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Research Article

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Modelling Laminar Film Condensation on Horizontal Flat Plate; Film Thickness, Velocity and Temperature Profile

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Abstract The study reports modeling of laminar film condensation of horizontal flat plate with hot vapor film exists above the plate with higher temperature (saturation temperature). In this study mathematical expressions for film thickness, velocity and temperature profile were obtained analytically. Models confirm that velocity and temperature depend on the distance from plate center (x), from plate surface (δ) and depend also on Ra and Ja numbers.

Keywords Laminar film condensation, horizontal plate, infinite-size, film thickness, film velocity.

1. Introduction

Laminar film condensation occurs due to heat transfer and it is common in two-phase flow applications such as heat exchangers and air conditioner. Therefore modeling this phenomenon is essential to design equipment that involves two-phase flow. There are two types of film condensation (1) drop wise condensation and (2) film condensation, the latter one involve film movement due to hydrostatic pressure gradient. Thus, film velocity is also a common feature of laminar film condensation and need to be specified for design purposes.

Heat transfer in laminar film condensation of horizontal flat plate was studied in many configurations (pipes, micro-channels …etc.). Laminar film condensation on flat plate with different orientation was studied in literature, for example, inclined plate [1]. In addition, Shigechi et al. (1990) obtained film thickness by adjusting inclined angle of the vapor-liquid interface at the plate edge [2].The earliest attempt to study the condensation heat transfer rate on a horizontal surface was experimentally done by Popov in 1951[3].Moreover, underside condensation was investigated by Gerstmann and Griffith (1976) theoretically and by experiments [4]. Nimmo and Leppert (1970) studied the upper side condensation on flat plate early [5]. In such case, condensate flows across the plate under the effect of hydrostatic pressure gradient, accordingly film thickness and velocity will not remain constant. The study indicated that the film thickness is to be assumed or specified by boundary conditions. In addition, a recent study considered flat plate, which was subjected to cooling source [6]. Numerical prediction of film velocity, thickness and temperature gradient was performed and found to be in an agreement with the analytical approach. In many cases, the concept of minimum mechanical energy was used to develop an expression for film thickness. This concept was developed early by Bakhmeteff (1966) and was implemented to obtain the condensate thickness on horizontal flat plate with finite-size [7]. Laminar film condensation of horizontal flat plate with finite-size was also investigated for porous and non-porous media [8].

The objective of the present study is to model film thickness, velocity and temperature profile on finite-size flat plate, based on the concept of minimum mechanical energy. Determination of these two parameters will further help in calculation of heat transfer coefficient and other parameters, which will be used ultimately for design purposes.

2. Analysis

Consider a finite-size flat plate with wall temperature T_p adjacent to saturated pure vapor at uniform temperature T_{sat}. Since the plate temperature lower than vapor temperature, condensation will occur creating two-phase film between plate surface and the vapor zone. A schematic diagram of the physical model and coordinate system is shown in figure (1) bellow. Under steady-state conditions, the liquid film boundary layer will be established with maximum depth existing at the center of plate and it will decreases gradually. The film at the plate center (maximum film thickness) is assumed, measured experimentally or determined by boundary conditions.

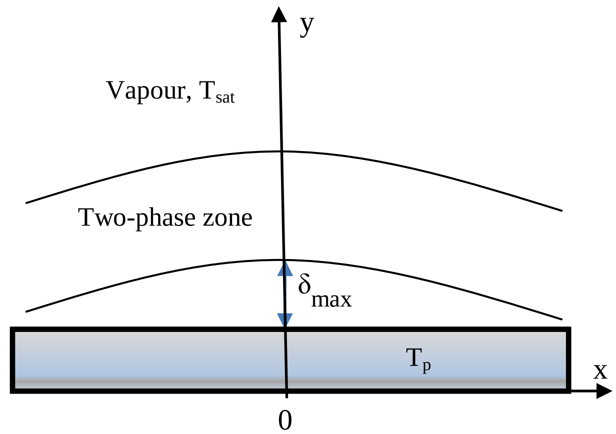

Figure 1: *Problem Graph.*

2.1. Assumptions

Laminar film condensation was analyzed to model the film thickness and velocity, the model is based on the following assumptions:

1. The flow steady and laminar. The inertia within the film is negligible,
2. The inertia within the film is negligible,
3. Constant and uniform vapor and plate temperatures,
4. Negligible film kinetic energy,
5. Darcy's law is applicable to the liquid film in the porous medium. And the properties of porous medium, dry vapor and condensate are kept constant.
6. Very low vertical velocity and constant temperature in x direction,
7. Absence of any porous media.

2.2. Model formulation

Continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow (1)$$

For vapor condensation the vertical velocity can be negligible (v≈0), thus $\frac{\partial v}{\partial y} = 0$ that gives:

$$\frac{\partial u}{\partial x} = 0$$

Momentum Equation

x- direction:

$$0 = -\frac{\partial P}{\partial x} - \frac{\mu u}{K} + \mu \frac{\partial^2 u}{\partial y^2} \rightarrow (2)$$

In this equation shear force are considered (surface tension effect).Where K is intrinsic permeability. For vapor phase and absence of porous media is very large K→∞ (high permeability) and equation (2) becomes:

$$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \rightarrow (3)$$

y- direction:

$$0 = -\frac{\partial P}{\partial y} - \rho g \rightarrow (4)$$

Where $\frac{\partial P}{\partial y}$ is the pressure gradient in y direction. ρg is the body force .

$$-\int_{P}^{P^{sat}} dP = \int_{y}^{\delta} \rho g dy,$$

$$P - P^{sat} = \rho g(\delta - y) or$$

$$P = P^{sat} + \rho g(\delta - y) \rightarrow (5)$$

Differentiate equation (5) with respect to x that gives:

$$\frac{dp}{dx} = \rho g \frac{d\delta}{dx} \rightarrow (6)$$

Substitute equation (6) into (3) that gives:

$$\mu \frac{\partial^2 u}{\partial y^2} = \rho g \frac{d\delta}{dx} \rightarrow (7)$$

By integration:

$$\mu \frac{\partial u}{\partial y} = \rho g \frac{d\delta}{dx} y + C_1 \rightarrow (8)$$

$$u = \frac{\rho g}{\mu} \frac{d\delta}{dx} \left(\frac{y^2}{2}\right) + C_1 y + C_2 \rightarrow (9)$$

B.C.1 u=0 at y=0 due to no-slip condition:

$$C_2 = 0$$

B.C.2$\frac{\partial u}{\partial y} = 0 \; at \; y = \delta$ from equation (8):$C_1 = -\frac{\rho g}{\mu} \frac{d\delta}{dx} \delta$

Thus; $u = \frac{\rho g}{\mu} \frac{d\delta}{dx} \left[\left(\frac{y^2}{2}\right) - \delta y\right] \rightarrow (10)$

At the interface for no-slip condition:

$$u^v|_{y2=0} = u|_{y=\delta}$$

$$\frac{\partial u^v}{\partial x}|_{y2=0} = \frac{\partial u}{\partial x}|_{y=\delta}$$

There is an expression for porous media u^v obtained by Udell, K.S (1983) [9]. But for absence of porous media u^v can be considered a constant.

2.3. Temperature Profile:

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_e \frac{\partial^2 T}{\partial y^2} \rightarrow (11)$$

Where the L.H.S is the convective heat transfer, while the R.H.S is the conduction heat transfer andα_e is the effective thermal diffusivity. For v=0 and T= constant in x-direction equation (11) becomes:

$$0 = \frac{\partial^2 T}{\partial y^2} \rightarrow (12)$$

Integrating two times:

$$T = C_1 y + C_2 \rightarrow (13)$$

By introducing boundary conditions, temperature profile cab be obtained

B.C.1: At y=0,T=T_p (plate temperature), B.C. 2: at y=δ,T=T_{sat}

$$C_2 = \text{Tp and } C_1 = \frac{T_{sat} - T_p}{\delta}$$

$$T = T_p + (T_{sat} - T_p)\left(\frac{y}{\delta}\right) \rightarrow (14)$$

2.4. Film thickness δ:

Overall energy balance equation:

$$\frac{d}{dx}\left[\int_0^\delta \rho u\left(h_{fg} + C_p(T_s - T)\right)dy\right]dx + \rho h_{fg} v_2 dx = \frac{k_e \Delta T}{\delta}dx \rightarrow (15)$$

Where the R.H.S represents the heat transferred from the liquid film to the plate. The L.H.S $\frac{k_e \Delta T}{\delta}dx$ represents the net energy flux across the film.

With:

i- linear temperature profile and $\left(h_{fg} \gg C_p(T_s - T)\right)$.

ii- v_2 is very low or negligible.

based on these approximations ,equation (15) can be written as;

$$\frac{d}{dx}\left[\int_0^\delta \rho u h_{fg}\, dy\right] = \frac{k_e \Delta T}{\delta} \rightarrow (16)$$

Substitute for u from equation (10):

$$\int_0^\delta u dy = \int_0^\delta \frac{\rho g}{\mu}\frac{d\delta}{dx}\left[\left(\frac{y^2}{2}\right) - \delta y\right]dy = -\frac{\rho g}{\mu}\frac{d\delta}{dx}\frac{\delta^3}{3} \rightarrow (17)$$

Equation (17) becomes:

$$\frac{d}{dx}\left[-\frac{\rho^2 g h_{fg}}{\mu}\frac{d\delta}{dx}\frac{\delta^3}{3}\right] = \frac{k_e \Delta T}{\delta} \rightarrow (18)$$

$$-\frac{\rho^2 g h_{fg}}{\mu}\frac{d}{dx}\left[\frac{d\delta}{dx}\frac{\delta^3}{3}\right] = \frac{k_e \Delta T}{\delta}$$

$$A = -\frac{\rho^2 g h_{fg}}{3\mu}$$

$$A\frac{d}{dx}\left[\frac{d\delta}{dx}\delta^3\right] = \frac{k_e \Delta T}{\delta}$$

$$A\left[\delta^3\frac{d^2\delta}{dx^2} + 3\delta^2\left(\frac{d\delta}{dx}\right)^2\right] = \frac{k_e \Delta T}{\delta} \rightarrow (19)$$

$$\left[\delta^3\frac{d^2\delta}{dx^2} + 3\delta^2\left(\frac{d\delta}{dx}\right)^2\right] = \frac{k_e \Delta T}{A\delta} = \frac{B}{\delta} \rightarrow (20)$$

$$B = \frac{k_e \Delta T}{A}$$

$$\left[\frac{d^2\delta}{dx^2} + 3\delta^{-1}\left(\frac{d\delta}{dx}\right)^2\right] = \frac{B}{\delta^4} \rightarrow (21)$$

W=f(δ) , W=$\left(\frac{d\delta}{dx}\right)^2$ $and\ W' = \frac{dW}{d\delta} = 2(\frac{d^2\delta}{dx^2})$

Substitute in (21)

$$W' + \frac{6}{\delta}W = \frac{2B}{\delta^4} \rightarrow (22)$$

Using the integral factor method:

$$p(\delta) = \frac{6}{\delta} \qquad Q(\delta) = \frac{2B}{\delta^4}$$

$$I = e^{\int p(\delta)d\delta} = e^{\int \frac{6}{\delta}d\delta} = e^{ln\,\delta^6} = \delta^6$$

$$\frac{d}{d\delta}(w\delta^6) = \frac{2B\delta^6}{\delta^4} \rightarrow (23)$$

$$w\delta^6 = \frac{2B\delta^3}{3} + C_1 \rightarrow (24)$$

B.C. 1:

W=$W' = 0\ at\ \delta = 0\ at\ the\ plate\ edge$ that requires very large or infinite size plate.

$$C_1 = 0$$

$$\left(\frac{d\delta}{dx}\right)^2 = \frac{2B}{3\delta^3} \rightarrow (25)$$

$$\left(\frac{d\delta}{dx}\right) = \left(\frac{2B}{3\delta^3}\right)^{\frac{1}{2}} \rightarrow (26)$$

$$d\delta.\,\delta^{1.5} = \int \left(\frac{2B}{3}\right)^{.5} dx + C_2 \rightarrow (27)$$

$$\frac{1}{2.5}\delta^{2.5} = \left(\frac{2B}{3}\right)^{.5} x + C_2 \rightarrow (28)$$

B.C 2:

at x=0 ,$\delta = \delta_{max}$

$$\frac{1}{2.5}\delta_{max}^{2.5} = C_2$$

Equation (28) becomes;

$$\delta = 1.443\left(\frac{2B}{3}\right)^{0.2} x^{0.4} + \delta_{max} \rightarrow (29)$$

Recall A and B

$$\delta = \delta_{max} - 1.657\left[\frac{\mu k_e \Delta T}{\rho^2 g h_{fg}}\right]^{0.2} x^{0.4} \rightarrow (30)$$

And by introducing Ra, Pr and Ja

$$Pr = \frac{\mu C_p}{k_e}, Ja = \frac{C_p \Delta T}{h_{fg}}\ and\ Ra = \frac{\rho^2 L^3 Pr}{\mu^2}$$

Equation (30) becomes:

$$\delta = \delta_{max} - 1.66\left[\frac{Ja \times L^3}{Ra}\right]^{0.2} x^{0.4} \rightarrow (31)$$

2.5. Velocity profile

By substituting equation (26) into equation (10) that gives:

$$u = -\frac{\rho g}{\mu}\left(\frac{2B}{3\delta^3}\right)^{0.5}\left[y\delta - \frac{y^2}{2}\right] \rightarrow (32)$$

With; $A = -\frac{\rho^2 g h_{fg}}{3\mu}$,$B = \frac{k_e \Delta T}{A}$

$$u = 1.41\left(\frac{k_e \Delta T g}{\delta^3 \mu\, h_{fg}}\right)^{0.5}\left[y\delta - \frac{y^2}{2}\right] \rightarrow (33)$$

3. Results and Discussion

Figure (2) shows temperature profile over the distance from plate surface (y=0) to film-vapor interface δ in term of ration of (y/δ). Temperature profile was found to be linear and depends only on vapor and plate temperature together with the distance from plate surface. For finite-size plate under certain assumptions film thickness is obtained (equation 31) as function of distance from center x, and it is maximum at the edge of the plate. Depending on Ja, Pr and distance x, δ decreases until it becomes negligible at the edge.

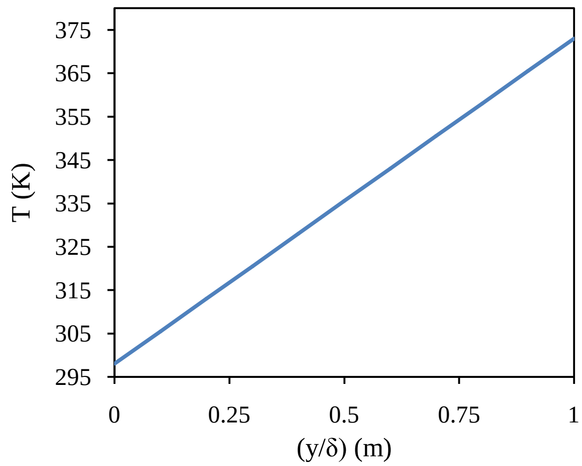

Figure 2: *Temperature variation with film thickness (δ_{max}=0.0005 m , L= 1 m)*

Figure (3) shows film thickness for case of a plate with 2 m total length and for different values of Jaat constant Ra number of 10^{10}. It can be noticed that film thickness reaches almost zero starting from x>.01 m, which is very short distance in comparison with the plate length (L). Furthermore, increasing Ja number contributes to decreasing film thickness as can be seen from figure (3).

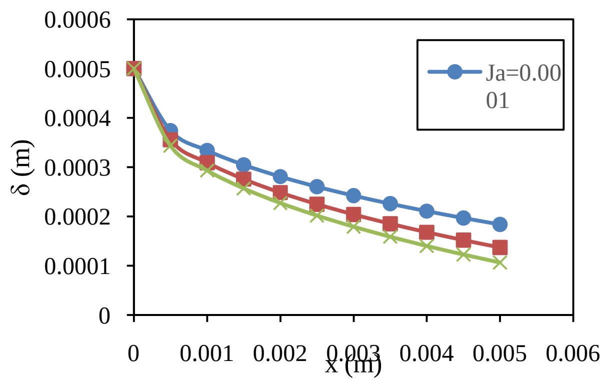

Figure 3: *Film thickness at different values ofJanumber (Ra=10^{10},δ_{max}=0.0005 m, L=1 m).*

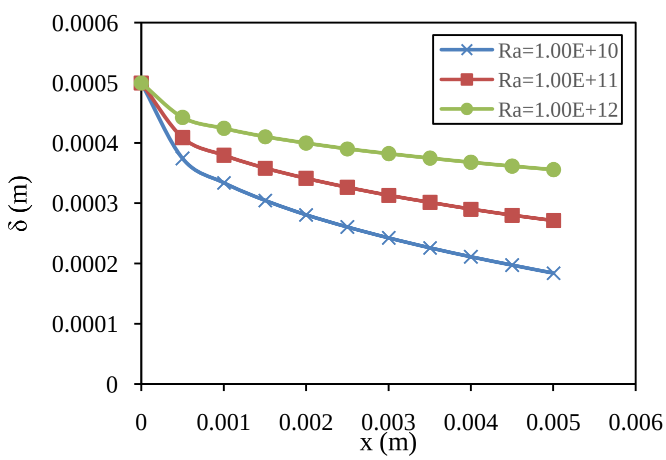

Figure 4: *Film thickness evaluated at different values of Ra (Ja=0.0001,δ_{max}=0.0005 m, L=1 m).*

Since Ja number is the ration of sensible heat absorbed to latent heat during condensation, it is reasonable to for film thickness to be decreased as less vapor condensation occurred [10]. On the other hand, figure (4) shows film thickness profile at various values of Ra and constant Janumber. Increasing Ra number increases film thickness at the edges as can be seen from figure (4). This is because Ra number in an indicator of the convection heat transfer strength and therefore increasing Ra value will increase the condensation process [11].

Velocity profile wasplotted using equation (33) as function of film thickness and the general shape is parabola curve with maximum value at y equal to the maximum film thickness (δ_{max}), and zero at y equal to zero. If we consider the case of pure water vapor, (ke=0.58 (w/m.k),h_{fg}=2443.889 j/kg and μ=0.8903 cP), withplate temperature of T_p=298 andmaximum film thickness δ_{max} = 0.005 m the profile appears as in figure(5). If we increase the maximum film thickness to 0.05 m velocity increase at the vapor-film interface. As mentioned earlier, velocity depends on hydrostatic pressure gradient with will be high as more vapor condensation occurs with increases film thickness.

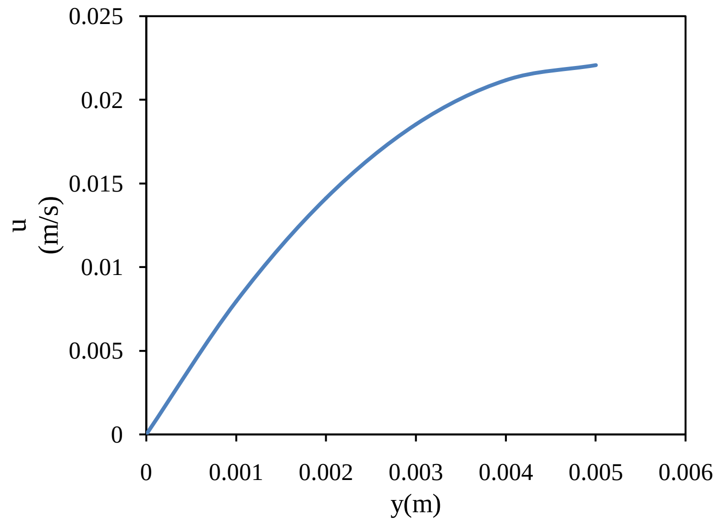

Figure 5: *Velocity Profile for δ_{max}=0.005 m*

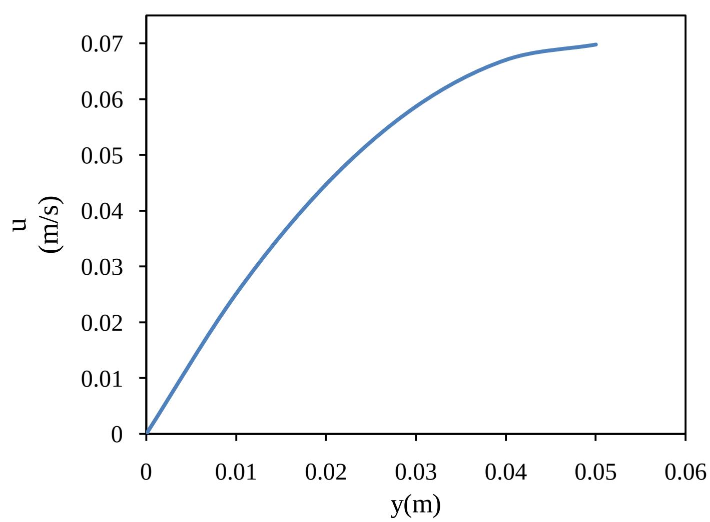

Figure 6: *Velocity Profile for δ_{max} = 0.05 m*

4. Conclusion

Laminar film condensation on horizontal flat pat was modeledusing minimum mechanical energy concept. Simple mathematical expressions were developed based on particular boundary conditions. Film thickness (δ) was found to be maximum at plate center and decreases with the horizontal distance x and depends of Ra and Ja numbers. Temperaturevelocityand profile were maximum value at maximum film thickness as expected and depends on (δ).

5. Acknowledgement

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Nomenclature

C_p Specific heat at constant pressure
g acceleration of gravity (m/s^2)
h heat transfer coefficient ($W/(m^2.K)$)
h_{fg} heat of vaporization (kJ/kg)
J_aJakob number.
k thermal conductivity (W/(m.K)
K permeability of the porous medium (m^2)
L half of plate width (m)
P pressure (Pa)
R_a Rayleigh number.
T temperature (^oC or K)
ΔT saturation temperature minus plate temperature (^oC or K)
u, v horizontal and vertical velocity components (m/s)

Greek symbols

δ condensate film thickness (m)
δ_{max}maximum film thickness(m)
μ liquid viscosity (m/s^2)
ρ liquid density (kg/m^3)
α thermal diffusivity (m/s^2)

Superscripts

p quantity at plate surface .
e effective property.
sat saturation conditions.

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