MACHINING METHOD WITH EVENLY DISTRIBUTED ALLOWANCE BASED ON THE NORMAL LINKAGE MODEL OF NON-CIRCULAR GEAR SHAPING
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基于法向插削联动模型的非圆齿轮齿面余量匀化加工方法

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ABSTRACT
Non-circular gears are important parts of agricultural machinery, such as transplanters and plant setters. The uneven distribution of allowance in the machining process seriously affects the machining accuracy and working performance of non-circular gears. Therefore, this study considered non-circular gear shaping as the research subject to solve the uneven distribution of allowance caused by the change of the curvature of the gear blank's pitch curve in non-circular gear machining. The influence of the radial motion on allowance distribution was determined by analyzing the geometric relationship between the shaper cutter and the gear blank in the machining process. Then, a normal linkage model of non-circular gear shaping was established, and the virtual and actual machining experiments were conducted. Finally, the machining method with evenly distributed allowance was simulated and validated. Results of the machining and testing show that the average unevenness of the allowance can be reduced from 110.21% to 4.95% through the machining method with evenly distributed allowance, meanwhile, the surface roughness S_a can be reduced from 3.18 μm to 1.26 μm. The proposed machining method can evenly distribute the allowance in the machining process, as well as remarkably improve the machining accuracy and surface quality of non-circular gears. The conclusions of this study provide significant references for the optimization and performance evaluation of non-circular gear machining methods.

摘要
非圆齿轮是取苗机、移栽机等农用机械的重要零件，其在加工过程中存在齿面余量分布不均问题，严重影响齿轮的加工精度与工作性能。因此，针对非圆齿轮加工中由齿坯节曲线曲率变化所引发的齿面余量分布不均问题，本文以非圆齿轮插齿加工为例，分析了加工过程中刀具与齿坯的几何位置关系，得到了径向进给运动对齿面余量分布的影响关系，建立了非圆齿轮法向插削联动模型，并进行了虚拟加工和实际加工试验，验证了齿面余量匀化加工方法。加工与检测结果表明，采用齿面余量匀化加工方法可将齿面余量不均匀度均值由110.21%降低至4.95%，齿面粗糙度S_a由3.18μm 降至1.26μm，该方法能够有效均布非圆齿轮齿面余量，并显著提高非圆齿轮加工精度与表面质量。研究成果可以为非圆齿轮加工方法的优化和性能评估提供参考依据。

INTRODUCTION
Non-circular gears synthesize the advantages of circular gears and cam mechanisms and can be designed according to the working requirements to realize the variable ratio drive; these gears have the advantages of compact structure, stable transmission, and easy realization of dynamic balance, all of which are important parts of agricultural machinery *(G.H. Yu et al., 2015; M. Zhou et al., 2014)*. The machining accuracy of non-circular gears determines the accuracy and durability of the transmission mechanism, so that improving the accuracy of non-circular gears is an important method for ensuring the working accuracy and service life of agricultural machinery products *(Pathak S et al., 2014)*.

Gear hobbing and shaping are the common machining methods of non-circular gears. Gear hobbing is efficient but can only be used for non-circular gears with full convex pitch curves and is easy to undercut *(B. Li et al., 2016)*. By contrast, gear shaping is a general method of non-circular gear machining that can overcome the limitations of gear hobbing. It can be used for non-circular external gears with concave pitch curves and non-circular internal gears and has high machining efficiency. In non-circular gear shaping, the pitch curve of the gear blank and the pitch circle of the shaper cutter in actual rough shaping are changed because of the constant change of the curvature of the gear blank's pitch curve *(X.T. Wu and H.G. Wang, 1997)*, thus, the uneven machining allowance appears. In actual machining, the uneven allowance on each tooth surface can cause unequal cutting force and cutting vibration, which has a serious influence on the service life of machines and the machining quality of gears *(Y. Zhang, 2015)*. If the allowance on one side of the tooth surface is small or non-existent, then finish machining (finish shaping, grinding, and skiving) cannot be performed efficiently and the gear blank will likely be scrapped *(X.T. Wu and H.G. Wang, 1997)*.

Based on this scenario, certain researchers conducted numerous analyses and studies on the machining principle and process of non-circular gear shaping. Litvin analyzed the generation process of the enveloping surface of an elliptical gear and proposed a matrix method to solve the involute equation. Then, he used a rack cutter, a hob, and a shaper as examples to illustrate the proposed method and established a machining theory of non-circular gears. However, the performances of the machining methods are disregarded *(Litvin F et al., 2007)*. Zheng established the linkage model of non-circular gear shaping based on three-axis machine tools, discussed the machining process of non-circular gears, and suggested a general shaping method. However, the different linkage models are not analyzed comparatively *(F. Zheng et al., 2017; F. Zheng et al., 2016)*. Xia deduced the mathematical model of non-circular gear meshing based on pure rolling contact theory and conducted the equal pole angle and arc length shaping simulations by setting different pole angles. However, the performances of the shaping methods are also ignored *(L. Xia et al., 2013)*. Liu proposed a variety of shaping strategies by adjusting the speed relationship between the shaper cutter and the gear blank, conducted the machining experiments, and then selected the optimal speed linkage model. However, the influences of the machining methods on machining accuracy are disregarded *(Y.Y. Liu, 2014)*. He J. installed the form-grinding wheel on a three-axis gear shaper and proposed a machining strategy to control the form-grinding process according to the linkage model of non-circular gear shaping. His work provided a method for realizing the finish machining on the non-circular gear shaper. However, this strategy is unverified by simulation or machining experiments *(J.L. He et al., 2007)*. Erkorkmaz proposed a linkage model that could accurately predict the chip shape and the cutting force. Therefore, the contact between the shaper cutter and the cylindrical gear was predicted based on the DEXEL model， and the linkage model was verified by kinematics modeling. However, the meshing process of non-circular gears is more complicated, and optimizing the machining accuracy of non-circular gears through this method is difficult *(Erkorkmaz K et al., 2016)*. Tarapanov analyzed the kinematic parameters of non-involute internal gear in the shaping process and studied the effect of the tool root contour on machining accuracy in terms of cutting thickness, cutting force component and surface roughness. However, the method for improving machining accuracy is not proposed *(Tarapanov A et al., 2015)*.

All these research results are mainly concerned with the realization of non-circular gear machining and the processing technology of cylindrical gear machining. However, research on the processing technology of non-circular gear machining is minimal, especially the influences of the machining methods on distributing allowance and surface quality. Therefore, the existing research results can hardly improve the machining accuracy of non-circular gears. Machining accuracy and surface quality of non-circular gears have important influences on the stability, accuracy, and service life of the transmission mechanism of agricultural machinery. Thus, studying the processing technology of non-circular gear machining and improving the quality of non-circular gears are important means of improving the working performance of agricultural machinery. Predicting the influence of the linkage model on distributing allowance evenly is an urgent problem that needs to be addressed.

This study proposes a machining method with evenly distributed allowance. First, the origin of the uneven distribution of allowance is analyzed by deducing the position relationship between the shaper cutter and the gear blank in the shaping process. Second, a normal linkage model of non-circular gear shaping is established for the allowance to be distributed evenly by adjusting the geometric position relationship between the shaper cutter and the gear blank in real time. Third, the accuracy and feasibility of the machining method with evenly distributed allowance are verified by virtual and actual machining experiments. Finally, the non-circular gears obtained by the actual machining experiments are measured via micro-topography analysis. Thus, the effectiveness of the proposed method is verified, and the influences of the machining methods on the surface quality of non-circular gears are analyzed, providing references for the optimization and performance evaluation of non-circular gear machining methods.

MATERIAL AND METHODS

Analysis of the origin of uneven allowance

Non-circular gear shaping is performed by rough shaping with multiple cutting cycles and (semi) finish shaping in the last cutting cycle. Fig. 1 shows a set-up of the coordinate system $S(O_c XY)$, where the rotation centres of the gear blank and the shaper cutter are O_c and O_b respectively, the rotation speeds of the gear blank and the shaper cutter are ω_c and ω_b respectively, and the velocity of the shaper cutter along O_bO_c is v_x.

In finish shaping, the starting point is A. At any time, the pitch circle of the shaper cutter and the pitch curve of the gear blank are tangent at point P. The angle between the polar radius r and the tangent of the pitch curve of the gear blank is μ. The angle between the polar radius of the gear blank r and O_bO_c is α, and the angle between the radius of the shaper cutter r_c and O_bO_c is β, as expressed in Eq. (1):

$$\begin{cases} \alpha = \arctan \dfrac{r_c \cos\mu}{r + r_c \sin\mu} \\ \beta = \arctan \dfrac{r \cos\mu}{r \sin\mu + r_c} \end{cases} \tag{1}$$

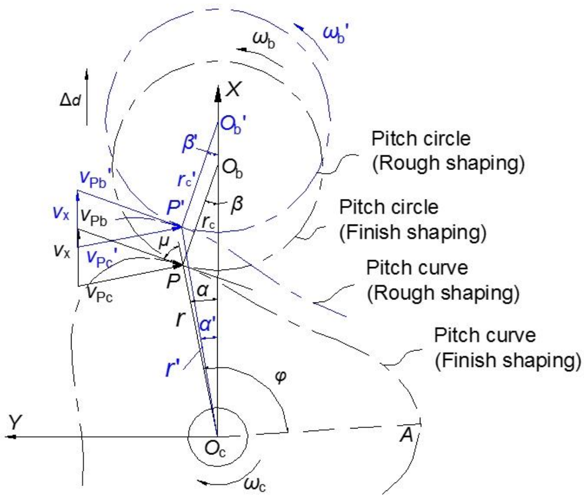


Fig.1 - Geometric relationship between shaper cutter and gear blank in non-circular gear shaping

Point P is the relative velocity instantaneous centre according to the purely rolling between the pitch circle of the shaper cutter and the pitch curve of the gear blank. The velocity of point P relative to point O_b is v_{Pb}, and the velocity of point P relative to point O_c is v_{Pc}. From the velocity triangle in Fig. 1:

$$v_x = r_c \omega_b \sin\beta + r\omega_c \sin\alpha \tag{2}$$

In Fig. 1:

$$r_c \sin\beta = r \sin\alpha \tag{3}$$

Eq. (3) is substituted in Eq. (2), and thus:

$$v_x = r\sin\alpha\left(\omega_b + \omega_c\right) = r_c \sin\beta\left(\omega_b + \omega_c\right) \tag{4}$$

In Fig. 1, the distance between the shaper cutter and the gear blank in rough shaping increases Δd along O_bO_c compared with that in finish shaping. The rotation centre of the shaper cutter is point $O_b^{'}$, the relative velocity instantaneous centre is point $P^{'}$. The angle between the polar radius of the gear blank $r^{'}$ and $O_b^{'}O_c$ is $\alpha^{'}$, and the angle between the radius of the shaper cutter $r_c^{'}$ and $O_b^{'}O_c$ is $\beta^{'}$. From the velocity triangle in Fig. 1:

$$v_x = r_c^{'} \omega_b \sin\beta^{'} + r^{'} \omega_c \sin\alpha^{'} \tag{5}$$

In Fig.1:

$$r_c^{'} \sin\beta^{'} = r^{'} \sin\alpha^{'} \tag{6}$$

Eq. (6) is substituted in Eq. (5), and thus:

$$v_x = r^{'} \sin\alpha^{'}\left(\omega_b + \omega_c\right) = r_c^{'} \sin\beta^{'}\left(\omega_b + \omega_c\right) \tag{7}$$

From Eqs. (4) and (7):

$$\begin{cases} r\sin\alpha = r^{'} \sin\alpha^{'} \\ r_c \sin\beta = r_c^{'} \sin\beta^{'} \end{cases} \tag{8}$$

From Eq. (8), point $P^{'}$ is above point P on the *X*-axis. The radial motion deviates from the normal direction of the gear blank's pitch curve. Accordingly, the uneven allowance appears after each cutting cycle, even the small allowance or the non-allowance, as depicted in Fig. 2.

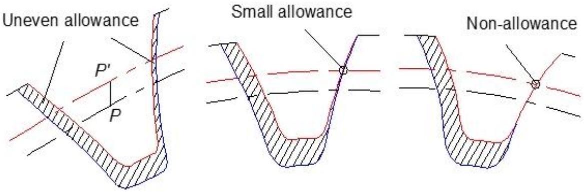


Fig. 2 - Distribution of allowance

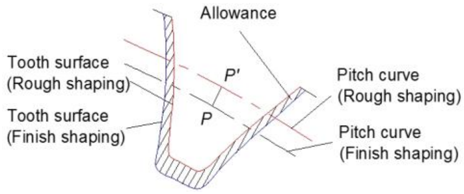


Fig. 3 - Principle of the machining method

Machining method with evenly distributed allowance of non-circular gear

In rough shaping, each position of the shaper cutter has a one-to-one correspondence with the position in finish shaping along the normal direction of the gear blank's pitch curve by changing the generation motion to ensure that the shaper cutter is constantly machining in the normal direction of the gear blank's pitch curve. The principle of the machining method with evenly distributed allowance is illustrated in Fig. 3.

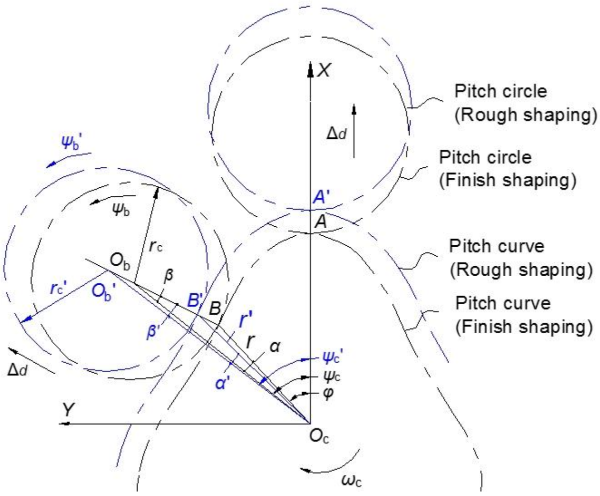


Fig. 4 - Normal linkage model of non-circular gear shaping

In Fig. 4, the coordinate system $S(O_c XY)$ is set up and the rotation centres of the gear blank and the shaper cutter are O_c and O_b respectively. In rough shaping, the distance between the shaper cutter and the gear blank increases Δd along the normal direction of the gear blank's pitch curve in each position compared with that in finish shaping. The rotation centre of the shaper cutter is point $O_b^{'}$, the relative velocity instantaneous centre is point $B^{'}$. The angle between the polar radius of the gear blank $r^{'}$ and $O_b^{'}O_c$ is $\alpha^{'}$, and the angle between the radius of the shaper cutter $r_c^{'}$ and $O_b^{'}O_c$ is $\beta^{'}$, as expressed in Eq. (9):

$$\begin{cases} \alpha^{'} = \arctan \dfrac{(r_c + \Delta d)\cos\mu}{r + (r_c + \Delta d)\sin\mu} \\ \beta^{'} = \arctan \dfrac{r\cos\mu}{r\sin\mu + (r_c + \Delta d)} \end{cases} \tag{9}$$

The rotation angle of the shaper cutter around point O_b is $\psi_b^{'}$ ($\psi_b^{'} = \psi_b$), the rotation angle of the gear blank around point O_c is $\psi_c^{'}$, and the distance between points $O_b^{'}$ and O_c is $l^{'}$, as follows:

$$\begin{cases} \psi_b^{'} = S_{AB}/r_c - \beta \\ \psi_c^{'} = \varphi - \alpha^{'} \\ l^{'} = r\cos\alpha^{'} + (r_c + \Delta d)\cos\beta^{'} \end{cases} \tag{10}$$

The arc length S_{AB} from points A to B is as follows:

$$S_{AB} = \int_0^{\varphi} \sqrt{r^2 + (dr/d\varphi)^2}\, d\varphi \tag{11}$$

Eqs. (1), (9), and (11) are substituted in Eq. (10). Thus, the normal linkage model of non-circular gear shaping is as follows:

$$\begin{cases} \psi_b^{'} = \dfrac{\int_0^{\varphi}\sqrt{r^2+\left(dr/d\varphi\right)^2}d\varphi}{r_c} - \arctan\dfrac{r\cos\mu}{r\sin\mu + r_c} \\ \psi_c^{'} = \varphi - \arctan\dfrac{\left(r_c+\Delta d\right)\cos\mu}{r+\left(r_c+\Delta d\right)\sin\mu} \\ l^{'} = r\cos\left[\arctan\dfrac{\left(r_c+\Delta d\right)\cos\mu}{r+\left(r_c+\Delta d\right)\sin\mu}\right] + \\ \qquad \left(r_c+\Delta d\right)\cos\left[\arctan\dfrac{r\cos\mu}{r\sin\mu+\left(r_c+\Delta d\right)}\right] \end{cases} \tag{12}$$

RESULTS

Validation example

A third-order elliptical gear is used as an example for verification. The equation of the pitch curve is as follows:

$$r\left(\varphi\right) = \frac{A\left(1-k^2\right)}{1-k\cos3\varphi} \tag{13}$$

where A is the calculated semi-major axis, k is the eccentricity, and φ is the polar angle.
The parameters are listed in Table 1.

Table 1

Parameters of third-order elliptical gear and shaper cutter

Parameters of gear	Value	Parameters of shaper cutter	Value
Number of teeth z	42	Number of teeth z_0	20
Normal module m_n [mm]	3	Normal module m_0 [mm]	3
Pressure angle α [°]	20	Pressure angle α_0 [°]	20
Order n	3	Helix angle β_0 [°]	0
Eccentricity k	0.1348		
Semi-major axis A [mm]	61.11		
Tooth width B [mm]	30		

Verification of the machining method with evenly distributed allowance based on virtual machining

Given that the rotation angle $\psi_c^{'}$ of the gear blank is known, the polar angle φ of the gear blank can be obtained by Eq. (12) with the Steffensen iteration. If each cutting cycle is completed with N steps, the rotation angle of the gear blank in each step will be $\Delta\psi_c^{'} = 2\pi/N$, and the rotation angle of the gear blank in non-circular gear shaping will be $\psi_c^{'}\left(i\right) = \left(i-1\right)\Delta\psi_c^{'}$, where $i = 1,2,\cdots,N+1$. Thus, $\psi_b^{'}\left(i\right)$ and $l^{'}\left(i\right)$ can be obtained. The rotation and moving amounts of the shaper cutter in each step are $\Delta\psi_b^{'}\left(i\right) = \psi_b^{'}\left(i+1\right) - \psi_b^{'}\left(i\right)$ and $\Delta l^{'}\left(i\right) = l^{'}\left(i+1\right) - l^{'}\left(i\right)$ respectively, where $i = 1,2,\cdots,N$.

The software platform used in this study is SolidWorks and the control program is written to call the application program interface functions (API) by using visual basic for applications (VBA) to develop the CAM system of non-circular gear shaping. Then, the virtual machining is performed. Fig. 5 shows the flow of virtual machining, as follows: the shaper cutter and the non-circular gear blank are modelled, the two parts are controlled to move progressively with the motion data, and the Boolean difference is performed after each step *(D.W. Liu et al., 2015D.Z. Li et al., 2014)*. The shaper cutter envelops all the teeth of the non-circular gear after a single rotation by the gear blank. The process of the virtual machining is demonstrated in Fig. 6.

The parameters of the gear and the shaper cutter are inputted. As shown in Fig. 7, when $\Delta d = 2$ mm, the virtual machining will be performed according to the traditional and normal linkage models of shaping, in turn, the models of rough shaping are obtained, when $\Delta d = 0$, the model of finish shaping is obtained.

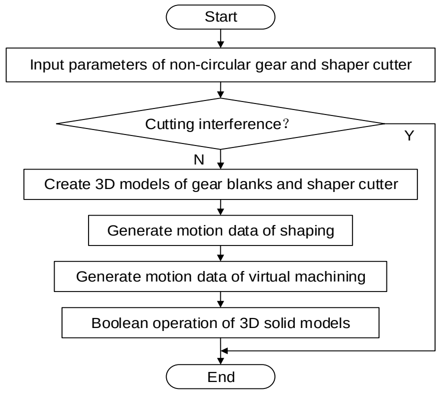


Fig.5 - Flowchart of virtual machining

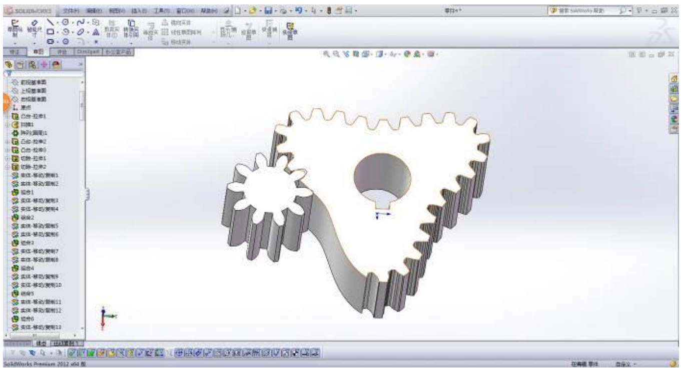
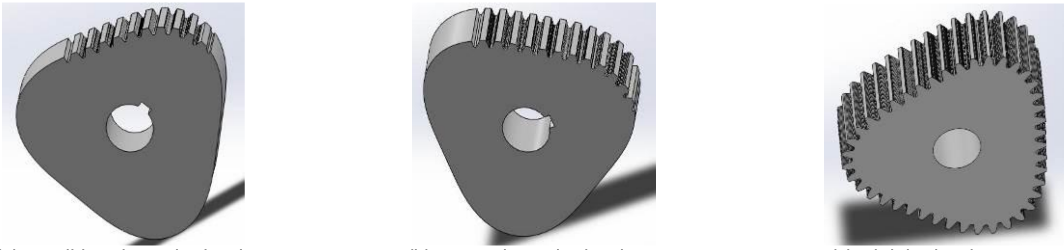


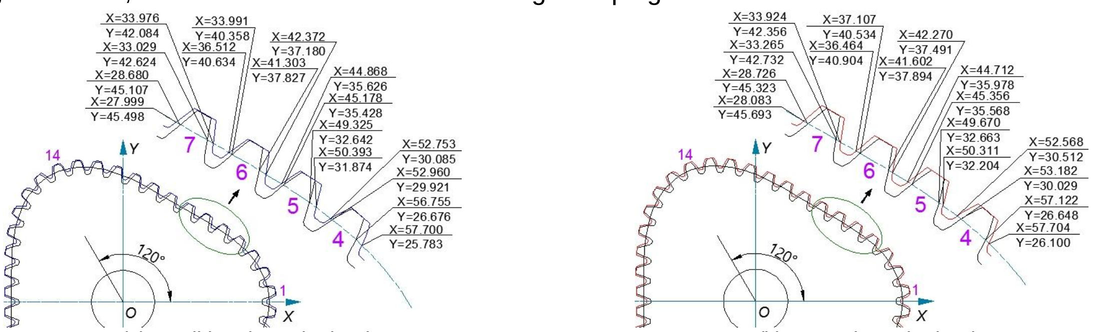
Fig.6 - Process of virtual machining



(a) Traditional rough shaping (b) Normal rough shaping (c) Finish shaping

Fig. 7 - Gear models obtained by virtual machining

The models of rough shaping are compared with the model of finish shaping, and the distribution of the allowance is analyzed as illustrated in Fig. 8. The distribution of the allowance after the traditional rough shaping is uneven, whereas that after the normal rough shaping is even.



(a) Traditional rough shaping (b) Normal rough shaping

Fig. 8 - Distribution of allowance after rough shaping

In Fig. 9, points A and D are the intersections of the pitch curve with the rough model, points B and C are the intersections of the pitch curve with the finish model, while the chord lengths of AB and CD are defined as the allowance of the left and right tooth surfaces respectively. Thus, the unevenness of the allowance δ is as follows:

$$\delta = 2\frac{\left|L_{AB} - L_{CD}\right|}{L_{AB} + L_{CD}} \cdot 100\% \tag{14}$$

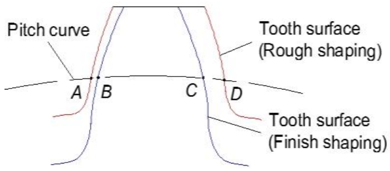


Fig. 9 - Definition of the unevenness of allowance

The coordinates of the intersection points are derived by the CAM system of non-circular gear shaping, a part of these coordinates are displayed in Table 2.

Table 2

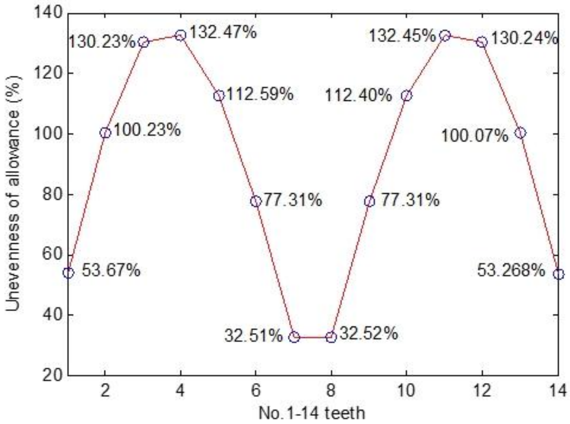
Coordinates of intersection points

Teeth	Points	Traditional shaping [mm]	Normal shaping [mm]
1	A	(69.040, 7.627)	(69.216, 8.005)
	B	(69.165, 7.146)	(69.426, 7.223)
	C	(69.925, 2.212)	(70.208, 2.114)
	D	(69.977, 1.352)	(70.257, 1.273)
2	A	(65.424, 16.158)	(65.437, 16.632)
	B	(65.589, 15.875)	(65.845, 15.942)
	C	(67.781, 11.387)	(68.108, 11.283)
	D	(68.135, 10.467)	(68.397, 10.525)

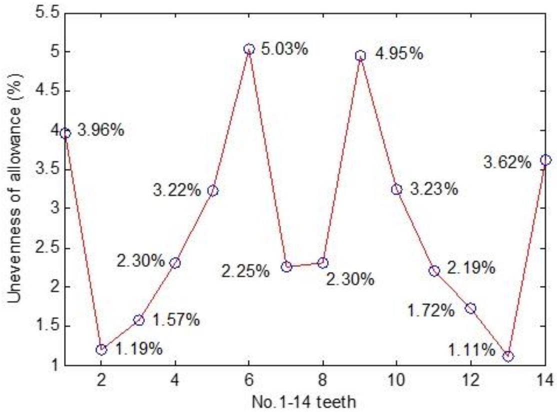
The teeth in the first period $[0, 2\pi/3]$ are selected for analysis because of the periodical symmetry of the pitch curve of the third-order elliptical gear. The coordinates of the intersections are substituted in Eq. (14) and the unevenness of the allowance is presented in Fig. 10.

The maximum and average unevenness of the allowance after the traditional rough shaping are 132.47% and 91.2344%. The maximum and average unevenness of the allowance after the normal rough shaping are 5.03% and 2.7606%; compared with the traditional shaping, the maximum and average unevenness are reduced by 127.44% and 88.4738% respectively.

The results show that the machining method with evenly distributed allowance can greatly reduce the unevenness of the allowance in non-circular gear shaping; the method is correct and feasible.



(a) Traditional rough shaping



(b) Normal rough shaping

Fig. 10 - Unevenness of allowance in virtual machining

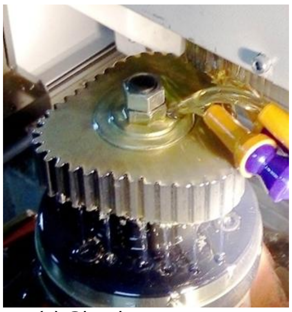
Verification of the machining method with evenly distributed allowance based on machining experiments

ARM+DSP+FPGA is used as the hardware platform of the CNC system, the code G84 for high-order elliptical gear shaping is developed by using the flexible electronic gearbox technology *(X.Q. Tian et al., 2015)*. G84 is defined as *G84 A_ N_ K_ R_ D_ S_*, where *A* is the semi-major axis, *N* is the number of the order, *K* is the eccentricity, *R* is the radius of the pitch circle of the shaper cutter, *D* is the polar angle of the start point, and *S* is the residual feeding amount.

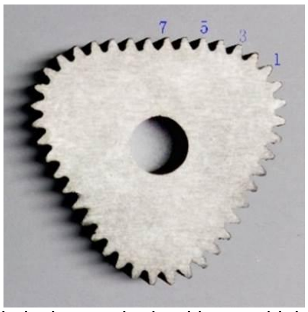
The elliptical gear used in the transplanting mechanism of a high-speed transplanter is taken as an example and the parameters are presented in Table 1.

The machining experiments are conducted on the YKS5132B CNC gear shaper.

The traditional and normal shaping is performed as illustrated in Fig. 11.



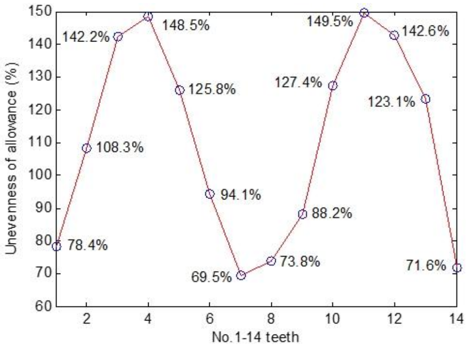
(a) Shaping process



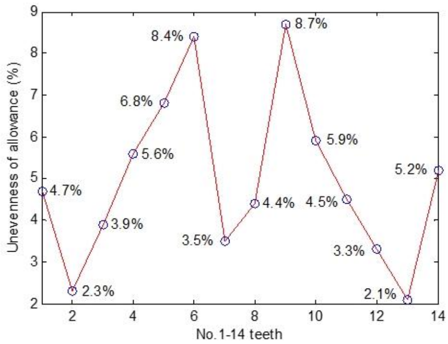
(b) Third-order elliptical gear obtained by machining experiments

Fig. 11 - Machining experiments of third-order elliptical gear

The non-circular gears obtained by the machining experiments are placed on the TESA Micro-Hite 3D coordinate measuring machine (CMM). The tooth profiles in the first period $[0,2\pi/3]$ are reconstructed according to the discrete points obtained from the CMM. The coordinates of the intersections of the tooth surfaces with the pitch curve are substituted in Eq.14, the unevenness of the allowance is presented in Fig.12.



(a) Traditional rough shaping



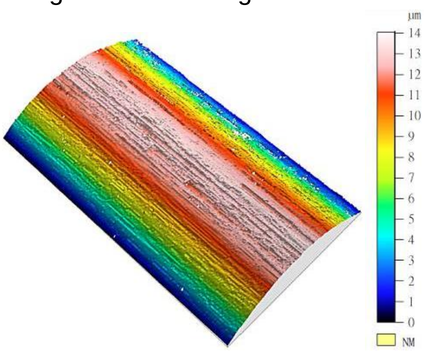
(b) Normal rough shaping

Fig. 12 - Unevenness of allowance in machining experiments

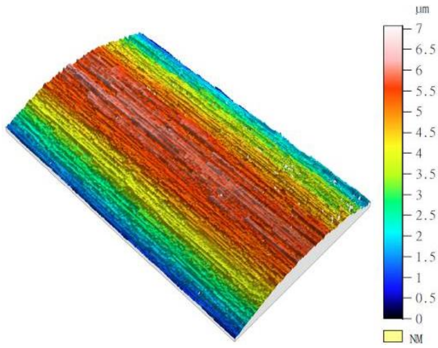
The results show that the machining method with evenly distributed allowance can reduce the average unevenness of the allowance from 110.21% to 4.95% in non-circular gear shaping, and thus the uneven distribution of the allowance can be improved effectively.

Measurement and analysis of the micro-topographies of non-circular gears

The No. 3 teeth of the non-circular gears obtained by the machining experiments are placed on the TRMOS TR-SCAN premium surface analysis instrument, and then 3D topography measurements are conducted. In Fig.13, the cut marks on the tooth surface of the non-circular gear obtained by the traditional shaping is uneven, additional pits and scratches appear, and the roughness is larger. The cut marks on the tooth surface of the non-circular gear obtained by the normal shaping is uniform, has no obvious defect, and the roughness is smaller. The results show that the micro-topography of the non-circular gear can be improved through the machining method with evenly distributed allowance.



(a) Traditional rough shaping



(b) Normal rough shaping

Fig. 13 - Micro-topographies of non-circular gears obtained by machining experiments

The micro-performances of the traditional and normal shaping are compared and analyzed according to ISO25178 and EUR15178N. S_a (arithmetical mean height) is the extension of R_a (arithmetical mean deviation) to a surface, which is expressed as the average of the absolute value of the difference in the height of each point compared with the arithmetical mean of the surface. S_a is used to evaluate the surface roughness. In Fig. 14, the parameter S_a of the non-circular gear obtained by the normal shaping is reduced from 3.18 μm to 1.26 μm, which is smaller than that of the traditional shaping.

S_k (core roughness depth) is calculated as the difference of heights at the areal material ratio values 0% and 100% on the equivalent line. Specifically, it is a value obtained by subtracting the minimum height from the maximum height of the core surface. S_{pk} (reduced peak height) represents the mean height of peaks above the core roughness, and S_{vk} (reduced valley depth) represents the mean depth of valleys below the core roughness. They are used to evaluate the functional features of surface roughness *(Jolivet S et al., 2014)*. In Fig. 15, the parameters S_k and S_{vk} of the non-circular gear obtained by the normal shaping are relatively smaller; meanwhile, the parameter S_{pk} is relatively larger. Therefore, the roughness profile has higher wear resistance and less initial wear quantity. The surface quality of the non-circular gear obtained by the normal shaping is also enhanced.

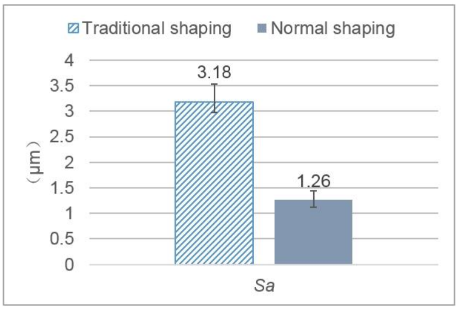


Fig. 14 - Contrast chart of S_a

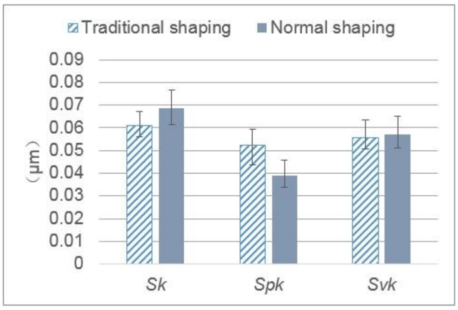


Fig. 15 - Contrast chart of S_k, S_{pk}, and S_{vk}

The results show that the machining accuracy and surface quality of non-circular gears can be improved through the machining method with evenly distributed allowance. Therefore, the working performance and service life of agricultural machinery products can be optimized.

CONCLUSIONS

The working performance and service life of agricultural machinery are mainly determined by the machining accuracy and surface quality of non-circular gears. This study analyzed the origin of the uneven distribution of allowance of non-circular gears, revealed the influences of the machining methods on the allowance distribution, and established the normal linkage model of non-circular gear shaping. Then, the machining method with evenly distributed allowance was verified by virtual and actual machining experiments, and the relationship between the machining methods and the surface quality of non-circular gears was analyzed by micro-topography measurements. The main conclusions are as follows:

(1) The degree of uneven distribution of the allowance of non-circular gears is proportional to the degree of the radial motion deviating from the normal direction of the gear blank's pitch curve. The normal linkage model of non-circular gear shaping set up in this study can adjust the geometric relationship between the shaper cutter and the gear blank in real time, and then, the even distribution of the allowance in the shaping process is realized.

(2) Theoretically, the average unevenness of the allowance can be reduced from 91.23% to 2.76% through the machining method with evenly distributed allowance. Meanwhile, the average unevenness of the allowance can be reduced from 110.21% to 4.95% in the actual machining. The method can evenly distribute the allowance of non-circular gears.

(3) The non-circular gears obtained by the machining method with evenly distributed allowance have uniform cut marks, no defects, and smooth micro-topography. Meanwhile, the surface roughness is smaller and the micro-performance of the roughness profile is better than that of the traditional shaping. The method can significantly improve the surface quality.

The conclusions in this study can effectively improve the uneven distribution of allowance caused by the change of the curvature of the gear blank's pitch curve and have an important guiding significance for improving the machining accuracy and surface quality of non-circular gears. In future studies, the machining method with evenly distributed allowance can be applied to finish machining methods with similar machining principles, such as the gear form-grinding based on the shaping linkage model and gear skiving, to further improve the machining accuracy and surface quality of non-circular gears. The conclusions in this study are crucial to improving the working accuracy and service life of agricultural machinery products.

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