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Optimal Routing and Energy Allocation for Lifetime Maximization of Wireless Sensor Networks With Nonideal Batteries

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Abstract:

An optimal control advance is used to solve the problem of routing in sensor networks where the goal is to maximize the network's lifetime. In our analysis, the energy sources (batteries) at nodes are not assumed to be "ideal" but quite behaving according to a dynamic energy use model, which captures the nonlinear behavior of actual batteries. We show that in a permanent topology case there exists an optimal policy consisting of time-invariant routing probabilities, which may be obtained by solving a set of relatively simple nonlinear programming (NLP) problems. We also show that this optimal policy is, under very mild conditions, robust with admiration to the battery model used. Further, we consider a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective. We show that the solution to this problem is given by a policy that depletes all node energies at the same time and that the Equivalent energy allocation and routing probabilities are obtained by solving an NLP problem. Numerical examples are included to display the optimality of the time-invariant policy and its robustness with admiration to the battery model used.

Key words—**Optimal control, power-imperfect system, routing, sensor network.**

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I. INTRODUCTION

A WIRELESS SENSOR NETWORK (WSN) is a spatially spread wireless network consisting of low-cost independent nodes, which are mainly battery powered and have sensing and wireless message capabilities. Applications of such networks include examination, surveillance, and environmental monitoring. Power use is a key issue in WSNs, since it directly impacts their lifetime in the likely nonattendance of human involvement for most applications of interest. Since the majority of power use is due to the radio component , nodes rely on short-range message and form a multi hop network to deliver in order to a base station. Routing schemes in WSNs aim to deliver data from the data sources (nodes with sensing capabilities) to a data sink in an energy-efficient and consistent way. A survey of the state-of-the-art routing algorithms is provided in [1].

We focus on the problem of routing in a WSN with the objective of optimizing presentation metrics that reproduce the imperfect energy resources of the network while also preventing ordinary security vulnerabilities. Most future outing Protocols in WSNs are based on straight path algorithms. Such algorithms usually need each node to maintain a global cost (or state) in order table, which is a significant burden for resource-constrained WSNs. proposed a multipath routing algorithm, so that a breakdown on the main path can be recovered without initiating a network-wide flooding process for path rediscovery.

On the other hand, judgment multiple paths and sending packets through them also consumes energy, thus adversely impacting the Life time of the network if there are no failures. The routing policies mentioned earlier may indirectly reduce energy usage in WSNs, but they do not explicitly use energy use models to address optimality of a routing policy with admiration to energy-aware metrics. Such “energy awareness” has motivated a number of minimum-energy routing algorithms, which typically seek paths minimizing the energy per packet consumed (or maximizing the residual node energy) to reach a destination. However, seeking a minimum energy path can rapidly deplete energy from some nodes and ultimately reduce the full network’s lifetime by destroying its connectivity. Thus, an alternative presentation metric is the *network lifetime*. The definition of the term “life-time” for WSNs varies. Some researchers, e.g., [7], define the network lifetime as the time until the first node depletes its battery; however, this may just as well be defined as the time until the data source cannot reach the data sink [5]. In what follows, we will adopt the former definition, i.e., the time until the first node depletes its battery. As our results will show, it is often the case that an optimal policy controlling the WSN’s resources leads to individual node lifetimes being the same or almost the same as those of others,

hence this definition is a good characterization of the overall network’s lifetime. Along the lines of energy-aware routing, Shah and Rabaey proposed an Energy Aware Routing (EAR) policy, which does not attempt to use a single optimal path, but quite a number of suboptimal paths that are probabilistically selected with the intent of extending the network lifetime by “spreading” the traffic and forcing nodes in the network to deplete their energies at the same time. In a similar problem is studied with the inclusion of uncertainties in several WSN parameters. From a network security viewpoint, deterministic routing policies are highly vulnerable to attacks that In order to reduce the effect of such attacks, probabilistic routing is an interesting alternative, since this makes it difficult for attackers to identify an “ideal” node to take over. In this sense, the EAR policy is attractive because of its probabilistic routing structure, even though it does not attempt to provide optimal routing probabilities for network lifetime maximization. It is worth mentioning, however, that a routing policy based on probabilities can easily be implemented as a deterministic policy as well by transforming these probabilities to packet flows over links and using simple mechanisms to ensure that flows are maintained over time.

The network lifetime maximization problem studied in [7] is based on two assumptions. First, it assumes that the energy in a battery depletes linearly with admiration to the quantity of information forwarded, and does not depend on the physical dynamics of the battery itself. Second, it seeks permanent routing probabilities over time, even though the dynamic behavior of the WSN may in fact imply that a time-dependent routing policy may be optimal. More generally, routing problems in WSNs are based on ideal battery models where a battery maintains a stable voltage during the discharge process and a stable capacity for all discharge profiles, neither of which is generally true. This dynamic behavior also leads to the conjecture that an optimal routing policy should consider the battery state over time and should, therefore, be time-dependent quite than permanent. Thus, an optimal control problem formulation for the network lifetime maximization problem seems to be a natural setting.

We adopt an optimal control setting with the goal of determining routing probabilities so as to maximize the lifetime of a WSN subject to a *dynamic* energy use model for each node. In particular, we will use a *Kinetic Battery Model* (KBM) , which has productivey been applied in other power management applications. We will then show that in a permanent network topology case there exists an optimal policy consisting of *time-invariant* routing probabilities. We subsequently show that the optimal control problem may be converted into a set of relatively simple nonlinear programming (NLP) problems. Moreover, under a very mild condition, this optimal routing policy is in fact robust with admiration to the battery model used, the routing probabilities are not affected by the battery model used, although naturally the estimated WSN lifetime itself is significantly longer under a nonideal battery model, primarily due to the recovery effect mentioned earlier. We also consider an alternative problem where, in addition to routing, we allocate a total initial energy over the network nodes with the same network lifetime maximization objective; the idea here is that a proper allocation of energy can further increase the network lifetime. That the solution to this problem is given by a policy that depletes all node energies *at the same time* and that the Equivalent energy allocation and routing probabilities are obtained by solving again an NLP problem. when the battery behavior is reduced to a simple idealized model, our setting recovers that of and where it was shown that the set of NLP sub problems can in fact be transformed into the linear programming (LP) formulation in [7]. It was also that the initial energy allocation problem can be reformulated into a shortest path problem on a graph where the arc weights equal the link energy costs.

We formulate the maximum lifetime optimization problem using non ideal energy sources at nodes that have their own dynamics. We adopt a standard energy use model along with the aforementioned KBM. That for a permanent network topology there exists an optimal routing policy which is time invariant and identify a set of NLP problems, which can be solved to obtain an explicit permanent optimal routing vector and the Equivalent WSN lifetime. We also derive sufficient conditions under which this optimal policy is robust with admiration to the battery model used. It is optimal to set a routing vector and initial node energies, so that all nodes have the same lifetime. An explicit solution can again be obtained by solving an NLP problem. Numerical examples are included to display our analytical results.

II. OPTIMAL CONTROL PROBLEM FORMULATION

In order to simplify our analysis, we will consider a WSN with a single source node and one base station and will assume a permanent topology. It will become clear that our methodology can be extended to multiple sources and one base station, as well as time-varying topologies, although the main permanent optimal routing result will obviously no longer hold in general.

A. *Network Model:*

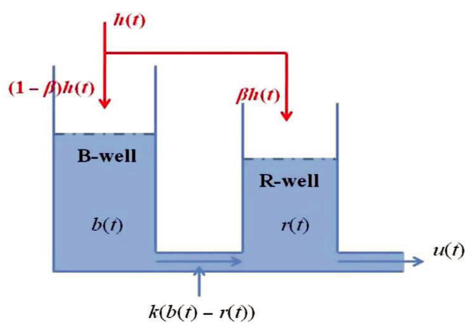
Consider a network with $N+1$ nodes, where 0 and N denote the source and destination (base station) nodes, respectively. Except for the base station whose energy supply is not constrained, a imperfect amount of energy is available to all other nodes.

B. Energy Use Model:

Under the assumption that an electrochemical battery cell is "ideal," a stable voltage during the discharge process and a stable capacity for all discharge profiles are both maintained over time. However, in real batteries, the *rate capacity effect* [12] leads to the loss of capacity with increasing load current, and the *recovery effect* makes the battery appear to regain portions of its capacity after some resting time. Due to these phenomena, the voltage as well as energy amount delivered by the battery heavily rest on the discharge profile. Therefore, when dealing with energy optimization, it is necessary to consider this along with nonlinear variations in a battery's capacity. As a result, there are several proposed models to describe a non ideal battery; a detailed overview. Accordingly, models are broadly classified electro chemical [12], circuit-based [8], stochastic [10], [9], [11], and analytical. Electrochemical models possess the highest accuracy, but their complexity makes them impractical for most real-time applications.

Electrical-circuit models are much simpler and, therefore, computationally less expensive but their accuracy leads to errors, which may be reduced at the expense of added complexity [8]. Stochastic models use a discrete time Markov chain with $N+1$ states to represent the number of charge units available in the battery. Since $N$is large (in the order of 10_7), these models are also imperfect by high computational requirements. Last but not the least, analytical models, including diffusion-based models [2] and the KBM, use only a few equations to capture the battery's main features. While diffusion-based models are hard to combine with a presentation model a KBM combines speed with sufficient accuracy, as reported, for instance, in embedded system applications. Experimental proof for the accuracy of the KBM is also provided in . The KBM was productivey used to study problems of optimal single and multi battery power control in with results consistent with the use of a more elaborate linear state space model derived from the popular RVW diffusion-based model. In what follows, we briefly review the KBM.

The KBM models a battery as two wells of charge, as shown in Fig. 1. The available-charge well (R-well) directly supplies



Electrons to the load, whereas the bound-charge well (B-well) only supplies electrons to the R-well. The energy levels in the two wells are denoted by $r(t)$ and $b(t)$,respectively. The rate of energy flow from the B-well to the R-well is $k(b(t)-r(t))$.The output $u(t)$ is the workload of the battery at time t. Three factors (e.g., see [4], [3]): the energy needed to sense a bit , the energy needed to receive a bit E_{r} and the energy needed to transmit a bit E_{t} . If the distance between two nodes is d, we have

$$E_{tx} = p(d), \quad E_{rx} = C_r, \quad E_{sense} = C_e$$

where C_r, C_e are given stables dependent on the communication and sensing characteristics of nodes, and $p(d) \geq 0$ is a function monotonically increasing in d; the most common such function is $p(d) = C_f + C d^{\beta}$ where C_f, C are given stables and β is a stable dependent on the medium involved. Use this energy model but ignore the sensing energy, i.e., set $C_e = 0$. Clearly, this is a relatively simple energy model that does not consider the channel quality or the Shannon capacity of each wireless channel. The ensuing optimal control analysis is not critically dependent on the exact form of the energy use model attributed to message, although the ultimate optimal value of $\mathbf{w}(t)$ obviously is. Before proceeding, it is convenient to define the following stables:

$$k_{i,j} \quad p(d_{i,j}) \quad p(d_{i,N}), \quad i < j < N$$

$$k_{0,N} = p(d_{0,N})$$

$$k_{i,N} = C_r + p(d_{i,N}), \quad i = 1, \ldots, N-1$$

where d_{ij} is the distance between nodes i and j. Note that we may allow these stable to be time-dependent if the network topology is not permanent, i.e., $d_{ij}(t)$ is time-varying. Let us now combine the KBM model above with (4). Although the ability to recharge a battery offers an interesting possibility for routing control, we shall not consider it in this paper, i.e., set $c_2 = 0$ in (2) and (3). Moreover, for simplicity, we set $c_1 = 1$. Then, starting with node 0, the work load $u_0(t)$ at that node. where we have used the fact that $G_0(\mathbf{w}) = 1$Similarly, for any node $i = 1 \ldots N-1$, where we

must include the energy for both receiving and transmitting data packets.

C. Optimal Control Problem Formulation:

Our objective is to maximize the WSN lifetime by control-ling the routing probabilities $w_{ij}(t)$. The WSN lifetime is defined as $T = \min_{0 \leq i < N} T_i$,

This is a classic minimum (maximum) time optimal control problem except for two complicating factors the boundary condition (16), which involves the non differentiable min function, and 2) the control constraints (15). In what follows, we will use $\mathbf{w}^*(t)$ to denote the optimal routing vector, which provides a solution to this problem.

Remark 1: Note that there is an additional state constraint imposed by the capacity of every node battery, i.e., $b_i(t) \leq B_i$. However, it is easy to show (see [33]) that as long as $R_i < B_i$, it is always true that $r_i(t) < b_i(t) < B_i$ for all $t > 0$, so that this nstraint is never active in our problem Moreover, if $B_i = R_i$, then $r_i(t) < b_i(t) < B_i$ as long as $G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t)) > 0$ for all $t > 0$.

observe that when the battery is "at rest," i.e., there is no load in (11), it is easy to show that $\lim_{t\to\infty}(b_i(t) - r_i(t)) = 0$. Therefore, we normally set initial conditions, so that $B_i = R_i$.where $r_i(t) b_i(t)$ are the state variables representing node i's instantaneous battery energy level, $i = 0 \ldots N-1$. Control constraints are specified through (15), where the first inequality follows from the fact that $\sum_{<} w_{ij}(t) + w_{iN}(t) = 1$. Finally, (16) provides boundary conditions for $r_i(t)$, $i=0...N-1$, at $t=T$ requiring that the terminal time is the earliest instant when $r_i(t)=$for any node i.

III. OPTIMAL CONTROL PROBLEM SOLUTION

We begin with the Hamiltonian for this optimal control problem

$$H(\mathbf{w},t,\lambda) = -1 + \sum[\lambda_{i1}(t)(-G_i(\mathbf{w}(t))g_i(\mathbf{w}(t))$$

$$+k(b_i(t) \quad r_i(t))) \quad \lambda_{i2}(t)(k(b_i(t) \quad r_i(t)))]$$

To derive explicit expressions for $\lambda_{i1}(t)$, $\lambda_{i2}(t)$, it is necessary to use boundary conditions $\lambda_{i1}(T)$, $\lambda_{i2}(T)$. This is complicated by the nature of the state

boundary conditions in (16). Thus, we proceed by considering each possible case of a node dying first, which we will refer to as "scenario S_i" under which $0=r_i(T)\leq r_j(T) j\neq i$ for some permanent node i

Analysis of Scenario:

The property that under a permanent network topology, there exists a static optimal routing policy, i.e., there exists a vector $\mathbf{w}^*(t)$, which is time invariant. Under S_i, we have the terminal time constraints$r_i(T)=0$ and $r_j(T)\geq 0$ for all $j\neq i$. Consequently, all $r_j(t)$, $j\neq i$ are unconstrained at $t=T$. The next theorem

Theorem 1: If $0=r_i(T)<r_j(T)$, $j\neq i$, for some i and the network topology is permanent, i.e., $d_{i\,j}(t)=d_{i\,j}=\text{constant}$ for all $i\,j=0...N-1$, then there exists a time-invariant solution of (10)–(16):

$$\mathbf{w}^*(t) = \mathbf{w}^*(T).$$

Algorithm for Solving the Optimal Control Problem:

Based on our analysis thus far, if we focus on a permanent scenario S_i, the solution to the optimal control problem is simply the solution of the NLP problem $\mathbf{P}_i$. However, since we do not know which node will die first, determining the value of i such that $T^*(\mathbf{w})\leq T^*(\mathbf{w})$ for all $j\neq i$ requires solving all $\mathbf{P}_i$ problems and find the best policy among them. Since not all $\mathbf{P}_i$ problems have feasible solutions, we can use (25) to check for feasibility before solving the associated NLP problem. The complete algorithm, referred to as A1, to solve this optimal control problem is as follows.

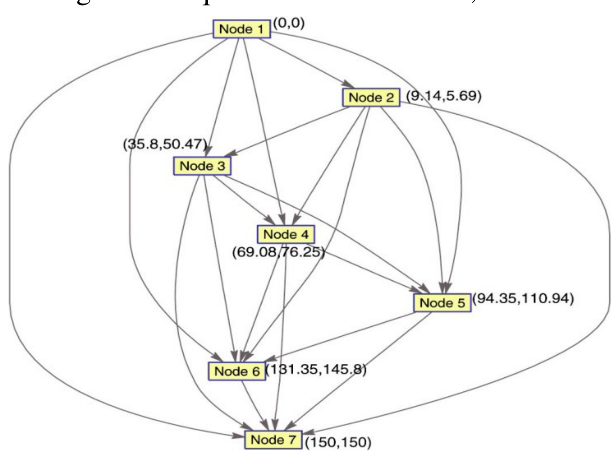
Algorithm A1:

1) Solve problem $\mathbf{P}_0$ to obtain $T^*(\boldsymbol{w})$.

2) For $0 < i < N$, if $\frac{R_i}{k_{iN}} > \frac{R_0}{k_{0j(0)}+k_{0N}-kB_0}$, set $T^*(\mathbf{w})=-1$ (no feasible solution exists); otherwise solve problem $\mathbf{P}_i$ and obtain $T^*(\mathbf{w})$ if it exists.

C. A Robustness Property of the Optimal Routing Policy:

The optimal routing vector $\boldsymbol{w}^*$ obtained through Algorithm A1 under the ideal battery assumption, $k=0$ in (11) and (12), is often unchanged when the non ideal battery model ($k > 0$) is used. The intuition behind such a robustness property lies in the nature of the NLPs $\mathbf{P}_i$ observe that the solution depends on the values of $G_i(\mathbf{w})g_i(\mathbf{w})$ and the associated constraints (13)–(15), while the only effect of the parameter k enters through the inequalities $T^*(\mathbf{w})\leq T^*(\mathbf{w})$, $j\neq i$



Therefore, if a solution is obtained under $k=0$ (a much easier problem which, as we have seen, can be reduced to an LP) and these inequalities are still satisfied when $k > 0$, then there is no need to re-solve the $\mathbf{P}_i$ NLPs. Naturally, when this property holds, the value of the resulting optimal network lifetime is generally different, but the actual routing policy remains unchanged. Let $\mathbf{w}^i(k)$ denote the solution of problem $\mathbf{P}_i$ when the KBM is invoked with parameter k, including the ideal battery case $k=0$. The Equivalent node lifetimes are denoted by$T^*(\mathbf{w}_i k)$. The robustness property we identify rests on the following lemma, which provides simple sufficient conditions under which $\mathbf{w}^i(0)=\mathbf{w}^i(k)$ for any $k > 0$.

Consider the NLP $\mathbf{P}_i$ with solution $\mathbf{w}^i(k)$ under battery parameter $k>0$. If the initial conditions for the node energies satisfy $B_j=R_j$ for all $j=0\ldots N-1$, then

$$\mathbf{w}^i(0) \quad \mathbf{w}^i(k)$$

Theorem 2: If the initial conditions for all node energies satisfy $R_i=B_i$, $i=0\ldots N-1$, then the optimal routing policy under an ideal battery model, $k=0$, is unaffected when $k > 0$:

$$\mathbf{w}^*(0) \quad \mathbf{w}^*(k), \; k > 0.$$

D. Simulation Examples:

display the results of our analysis, let us consider a 7-node network The optimal network lifetime in this case is 54.55 and all node lifetimes under the optimal routing policy (we do not provide specific units in our examples, but based on standard known data, distance units in feet and time units in months or weeks are reasonable). Note 1–5 die virtually simultaneously, whereas the lifetime of node 6 is considerably longer. This is because energy consumption at each node depends on both the inflow rate to that node and the transmitting distances to other nodes. Node 6 is located close to the base, hence using little energy in packet transmissions. In fact, by relocating node 6 to (120,120) and roughly doubling its distance from the base, it was observed that all 6 nodes die at the same time under the optimal policy. Another important observation in this example is that node 2 receives only 34% of the network inflow and this happens because there is no benefit in sending data packets to a relatively close relay node. The network topology and all energy model parameter values are taken from an example in where the routing problem was solved for the ideal battery case. Our results under $k=0$ recover almost the same routing probabilities and the exact same lifetimes. Moreover, contains a comparison of the WSN lifetime obtained here with the one obtained using a locally greedy policy, random routing, and the EAR policy it was shown that the former provides significant lifetime improvements over all three alternatives. Next, we revisit the same Problem with the KBM battery dynamics (11) and (12). Assuming and using AlgorithmA1, the optimal routing Probabilities and node lifetimes are respectively. It is interesting to observe that even such a small value of results in a lifetime improvement of approximately 3%, which is due to the recovery effect in the battery dynamics captured in (11) and(12). provide the resulting optimal routing probabilities and node lifetimes for two dditional larger values of showing considerable network lifetime improvements.

Note that the optimal routing probabilities for the ideal and nonideal battery cases are virtually identical, thus confirming our result in Theorem 2 . As a result, one can adopt in practice a simple ideal battery model, leading to a simple optimal routing solution through an LP as in [7] and Similar results are obtained for a symmetric network topology with the same positions for source and base nodes. As one would expect, all nodes die simultaneously due to this symmetry. We go a step beyond routing as a mechanism through which we can control the WSN resources by also controlling the allocation of initial energy over its nodes so as to maximize the lifetime. An application where this problem arises is in a network with rechargeable nodes. In this case, solving the joint optimal routing and initial energy allocation problem provides optimal recharging amounts maximizing the network lifetime, which may not correspond to full charges for all nodes. Let us define the total initial energy available as $\bar{R}$ and let $\boldsymbol{R} = [R_0, \ldots, R_{N-1}]$. From Theorem 1, we know that the optimal routing policy is fixed unless the topology of the network changes.

IV. A JOINT OPTIMAL ROUTING AND INITIAL ENERGY ALLOCATION PROBLEM

We go a step beyond routing as a mechanism through which we can control the WSN resources by also controlling the allocation of initial energy over its nodes so as to maximize the lifetime. An application where this problem arises is in a network with rechargeable nodes. In this case, solving the joint optimal routing and initial energy allocation problem provides optimal recharging amounts maximizing the network lifetime, which may not correspond to full charges for all nodes.

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Recalling Remark 1, we may assume that $B_i=R_i$ since all batteries are normally initialized at an equilibrium state. In this case, (29) holds. Otherwise, (29) becomes a condition we need to impose so as to ensure that $\frac{\partial T}{\partial R_i} > 0$, which will be used in the result that follows.

If the solution of problem (28) is $(\mathbf{w}^* \boldsymbol{R}^*)$, then $T^*(\mathbf{w}_* R^*)$ is the solution of (19) under this routing vector and initial energy at node i. The following theorem establishes the fact that this optimal solution is such that all nodes deplete their energy at the same time.

Proof: See Appendix.

Remark 3: In order to guarantee (30), it is necessary that $T^*(\mathbf{w}_* R^*) < \infty$. Looking at (19) and recalling that $g_i(\mathbf{w}) > 0$, this is equivalent to assuming that $G_i(\mathbf{w}) > 0$, i.e., no node is left unutilized.

Based on Theorem 3, we can simplify the NLP problem (28). In particular, we solve it in two steps. In Step 1, assuming a fixed routing policy $\mathbf{w}$, we determine the corresponding optimal initial energy distribution policy by solving the set of equations:

$$T_0^*(\mathbf{w}, R_0) = T_1^*(\mathbf{w}, R_1) = \cdots = T_{N-1}^*(\mathbf{w}, R_{N-1})$$

$$\text{s.t.} \sum_{i=0}^{N-1} R_i = \bar{R}. \qquad ($$

we can simplify the NLP problem (28). In particular, we solve it in two steps. Assuming a permanent routing policy $\mathbf{w}$.

defined to be $\boldsymbol{R}^*(\mathbf{w})$ with an associated lifetime $T^*(\mathbf{w})$. Then, in

We search over the feasible set of $\mathbf{w}$ given by (15) to determine the optimal $T^*(\mathbf{w})$ by using a standard nonlinear optimization solution procedure. We should point out, however, that solving problem (31) to obtain parametric solutions for $T^*(\mathbf{w})$ and $\boldsymbol{R}^*(\mathbf{w})$ is not a simple task and common solvers fail to accomplish it. Instead, we can proceed by selecting one of the parametric equations for $T^*(\mathbf{w} R_i)$ as an objective function and add (31) as constraints to a new NLP

Remark 4: our analysis can recover the ideal battery case by setting $k=0$ in (11) and (12), which implies that $T^*(\mathbf{w})=R_i[G_i(\mathbf{w})g_i(\mathbf{w})]_-$. This simplifies the solution of (31) as follows. Setting $K_i(\mathbf{w})=[G_i(\mathbf{w})g_i(\mathbf{w})]_-$, (31) implies that

$$R_i = \frac{K_0(\mathbf{w})}{K_i(\mathbf{w})} R_0, \quad i = 1, \ldots, N-1$$

$$R_0 = \bar{R}\left[1 + \sum_{i=1}^{N-1} \frac{K_0(\mathbf{w})}{K_i(\mathbf{w})}\right]^{-1} = \frac{\bar{R}}{K_0(\mathbf{w})}\left[\sum_{i=0}^{N-1} \frac{1}{K_i(\mathbf{w})}\right]^{-1}$$

A. Simulation Examples:

We consider a numerical example for the joint optimal routing and initial energy allocation problem. first the problem is solved for a network with ideal node batteries and then using the KBM dynamics (11) and (12). Let us consider the same network. The source node to $(-15, -15)$. The optimal routing probabilities and initial energies of all nodes under different values of k, including the ideal battery case where $k=0$ in (11) and (12). Note that the WSN lifetime with $k=0$ is 63.33, which considerably exceeds the value 54.55 . Even though the distance between the source and base nodes. Moreover, once again we observe that both optimal initial energies and routing probabilities are the same over different values of k. Finally, note the fact that the network lifetime coincides with all individual node lifetimes, which are the same by Theorem 3, and provides a strong justification for the definition of network lifetime being that of the first node to deplete its energy.

V. CONCLUSIONS AND FUTURE WORK

That an optimal routing policy for maximizing a permanent topology sensor network's lifetime is time invariant even when the batteries used as energy sources for the nodes are modeled so as to consider "nonideal" phenomena such as the rate capacity effect and the recovery effect. The associated permanent routing probabilities may be obtained by solving a set of relatively simple NLP problems. In addition, under very mild conditions, this optimal policy is independent of the battery parameter k, where $k=0$ for ideal batteries. Therefore, one can resort to the case of ideal atteries where the optimal routing problem is much simpler to solve and can be reduced to an LP problem. We have also considered a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective. In this case, the solu-tion to this problem is given by a policy that depletes all node energies at the same time and the Equivalent energy allocation and routing probabilities are obtained by solving an NLP problem.

Extensions of our analysis to networks with multiple sources and base stations are expected to be straightforward. The robustness property we have identified for the optimal routing policy with admiration to the battery dynamics assumed may no longer hold if different nodes use different battery characteristics (i.e., different parameters k_i). In addition, it remains to investigate whether different battery models used can still preserve the time-invariant nature of the optimal routing policy and the robustness property identified in Theorem 2. It is also interesting to explore how an optimal routing policy may depend on a changing network topology. Finally, the solutions we have obtained so far are centralized and require global location in order, so that an obvious direction to pursue is one seeking spread versions of the same optimal routing and energy allocation problem advancees.

APPENDIX

Proof of Theorem 1: Since $r_i(t)\geq 0$ for all i and $t\in[0T]$, the optimal control problem under S_i is state-unconstrained except for $r_i(T)=0$. Thus, the terminal state constraint function $\Phi(r(T)b(T))$ is Preduced to $r_i(T)$ and the costate boundary conditions [6] are given by

$$\begin{cases}\lambda_{i1}(T)=\nu\frac{\partial\Phi(r(T),b(T))}{\partial r_i}=\nu\\ \lambda_{i2}(T)=0\end{cases}\quad\begin{cases}\lambda_{j1}(T)=0\\ \lambda_{j2}(T)=0,\quad j\neq i\end{cases}$$

Observe that the control variables $w_{ij}(t)$ appear only in $G_i(\mathbf{w}(t))$ and $g_i(\mathbf{w}(t))$ in the problem formulation (10)–(16). Thus, we can set $U_i(t)=G_i(\mathbf{w}(t))g_i(\mathbf{w}(t))i=0\ldots N-1$ to be the effective control variables with $U_l\leq U_i(t)\leq U_u$, where $U_l\geq 0$ nd U_u are, admirationively, the lower bound and upper bound of $U_i(t)$ for all$t\in[0T]$. Note that both are stable since their determination depends exclusively on (13), (14) subject to (15), independent of the states $r_i(t)$ and $b_i(t)$. In particular, they depend on the permanent network topology and the values of the energy parameters k_{ij}, k_{iN} in (14). Applying the Pontryagin minimum principle to (34):

$$U_i^*(t)=\arg\min_{U_l\leq U_i(t)\leq U_u}H(U_i,t,\lambda^*)$$

implies that the optimal control is of bang-bang type:

$$U_i^*(t)=\begin{cases}U_u & \text{if }\lambda_{i1}(t)>0\\ U_l & \text{if }\lambda_{i1}(t)<0\end{cases}$$

with the possibility that there is a singular arc on the optimal trajectory if $\lambda_{i1}(t)=0$. Moreover, the optimal solution must satisfy the transversality condition [6] $(\lambda*\frac{d\Phi}{}+L)=0$, where $L=-1$ and we have seen that $\Phi(r(T)b(T))=r_i(T)$. Therefore

$$-1+\nu\dot{r}_i(T)=-1+\nu-G_i(\mathbf{w}(T))g_i(\mathbf{w}(T))+kb_i(T)]=0$$

and it follows that

$$\nu=\frac{1}{-G_i(\mathbf{w}(T))g_i(\mathbf{w}(T))+kb_i(T)}.$$

Observing that $\nu\neq 0$ and looking at (33), we can immediately exclude the singular case $\lambda_{i1}(t)=0$. Moreover, since $r_i(T)=0$ and $r_i(t)>0$ for all $t\in[0T)$, it follows that $\dot{r}_i(T)<0$ and (36) implies that $\nu<0$. Therefore, from (33), $\lambda_{i1}(t)<0$ during $[0T)$. Consequently, $U^*(t)=U_l$ for $t\in[0T]$ by (35). We conclude that the optimal control problem under S_i is reduced to the following optimization problem:

$$\begin{cases}\lambda_{i1}(T)=\nu\frac{\partial\Phi(r(T),b(T))}{\partial r_i}=\nu\\ \lambda_{i2}(T)=0\end{cases}\quad\begin{cases}\lambda_{j1}(T)=0\\ \lambda_{j2}(T)=0,\quad j\neq i\end{cases}$$

$$\min_{w(t)} G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t))$$

$$\text{s.t.} \quad (13)\ (15) \text{ and } 0 = r_i(T) \leq r_j(T), \quad j \neq i.$$

When $t = T$, the solution of this problem is $\mathbf{w}^*(T)$ and depends only on $r_j(T)$, $j \neq i$, and, as already argued, the permanent network topology and the values of the permanent energy parameters k_{ij}, k_{iN} in(14)

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