëshen tunnel site and ex-situ tests conducted. The factor of safety and the probability of failure for the tunnel supports have been calculated by using the Monte Carlo technique.

After performing the calculations, it was observed that by accepting the distribution for the performance function (in our case the factor of safety), leads to errors in the value of the probability of failure, compared with the “exact” value taken directly from Monte Carlo Simulations. For example, the Normal and LogNormal distributions give errors as 12.24% and 52.40%, respectively.

Monte Carlo Simulation is the only stochastic method which does not require assuming a distribution for the performance function. Analysts have to take into account the error obtained by accepting a distribution, which is obligatory if methods like FOSM, FORM, PEM are used.

Nomenclature

m_b - reduced value of material constant which depends on m_i, GSI and D.

m_i - material constant, depends on rock type.

s – constant for rock mass, which is calculated by using equation (2) for GSI ≥ 25, and s = 0 for GSI<25.

a – constant for rock mass, which is calculated by using equation (3) for GSI<25, and a = 0.5 for GSI ≥ 25.

D – is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses.

GSI – Geological Strength Index

P_i - Scaled internal Pressure

p_i - Uniform Internal Pressure (kPa)

σ_{ci} - Unconfined compressive strength of intact rock core specimens (kPa)

σ_o - Rock mass initial stress (kPa)

S_o - Scaled rock mass initial stress

p_i^{cr} - Critical internal pressure (kPa)

P_i^{cr} - Scaled critical internal pressure

u_r^{el} - Radial elastic displacement

u_r^{pl} - Radial plastic displacement

G_{rm} - Shear modulus for rock mass

E_{rm} - Young's modulus for rock mass

ν - Poisson's ratio

R - Tunnel radius

R_p - Tunnel plastic radius

K_ψ - Dilation coefficient $(K_\psi = 1\ for\ \psi = 0\ and\ K_\psi = 3\ for\ \psi = 30)$

p_{cr} - critical internal pressure

$U_{r,ILE}$ - inward radial elastic displacement

$U_{r,ELPLA}$ - inward radial elasto – plastic displacement

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