UDC 678.029.43:[621.791.042+678.073]

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IMPROVEMENT OF THE TECHNOLOGY OF THERMAL GLUING BY A MELT OF POLYMER ADDITIVE MATERIAL

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The paper reports the advanced technology of welding of various materials, components and assembly units by using polymer melt and appropriate polymer additive material in the shape of a tube (a hollow core). Ethylene copolymer with vinyl-acetic ester was used as a polymer additive material. The mathematical model of the process of melting was developed both for the offered tubular polymer additive material and for the traditional one (in the shape of a continuous rod). The dependence of the temperature field by the radius and length of an additive material during its movement in the welding device was investigated. The effectiveness of the developed technology, design of the device and the polymer additive material was shown. At the same time, working speed in the case the use of tubular additive material and an advanced design of the welding device is almost twice the working speed of the traditional rod additive material. The calculations showed that the force of pushing an additive material of a traditional form for the feeding speed of 6 mm/s and a tubular shape using a mandrel for the feeding speed of 15 mm/s is almost identical. This can be explained by the faster heating of the additive material of the proposed shape.

Keywords: thermal gluing, additive material, polymer, extrusion, melting.

Introduction

The bonding of elements of various products and designs by means of polymer additive materials is widely used when processing the most various materials, such as polymers, cardboard, paper, ceramics, fabric, wood, etc. Due to relative simplicity, high efficiency, ample technological capabilities, lack of solvents, resistance to the effect of water and weak acids, and also high quality this method of receiving one-piece connections can be often considered as the most preferable (for example, in comparison with pasting). At the same time, this type of bonding is most often used in the production of packing from the polymeric and combined materials and also when packing in it various production [1].

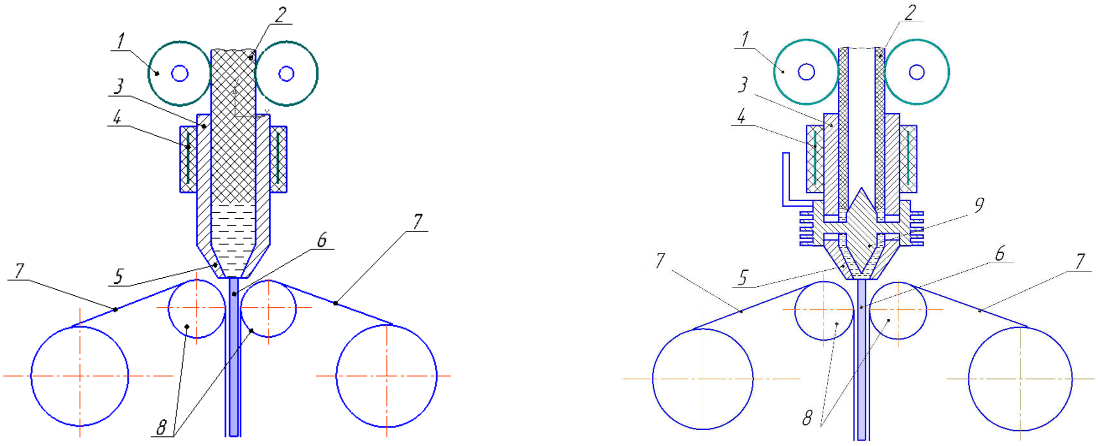
The technology of connection of various elements is based by the fusion of a polymer additive material using the heat of the melted polymer which moves between the connected surfaces of the elements. In the case of connection of elements made of thermoplastic polymers, the fusion of additive material transfers a part of the heat to the connected elements, partially melts them and gradually strengthens them; as a result, one-piece welded connection is formed. In the case of the connection between the parts made of infusible or hardly fusible elements, the connection is formed by a one-piece solder (or a glue). For this reason, polymer additive material is quite often called «thermal glue». At the same time, an additive material should be heated to temperature of 40–80^{0}C above the fluidity temperature of melt polymer to ensure reliable one-piece connection. Further process of the formation of one-piece connection by the specified method will be called «welding» for convenience.

The purpose of this work was to develop the mathematical model of the melting process of a polymer additive material for compounds of various materials (first of all, flexible packing materials), and also increase the efficiency of this process.

Experimental

Physical and mathematical models of the melting process an additive polymer material

During welding by a melted polymer additive material in the shape of a rod (stick), the melt comes to a welding zone as a result of the heating and melting of this rod in direct-flow type devices and its further expression through a nozzle (Fig. 1,a).



a **b**

Fig. 1. The schemes of the device of welding with melt: (a) traditional, (b) developed in this work. 1 – feeding mechanism; 2 – polymer rod; 3 – the channel for melting of a polymeric rod; 4 – heating block; 5 – nozzle; 6 – polymer melt; 7 – rolls of the connected material; 8 – clamping rollers; 9 – mandrel

In most of technological processes, the raw materials are used in the shape of granules [2] or a crumb and flakes [3]. The most widespread type of the equipment for the treatment of polymeric materials is the equipment on the basis of screw extruders [4,5], in which the process of melting is rather well investigated [6,7]. However, devices for the 3D press, in which the scheme of polymer is similar to those in melting processes by polymer melt welding, are widely adopted recently (Fig. 1,a). Some analytical [8] and numerical [9] models of the melting of polymeric rods in the cylindrical channel with a nozzle have been developed. However, the available reports do not consider a number of factors, in particular the temperature dependence of coefficient of friction of polymer on a wall of the channel [10].

Scaling of melting process which happens in an extruder of 3D printers in relation to the devices of the preparation of fusion for welding not always is effective, since the diameter of a polymeric rod in the latter case is several times as much (owing to a significant decrease in the intensity of the process of heat transfer and a considerable increase in the temperature gradient along the radius).

We offer a new scheme of the device of welding by fusion (Fig. 1,b), main distinctive features of which are the following:

– one-sided heating of the rod is replaced on two-sided heating (from the outside and from within) that increases the heating process and reduces the temperature gradient along the radius of the channel;

– axial movement of a mandrel in the channel and, respectively, the regulation of section of an the nozzle opening and volume expense of melt through the nozzle is provided.

The rod of polymer additive material moves through the feeder to the channel of cylindrical cross section where it is heated under the influence of the heat from the heating block and further begins to melt. The formed polymer melt belongs to pseudoplastic liquids and has the properties of incompressible non-Newtonian liquid. The mode of the melt movement in the channel and nozzle is laminar. At the same time, the sticking condition is realized on a wall of the channel. When passing between the heated walls of the channel, the intensity of the melting of the rod increases owing to its dissipative friction on the channel walls. The coefficient of friction of polymer on the wall is a function of temperature and viscosity, while the thermal-physical properties of polymer are functions of shear speed and temperature. The cylindrical channel comes to an end with the conic nozzle in the shape of a confusor. The feeding speed of polymer additive material is defined by the minimum temperature to which the melt of polymer at the exit from a nozzle should be heated to ensure required adhesive properties and specified fluidity. At the same time, the gravitational forces are not considered as their influence on the movement of fusion is insignificant.

According to the formulated physical model of an polymer additive material, the mathematical model of the rod melting process in the case of nonisothermal movement of polymer melt in the channel can be presented by the system of the equations which includes the continuity equation, the nonstationary equations of preservation of number of the movement and energy as follows [11]:

$$\begin{cases} \nabla\cdot v = 0 \\ \rho\left[\dfrac{\partial v}{\partial t}+\left(v\cdot\nabla\right)v\right]=-\nabla p+\nabla\cdot\overset{=}{\tau} \\ \rho\left[\dfrac{\partial v}{\partial t}+\nabla\cdot\left(vh\right)\right]=\nabla\cdot\left[\lambda\left(T\right)\nabla T\right]+\overset{=}{\tau}:\nabla v, \end{cases}, \quad (1)$$

where ∇ is the Hamilton operator, m^{-1}; v is the vector of speed, m/s; t is the time, s; ρ is the density, kg/m^3; p is the external hydrostatic pressure, Pa; $\overset{=}{\tau}=2\eta(\dot{\gamma})\dot{D}$ is the tensor of viscous tension of the second rank, Pa; $\dot{D}=\frac{1}{2}(\nabla v+v\nabla)$ is the deformation speed tensor, s^{-1}; $\eta(\dot{\gamma})$ is the viscosity of liquid as a function of the second invariant of speed of shift $\dot{\gamma}$ from $\dot{D}$, Pa·s; $\dot{\gamma}=\sqrt{\frac{1}{2}\dot{D}:\dot{D}}$ is the second invariantfrom $\dot{D}$, s^{-1}; $h=\int_0^T c_p(T)dT$ is the obvious mass enthalpy, J/kg; c_p is the mass isobaric thermal capacity, J/(kg·K); λ is the coefficient of heat conductivity, W/(m·K); $\overset{=}{\tau}:\nabla v$ is the term correspondings to the dissipation of mechanical energy, W/m^3; T is the absolute temperature, K; (:) is the operator of a double scalar product.

The power law which takes into account the temperature dependence of viscosity for various classes of liquids has the following form:

$$\eta(\dot{\gamma},T)=K\left(\xi\dot{\gamma}\right)^{n-1}H(T), \quad (2)$$

where K is the size of average viscosity of liquid, Pa·s; ξ is the relaxation period, s; n is the exponent defining a liquid class (according to physical model n<1 as the material belongs to pseudo-plastic liquids); $H(T)=\exp\left[\frac{E_a}{RT_a}\left(\frac{1}{T-T_0}\right)-\left(\frac{1}{T_a-T_0}\right)\right]$ is the activation energy of a current, J/mol; R is the universal gas constant, J/(mol·K); E_a is the absolute activation energy, J/(mol·K); T_0 is the absolute temperature of counting, K.

The distribution of fields a component of a vector of speed v_0, pressure p_0 and temperature T_0 in a point of time t=0 can be taken as initial conditions of the system (1):

$$\begin{cases} v_0=v\left(x,y,z\right); \\ p_0=p\left(x,y,z\right); \\ T_0=T\left(x,y,z\right); \end{cases} \quad (3)$$

where $(x,y,z)\in\Omega$ are Cartesian coordinates, m; Ω is the settlement area.

The boundary conditions for system of the equations (1) are defined as follows:

– the set of temperatures and normal of components of speed or mass consumption of material at the entrance to the channel:

$$\begin{cases} n\cdot v=v_{inlet}(t); \\ T=T_{inlet}(t); \end{cases} \vee \begin{cases} G=G_{inlet}(t); \\ T=T_{inlet}(t); \end{cases}, \quad (4)$$

where n is the vector of an external normal to a surface of entrance section of the channel; v_{inlet}, G_{inlet}, T_{inlet} are the speed (m/s), mass expense (kg/s) and absolute temperature (K) in the entrance section of the channel, respectively; $\vee$ is the logical sign «or»;

– the set of zero gradients of pressures and temperatures at the exit from the channel:

$$\begin{cases} n\cdot\nabla p=0; \\ n\cdot\nabla T=0; \end{cases} \quad (5)$$

– the wall shear tensions in the form of the generalized Navier law (the boundary conditions of Navier [12] which represent the equation of balance of forces, operating on a contact surface between two environments) on the surfaces of contact of melt with the walls of the channel, depending on temperature:

$$F_{slip}(T)\left(v_w-v_t\right)\cdot t+\left(\overset{=}{\tau}\cdot n\right)\cdot t=0\to$$
$$\to F_{slip}(T)\left(v_{wt}-v_{tt}\right)+\tau_{w\,sh\,str}\leftrightarrow\tau_{w\,sh\,str}=$$
$$=-F_{slip}(T)\left(v_{wt}-v_{tt}\right)\text{ at }T<T_{fluid}, \quad (6)$$

– or the sticking conditions:

$$V=0 \text{ at } T\geq T_{fluid}, \quad (7)$$

where $\tau_{w\,sh\,str.}=\left(\overset{=}{\tau}\cdot n\right)$ is the component of tangential tension in a wall layer, Pa; n and t are the single normal and tangential vectors to a surface of the channel, respectively; v_w, v_t are the vectors of a resultant and tangential speed on a surface of the channel (in a wall layer), respectively, m/s; v_{wt}, v_{tt}

are the tangential components of vectors of a resultant and tangential speed on a surface of the channel (in a wall layer), respectively, m/s; F_{slip} is the coefficient of sliding (friction) on a surface of the channel, kg/(m^2·s); T_{fluid} is the temperature of the fluidity (melting) of polymer, K;

– the conditions of convective heat exchange (Newton's law) on an external surface of the channel contacting to air:

$$n \cdot q = \alpha (T - T_\infty), \tag{8}$$

where α is the heat transfer coefficient on a surface of the channel which is defined experimentally, W/(m^2·K); T_∞ is the absolute ambient temperature, K; q is the vector of density of a thermal stream, W/m^2.

Verification of the developed mathematical model

To numerically solve the mathematical model (1)–(8), we used free program code OpenFOAM [13] which is intended for performing CFD-Calculations of both the Newtonian and non-Newtonian liquids and is constructed by a numerical finite volume method.

The sampling of geometrical model of the rod of additive material was carried out with the application of tetrahedron elements. To generate and validate a tetrahedron mesh standard utilities of OpenFOAM-blockMesh, we used checkMesh or free program code for the automated generation of a mesh of Gmsh. The parameters of the design mesh (quantity of design cells and nodes) during research of net convergence were determined on the basis of a method of double recalculation [14].

The sampling of the calculation area of the rod, got on the basis of double re-calculations, was used in numeral experiments. This sampling included 3235 and 3559 calculation cells, and also 1062 and 1168 nodes for traditional and modernized filler rods, respectively.

To visualize the results of the calculations, a free program code ParaView [15] was used.

The adequacy of the developed mathematical model was checked by comparison of the calculated and measured temperature of polymer melt at the exit from a nozzle. The temperature was measured by ThermoPoint TPT pyrometer 62 (Agema Infrared Systems AB, Sweden). The feeding speed of the polymer additive material rod was measured by means of the stopwatch with tags placed on rod surfaces through each millimeter.

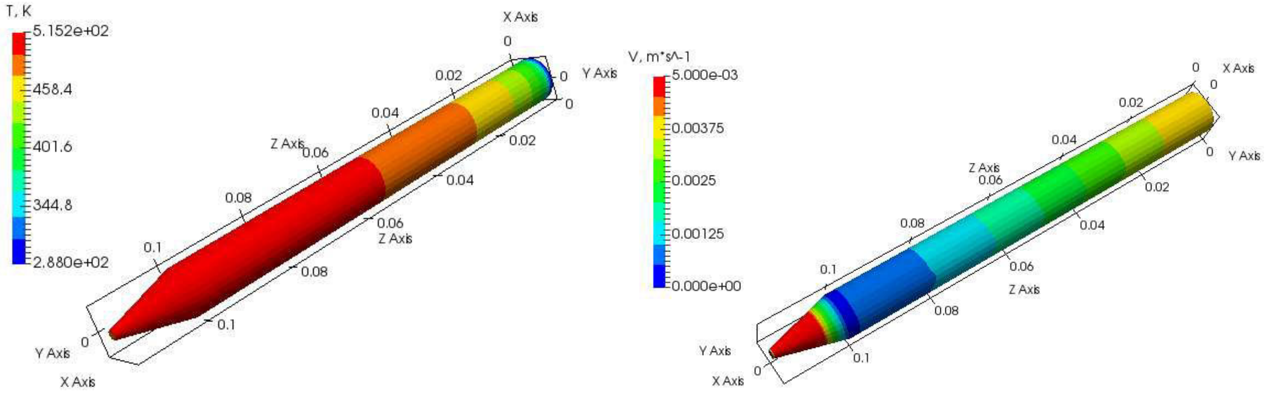
The divergence of the calculated and measured temperatures of polymer melt at the exit from a nozzle did not exceed 9% for all performed experiments.

Results and discussion

Further we give the results of numerical modeling of the melting process with polymer additive material according to the developed model.

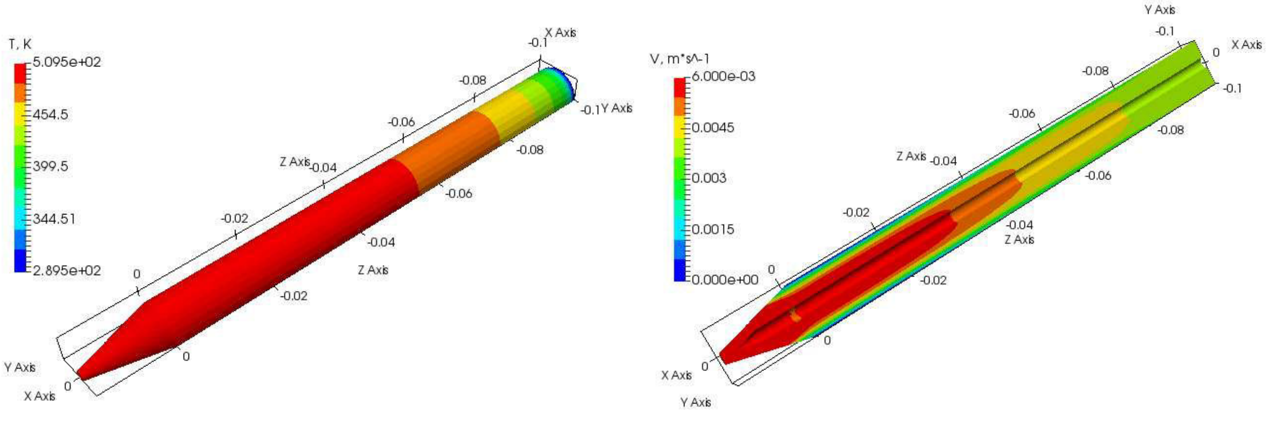
Figure 2 represents the distribution of temperature Fig. 2,a and speed Fig. 2,b in a wall of the cylindrical heating channel with a diameter of 11 mm and the nozzle with a diameter of 2 mm fixed at the exit from the channel, the entrance speed of a continuous polymeric rod being 4 mm/s. Ethylene copolymer with vinyl-acetic ester was used as polymer additive material.

It is visible in Fig. 2 that the surface temperature reaches the temperature of fluidity (390 K) at the distance of 30 mm from the beginning of the heating channel (at the speed of rod of 4 mm/s). At the same time, braking of polymer on the surface of the channel occurs gradually with increasing temperature, and almost full sticking of polymer is observed at the distance of 80 mm from the beginning of the heating channel.



a b

Fig. 2. The distribution of temperature (a) and speed (b) on the surface of additive material at feeding speed of a rod of 4 mm/s (the rod in the shape of a traditional continuous cylinder)



a

b

Fig. 3. The distribution of temperature on the surfaces of additive material (a) and speed of polymer material (b) at a feeding speed of a rod of 4 mm/s (the rod in the shape of the offered cylindrical tube)

Numerical experiments were made for various feeding speed of the additive material rod before achievement by temperature polymer at the exit, equal temperature of its fluidity. Expedient speed of feeding is such at which polymer temperature at the exit is equal recommended by the producer of additive material (for the studied material this temperature is equal to 450 K).

Also a series of numerical experiments for the offered tubular configuration of additive material and a nozzle with mandrel was carried out at the following basic data: external diameter of a tubular additive rod is equal to 11 mm, its internal diameter is equal to 3 mm, diameter of a nozzle is equal to 2 mm, a corner of conicity of the nozzle is equal to 15^0, the feeding (entrance) speed is equal to 3–15 mm/s (Fig. 3).

From Fig. 3,b it is visible that polymeric material on the wall of the heating channel heats up to temperature of fluidity (390 K) at distance of 20 mm from the beginning of the heating channel. At the same time braking of polymer on a surface of the channel happens gradually to temperature increase, and almost full sticking of polymer is observed at distance of 60 mm from the beginning of the heating channel.

The dependence of temperature along an axis of polymer additive material on the feeding speed is given in Fig. 4. It is seen that the feeding speed limit is 7 mm/s and the expedient feeding speed limit is 4 mm/s in the case of a traditional scheme (a continuous rod). At the same time, the speed limit for melting of an additive material of a tubular shape with application of the mandrel in the channel of the device is 15 mm/s, and the expedient feeding speed limit is 6 mm/s, which significantly exceeds the speed for a continuous rod and the device of the known design. Obviously, such a decrease in the temperature dependence of speed can be explained not only by the difference in shapes, but also by an increase in the effect of dissipation in the offered design (provided by the mandrel). The calculations showed that the forces of pushing additive material are almost identical for a traditional shape at the feeding speed of 6 mm/s and for a tubular form with application of the mandrel at the feeding speed of 15 mm/s. This testifies the expediency of the proposed design.

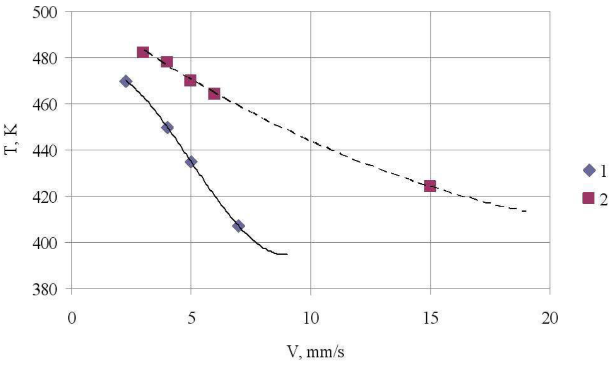


Fig. 4. Plot of temperature along the longitudinal axis of the rod vs. feeding speed: 1 – a continuous rod; 2 – a tubular rod

Conclusions

The mathematical model is developed for the process of melting of an additive polymer material with the corresponding initial and boundary conditions.

Numerical experiment allowed establishing the interdependence between the feeding speed and the temperature of additive polymer material at the nozzle outlet for various configurations of a rod taking into account dissipative effects. The dependence of rheological and thermal-physical properties of an additive material on temperature was determined too.

It was shown that the expedient feeding speed for the tubular additive material and a nozzle with the mandrel is almost twice the feeding speed of traditional continuous additive material. Thus, double-sided heating of an additive material is more effective than one-sided heating (from an external surface of additive material). At the same time, the force of pushing the additive material is almost identical for both schemes.

The received results will allow improving the equipment for the welding of elements of flexible packing by polymer additive materials.

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Received 12.06.2018

УДОСКОНАЛЕННЯ ТЕХНОЛОГІЇ ТЕРМОСКЛЕЮВАННЯ РОЗПЛАВОМ ПОЛІМЕРНОГО ПРИСАДКОВОГО МАТЕРІАЛУ

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У статті запропоновано удосконалену технологію зварювання різноманітних матеріалів, деталей і складальних одиниць розплавом полімеру, а також відповідний полімерний присадковий матеріал у вигляді трубки. Як присадковий матеріал було застосовано співполімер етилену с вінілацетатом (СЕВА). Розроблено математичну модель процесу плавлення як запропонованого трубчастого полімерного присадкового матеріалу, так і традиційного у вигляді суцільного стрижня. Досліджено залежність температурного поля з радіусу й довжини присадкового матеріалу в процесі його руху в пристрої для зварювання. Показано ефективність запропонованих удосконалень технологічного процесу, конструкції пристрою та полімерного присадкового матеріалу. При цьому робоча швидкість для трубчастого присадкового матеріалу й удосконаленої конструкції зварювального пристрою майже вдвічі перевищує робочу швидкість для традиційного стрижневого присадкового матеріалу. Крім того, як свідчать розрахунки, зусилля проштовхування присадкового матеріалу традиційної форми для швидкості подачі 6 мм/с і трубчастої форми із застосуванням дорна для швидкості подачі 15 мм/с є майже однаковим. Це може бути пояснено більш швидким прогріванням присадкового матеріалу пропонованої форми.

Ключові слова: термосклеювання, присадковий матеріал, полімер, екструзія, плавлення.

IMPROVEMENT OF THE TECHNOLOGY OF THERMAL GLUING BY A MELT OF POLYMER ADDITIVE MATERIAL

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Keywords: thermal gluing; additive material; polymer; extrusion; melting.

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