

## ON FUZZY $SU$ -IDEAL TOPOLOGICAL STRUCTURE

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ABSTRACT. This paper discuss Fuzzy  $SU$ -ideal Topological Structure on  $SU$ -algebras, by connecting the two notions  $SU$ -algebras and Fuzzy Topology.

### 1. Introduction

The concept of a fuzzy set was introduced by Zadeh [7], and it is now a rigorous area of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behavior studies. The study of fuzzy algebraic structures was initiated by Rosenfeld [5]. The notion of a fuzzy set provides a natural framework for generalizing many concepts of general topology to what might be called fuzzy topological spaces [1]. In [2], Foster combined the structure of fuzzy topological spaces with that of fuzzy groups to form the notion of fuzzy topological group.

In 1966, Imai and Iseki [3] introduced two classes of abstract algebras;  $BCK$ -algebras and  $BCI$ -algebra. It is known that the  $BCK$ -algebras are a proper sub class of the class of  $BCI$ -algebras. During 2011 Keawrahn and Leerawat [6] introduced new structured algebra:  $SU$ -Algebra. Recently Sukklin and Leerawat [4], discussed Fuzzy  $SU$ -subalgebras and Fuzzy  $SU$ -ideals.

With all these motivation, this paper, discusses the fuzzy  $SU$ -ideal topological structure on  $SU$ -algebras and investigate some of their properties.

### 2. Preliminaries

In this section, some basic definitions that are required are recalled in the sequel.

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DEFINITION 2.1. Let  $X$  be a non-empty set. A mapping  $\mu : X \rightarrow [0, 1]$  is called a fuzzy set of  $X$ .

DEFINITION 2.2. The union of two fuzzy sets  $\mu_1$  and  $\mu_2$  of a set  $X$ , is defined to be a fuzzy set of  $X$  by  $(\mu_1 \cup \mu_2)(x) = \max\{\mu_1(x), \mu_2(x)\}$  for any  $x \in X$ .

DEFINITION 2.3. The intersection of two fuzzy sets  $\mu_1$  and  $\mu_2$  of a set  $X$ , is defined to be a fuzzy set of  $X$  by  $(\mu_1 \cap \mu_2)(x) = \min\{\mu_1(x), \mu_2(x)\}$  for any  $x \in X$ .

DEFINITION 2.4. Any fuzzy sets  $\mu_1$  and  $\mu_2$  of  $X$ , if  $\mu_1 \subset \mu_2 \Rightarrow \mu_1(x) \leq \mu_2(x)$  for any  $x \in X$ .

DEFINITION 2.5. Let  $\mu$  be a fuzzy set of  $X$ . Then the complement of  $\mu$  denoted by  $\mu'$  is defined to be  $\mu'(x) = 1 - \mu(x)$  for any  $x \in X$ .

DEFINITION 2.6. A fuzzy topology is a family  $\tau$  of fuzzy sets in  $X$  which satisfies the following conditions:

- (1)  $\bar{0}, \bar{1} \in \tau$  where  $\bar{0} = \mu(x) = 0 \forall x \in X$  &  $\bar{1} = \mu(x) = 1$  for any  $x \in X$ .
- (2) If  $\mu_1, \mu_2 \in \tau$  then  $\mu_1 \cap \mu_2 \in \tau$ .
- (3) If  $\mu_i \in \tau$  for each  $i \in I$  then  $\bigcup_{i \in I} \mu_i \in \tau$  where  $I$  is an indexing set.

REMARK 2.1. If  $X$  is a set with a fuzzy topology  $\tau$  then  $(X, \tau)$  is called a fuzzy topological space and any element in  $\tau$  is called a  $\tau$ -open fuzzy set in  $X$ .

DEFINITION 2.7. Let  $(X, \tau)$  be a fuzzy topological space. Let  $\mu$  be a fuzzy set in  $X$ . A fuzzy set  $\nu \in \tau$  is said to a neighbourhood of  $\mu$  if there exists a  $\tau$ -open fuzzy set  $\alpha$  such that  $\mu \subset \alpha \subset \nu$ . i.e  $\mu(x) \subset \alpha(x) \subset \nu(x)$  for any  $x \in X$ .

DEFINITION 2.8. Let  $\mu$  and  $\nu$  be fuzzy sets in a fuzzy topological space  $(X, \tau)$ . Let  $\mu \supset \nu$ . Then  $\nu$  is called an interior of  $\mu$  if  $\mu$  is a neighbourhood of  $\nu$ . The union of all interior fuzzy sets of  $\mu$  is again an interior of  $\mu$  and is denoted by  $\mu^0$ .

DEFINITION 2.9. A  $SU$ -algebra  $(X, *, 0)$  is a non-empty set  $X$  with a constant  $0$  and a binary operation  $*$  satisfying the following axioms:

- (1)  $((x * y) * (x * z)) * (y * z) = 0$ ,
- (2)  $x * 0 = x$ ,
- (3) if  $x * y = 0 \Rightarrow x = y \forall x, y, z \in X$ .

EXAMPLE 2.1. The following Caley's table

|   |   |   |   |   |
|---|---|---|---|---|
| * | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 2 | 1 | 0 |

shows  $(X = \{0, 1, 2, 3\}, *)$  is a  $SU$ -algebra.

DEFINITION 2.10 (Fuzzy  $SU$ -sub algebra). A fuzzy subset  $\mu$  of a  $SU$ -algebra  $(X, *, 0)$  is called a fuzzy  $SU$ -Subalgebra of  $X$  if  $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$  for any  $x, y \in X$ .

DEFINITION 2.11 (Fuzzy  $SU$ -Ideal). A fuzzy subset  $\mu$  of a  $SU$ -algebra  $(X, *, 0)$  is called a fuzzy  $SU$ -Ideal of  $X$  if

- (1)  $\mu(0) \geq \mu(x)$  and
- (2)  $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y)\}$  for any  $x, y \in X$ .

### 3. Fuzzy $SU$ -ideal Topological Structure (FSTS)

This section introduces Fuzzy  $SU$ -ideal Topological Structure (FSTS) on a  $SU$ -Algebra. For the sake of simplicity the notations  $A_1, A_2, A_3$  etc. have been used to represent the elements in  $\tau$ . Also  $X$  is a  $SU$ -Algebra unless otherwise specified.

DEFINITION 3.1 (Fuzzy  $SU$ -ideal Topological Structure (FSTS)). Let  $X$  be a  $SU$ -algebra.  $(X, \tau)$  is said to be a Fuzzy  $SU$ -ideal Topological Structure (FSTS) on a  $SU$ - algebra if there is a family  $\tau$  of fuzzy  $SU$ -ideals in  $X$  which satisfies the following conditions:

- (1)  $\bar{0}, \bar{1} \in \tau$
- (2) If  $A_1, A_2 \in \tau$  then  $A_1 \cap A_2 \in \tau$ .
- (3) If  $A_i \in \tau$  for each  $i \in I$  then  $\bigcup_{i \in I} A_i \in \tau$  where  $I$  is an indexing set.

REMARK 3.1. Any element in  $\tau$  is called a  $\tau$ -open fuzzy set in the  $SU$ -Algebra  $X$ .

EXAMPLE 3.1. Consider the  $SU$ -algebra defined in Example 2.1. Define the fuzzy  $SU$ -ideals  $A_i, i = 1, 2, 3, 4, 5, 6, 7, 8$  on  $X$  as follows.

$$\begin{aligned}
 A_1(x) &= \begin{cases} .7 & ; = 0 \\ .5 & ; = 1 \\ .4 & ; = 2 \\ .3 & ; = 3 \end{cases} &
 A_2(x) &= \begin{cases} .6 & ; = 0 \\ .4 & ; = 1 \\ .3 & ; = 2 \\ .2 & ; = 3 \end{cases} &
 A_3(x) &= \begin{cases} .8 & ; = 0 \\ .6 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases} \\
 A_4(x) &= \begin{cases} .5 & ; = 0 \\ .3 & ; = 1 \\ .2 & ; = 2 \\ .1 & ; = 3 \end{cases} &
 A_5(x) &= \begin{cases} .9 & ; = 0 \\ .7 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases} &
 A_6(x) &= \begin{cases} .7 & ; = 0 \\ .6 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases}
 \end{aligned}$$

Then  $(X, \tau = \{\bar{0}, \bar{1}, A_1, A_2, A_3, A_4, A_5, A_6\})$  is a FSTS on a  $SU$ -Algebra.

DEFINITION 3.2 (Fuzzy neighbourhood). Let  $(X, T)$  be a FSTS on a  $SU$ -Algebra on a  $SU$ -Algebra. A fuzzy set  $U$  in  $(X, \tau)$ , is called a fuzzy neighbourhood of a fuzzy set  $A$  if there exist an  $\tau$ -open fuzzy set  $O$  such that  $A \subset O \subset U$  i.e.  $A(x) \leq O(x) \leq U(x)$  for any  $x \in X$ .

EXAMPLE 3.2. Consider the  $SU$ -algebra defined in Example 2.1 and the FSTS  $(X, \tau)$  defined in Example 3.1. Then  $A_1$  is a fuzzy neighbourhood of a fuzzy set  $A_2$  i.e.  $A_2(x) \leq A_4(x) \leq A_1(x)$ .

DEFINITION 3.3 (Fuzzy interior). Let  $A$  and  $B$  be fuzzy sets in a FSTS on a  $SU$ -Algebra.  $(X, \tau)$  and let  $A \supset B$  then  $B$  is called a fuzzy interior of  $A$  iff  $A$  is a

fuzzy neighbourhood of  $B$ . The union of all fuzzy interior sets of  $A$  is again a fuzzy interior of  $A$  and it is denoted by  $A^0$ .

EXAMPLE 3.3. Consider the  $SU$ -algebra defined in Example 2.1 and the FSTS  $(X, \tau)$  defined in Example 3.1. Then  $A_5$  is a fuzzy neighbourhood of a fuzzy sets  $A_2, A_4, A_4, A_6$  and

$$\begin{aligned} A_2(x) &\leq A_4(x) \leq A_5(x), \quad \forall x \in X, \\ A_3(x) &\leq A_1(x) \leq A_5(x), \quad \forall x \in X, \\ A_4(x) &\leq A_1(x) \leq A_5(x), \quad \forall x \in X, \\ A_6(x) &\leq A_1(x) \leq A_5(x), \quad \forall x \in X. \end{aligned}$$

From this follows that  $A_2, A_3, A_4, A_6$  are fuzzy interiors of  $A_5(x)$  and

$$A^0 = \cup\{A_2, A_3, A_4, A_6\} = \max\{A_2(x), A_3(x), A_4(x), A_6(x)\} = A_3(x)$$

which is also interior of  $A_5$ .

DEFINITION 3.4. Let  $(X, \tau)$  be a FSTS on  $X$ . A sequence of fuzzy sets

$$\{A_n : n = 1, 2, 3, \dots\}$$

in  $SU$ -algebra  $X$  and it is said to be eventually contained in a fuzzy set  $A$  iff there is an integer  $m$  such that  $n \geq m \Rightarrow A_n \subset A$ .

DEFINITION 3.5. Let  $(X, \tau)$  be a FSTS on  $X$ . Let  $\{A_n : n = 1, 2, 3, \dots\}$  be a sequence of fuzzy  $SU$ -ideals in  $X$ . The sequence is said to converge to a fuzzy  $SU$ -ideal  $A$  in  $X$  iff it is eventually contained in each fuzzy neighbourhood of  $A$ .

EXAMPLE 3.4. Consider the  $SU$ -algebra defined in Example 2.11. Define the fuzzy  $SU$ -ideals  $A_i$ ,  $i = 1, 2, 3, 4, 5, 6, 7, 8$  on  $X$  as follows.

$$\begin{aligned} A_1(x) &= \begin{cases} .7 & ; = 0 \\ .5 & ; = 1 \\ .4 & ; = 2 \\ .3 & ; = 3 \end{cases} & A_2(x) &= \begin{cases} .6 & ; = 0 \\ .4 & ; = 1 \\ .3 & ; = 2 \\ .2 & ; = 3 \end{cases} & A_3(x) &= \begin{cases} .8 & ; = 0 \\ .6 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases} \\ \\ A_4(x) &= \begin{cases} .5 & ; = 0 \\ .3 & ; = 1 \\ .2 & ; = 2 \\ .1 & ; = 3 \end{cases} & A_5(x) &= \begin{cases} .9 & ; = 0 \\ .7 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases} & A_6(x) &= \begin{cases} .7 & ; = 0 \\ .6 & ; = 1 \\ .5 & ; = 2 \\ .4 & ; = 3 \end{cases} \\ \\ & & A_7(x) &= \begin{cases} .4 & ; = 0 \\ .2 & ; = 1 \\ .1 & ; = 2 \\ 0 & ; = 3 \end{cases} & A_8(x) &= \begin{cases} .6 & ; = 0 \\ .5 & ; = 1 \\ .3 & ; = 2 \\ .2 & ; = 3 \end{cases} \end{aligned}$$

Then  $(X, \tau = \{\bar{0}, \bar{1}, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\})$  is a FSTS on a  $X$  and take

$$A = A_4(x) = \begin{cases} .5 & ; = 0 \\ .3 & ; = 1 \\ .2 & ; = 2 \\ .1 & ; = 3 \end{cases}$$

Consider the sequence  $A_n = \{A_2, A_3, A_6, A_7\}$ . Now

$$A_4(x) \leq A_8(x) \leq A_1(x) \quad \text{and} \quad A_4(x) \leq A_8(x) \leq A_5(x).$$

Thus  $A_1, A_5$  are neighbourhood of  $A$  and  $A_n$  is contained in  $A_1, A_5$ . Finally the sequence  $A_n$  converges to  $A$ .

**THEOREM 3.1.** *Let  $(X, \tau)$  be a FSTS on  $X$ . A fuzzy set  $A$  is  $\tau$ -open if and only if for each fuzzy set  $B$  contained in  $A$ ,  $A$  is a fuzzy neighbourhood of  $B$ .*

**PROOF.** Suppose a fuzzy set  $A$  is  $\tau$ -open. Let  $B$  be any fuzzy set contained in  $A$ . Since  $A$  is open, and  $A \subset B$  we have  $B \subset A \subset A$  and  $A$  is fuzzy neighbourhood of  $B$ .

Conversely, for each fuzzy set  $B$  contained in  $A$ ,  $A$  is a fuzzy neighbourhood of  $B$ . Since  $A \subset A$  by our assumption,  $A$  is a fuzzy neighbourhood of  $A$ . Hence there exists an open fuzzy set  $O$  such that  $A \subset O \subset A$ . Therefore  $A = O$  and  $A$  are open.  $\square$

**DEFINITION 3.6.** *Let  $(X, \tau)$  be a FSTS on a  $SU$ -Algebra. Let  $A$  be a fuzzy set in  $X$ . The set of all fuzzy neighbourhood  $A \in \tau$ ,  $\mathfrak{U}_A$  is defined to be the fuzzy neighbourhood system of  $A$ .*

**THEOREM 3.2.** *Let  $(X, \tau)$  be a FSTS on a  $SU$ -Algebra. Let  $A$  be a fuzzy set in  $X$ . Let  $\mathfrak{U}_A$  be the fuzzy neighbourhood system of a fuzzy set  $A$ . Then*

- (1) *The finite intersections of members of  $\mathfrak{U}_A$  belong to  $\mathfrak{U}_A$*
- (2) *Any fuzzy set of  $X$  which contain a member of  $\mathfrak{U}_A$  belong to  $\mathfrak{U}_A$*

**PROOF.** (1) Let  $(X, \tau)$  be a FSTS on  $X$ . Let  $A$  be a fuzzy set in  $X$ . Let  $\mathfrak{U}_A$  be a fuzzy neighbourhood system of  $A$ . Let  $R, S \in \mathfrak{U}_A$  Hence  $R$  and  $S$  are fuzzy neighbourhood of  $A$ . Thus there exists open fuzzy sets  $R_0$  and  $S_0$  Such that  $A \subset R_0 \subset R$  and  $A \subset S_0 \subset S$  respectively. Hence  $A \subset R_0 \cap S_0 \subset R \cap S$ . Follows  $R \cap S$  is a fuzzy neighbourhood of  $A$ . Hence the intersection of two members of  $\mathfrak{U}_A$  is again a member of  $\mathfrak{U}_A$ . It automatically follows that the intersection of any finite number of members of  $\mathfrak{U}_A$  is again a member of  $\mathfrak{U}_A$

(2) Let  $R$  be a fuzzy set that contains a member of  $\mathfrak{U}_A$  say  $U$ . Hence  $R$  contains a neighbourhood  $U$  of  $A$ , that is  $U \subset R$ ,  $U \in \mathfrak{U}_A$ . Since  $U$  is a fuzzy neighbourhood of  $A$ , there exists a  $T$ -open fuzzy set  $O$  such that  $A \subset O \subset U \subset R$ . Thus  $A \subset O \subset R$  and  $R$  is a fuzzy neighbourhood of  $A$ . Therefore  $R \in \mathfrak{U}_A$ .  $\square$

**THEOREM 3.3.** *Let  $(X, \tau)$  be a FSTS on  $X$ . Let  $A$  be a fuzzy set in  $X$ . Then*

- (1)  *$A^0$  is open and is the largest open fuzzy set contained in  $A$ .*
- (2) *The fuzzy set  $A$  is open iff  $A = A^0$*

PROOF. (1) Let  $(X, \tau)$  be a FSTS on a  $SU$ -Algebra. Let  $A$  be a fuzzy set in  $X$ . By def of fuzzy interior,  $A^0$  is again a fuzzy interior set of  $A$ . Hence there exist an  $T$ - open fuzzy set  $O$  such that  $A^0 \subset O \subset A$  but  $O$  is an fuzzy interior set of  $A, O \subset A^0$  Hence  $A^0 = O$ . Thus  $A^0$  is open and is the largest open fuzzy set contained in  $A$ .

(2) Suppose the fuzzy set  $A$  is open. If  $A$  is open, then  $A \subset A^0$  for  $A$  is an fuzzy interior set of  $A$ . Hence  $A = A^0$ .

Conversely, Suppose  $A = A^0$  By definition, the union of all fuzzy interior sets of  $A$  is called the interior of  $A$  and is denoted by  $A^0$ . Thus  $A$  is a neighbourhood of  $A^0$  and therefore fuzzy set  $A$  is open.  $\square$

**THEOREM 3.4.** *If the fuzzy neighbourhood system of each fuzzy set in a FSTS  $(X, \tau)$  is countable, then a fuzzy set  $A$  is open if and only if each sequence of fuzzy sets,  $\{A_n, n = 1, 2, 3, \dots\}$  which converges to a fuzzy set  $B$  contained in  $A$  is eventually contained in  $A$ .*

PROOF. Suppose  $A$  is open. Given each sequence of fuzzy sets,  $\{A_n, n = 1, 2, 3, \dots\}$  which converges to a fuzzy set  $B$ . Since  $A$  is open and  $B$  contained in  $A$ . Thus  $A$  is a neighbourhood of  $B$ . Hence  $\{A_n, n = 1, 2, 3, \dots\}$  is contained in  $A$ .

Conversely, for each  $B \subset A$ , let  $U_1, U_2, \dots, U_n, \dots$  be the neighbourhood system of  $B$ . Let  $V_n = \bigcap_1^n \{U_i\}$ . Then  $V_1, V_2, \dots, V_n, \dots$  is a sequence which is eventually contained in each fuzzy neighbourhood of  $B$ . Hence there is an  $m$  such that for  $n > m, V_n \subset A$ . Thus  $V_n$  are fuzzy neighbourhood of  $B$ . Therefore  $A$  is open.  $\square$

**DEFINITION 3.7.** *Let  $f$  be a function from  $X$  to  $Y$ . Let  $\lambda$  be a fuzzy set in  $Y$ . The inverse of function,  $f^{-1}$  is defined as*

$$\lambda_{f^{-1}(x)} = \lambda(f(x)) \text{ for all } x \in X.$$

Let  $\mu$  be a fuzzy set in  $X$ . The image of  $\mu$  is defined as

$$\mu(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

**DEFINITION 3.8.** *Let  $(X, \tau)$  and  $(Y, \sigma)$  be a FSTS on  $SU$ -Algebra  $X$  and  $Y$  respectively. A function  $f$  from  $(X, \tau)$  to  $(Y, \sigma)$  is called a  $F$ -continuous function if the inverse of each  $\sigma$ -open fuzzy set is  $\tau$ -open fuzzy set.*

**THEOREM 3.5.** *Let  $(X, \tau)$  and  $(Y, \sigma)$  be a FSTS on  $SU$ -Algebra  $X$  and  $Y$  respectively and  $f$  be a function from  $X$  to  $Y$ . Then the function  $f$  is  $F$ -continuous if and only if the inverse image of every closed fuzzy set is closed.*

PROOF. Suppose the function  $f$  is  $F$ -continuous. Then the inverse of each  $\sigma$ -open fuzzy set is  $\tau$ -open. Let  $\sigma'$  be the set of closed fuzzy set in  $Y$ . Then

$$\mu_{f^{-1}(\sigma')}(x) = \mu'_{\sigma'}(f(x)) = \mu_{\sigma'}(f(x)) = 1 - \mu_{\sigma}(f(x)) = 1 - \mu_{f^{-1}(\sigma)}(x) = \mu'_{f^{-1}(\sigma)}(x).$$

Thus  $f^{-1}(\sigma') = \{f^{-1}(\sigma)\}'$  for all  $x$  in  $X$ . Since  $f$  is  $F$ -continuous, the inverse of every closed fuzzy set is closed.

Conversely, let  $\sigma$  be the set of open fuzzy set in  $Y$ . Then  $\mu_{f^{-1}(\sigma)}(x) = \mu_{\sigma}(f(x))$  for all  $x$  in  $X$ . Since the inverse of every closed fuzzy set is closed. Hence the inverse of every open fuzzy set is open. Therefore  $f$  is  $F$ -continuous.  $\square$

**THEOREM 3.6.** *Let  $(X, \tau)$  and  $(Y, \sigma)$  be a FSTS on SU-Algebra  $X$  and  $Y$  respectively and  $f$  be a function from  $X$  to  $Y$ . Then for each fuzzy set  $A$  in  $X$ , the inverse of every neighbourhood of  $f(A)$  is a neighbourhood of  $A$  if and only if for each fuzzy set  $A$  in  $X$  and each neighbourhood  $v$  of  $f(A)$ , there is neighbourhood  $w$  of  $A$  such that  $f(w) \subset v$ .*

**PROOF.** Let  $\mathcal{A}$  be the fuzzy sets of  $X$ . Let  $\mathcal{U}, \mathcal{I}$  be the family of neighbourhoods of fuzzy sets and their image. Let  $A \in \mathcal{A}$ . Suppose  $v \in \mathcal{I}$ ,  $w \in \mathcal{U}$  is the neighbourhood of  $f(A)$  and  $A$ . Since the inverse of every neighbourhood of  $f(A)$  is a neighbourhood of  $A$ . Thus  $f(w) = f(f^{-1}(v)) \rightarrow 1$ . If  $f^{-1}(y)$  is not empty, then

$$\mu_{f(f^{-1}(v))}(y) = \sup_{z \in f^{-1}(y)} \mu_{f^{-1}(v)}(z) = \sup_{z \in f^{-1}(y)} \{\mu_v(f(z))\} = \mu_v(y)$$

for all  $y$  in  $Y$ . If  $f^{-1}(y)$  is not empty, then  $\mu_{f(f^{-1}(v))}(y) = 0$ . Hence  $\mu_{f(f^{-1}(v))}(y) \leq \mu_v(y)$  for all  $y$  in  $Y$  and  $f(f^{-1}(v)) \subset v$ . Thus  $1 \Rightarrow f(w) \subset v$ .

Conversely, let  $V$  be a neighbourhood of  $f(A)$ . Since there is a neighbourhood  $w$  of  $A$  such that  $f(w) \subset v$ . Hence  $f^{-1}(f(w)) \subset f^{-1}(v) \rightarrow 2$ . Then for any  $x \in X$  we have

$$\mu_{f^{-1}(f(w))}(x) = \mu_{f(w)}(f(x)) = \sup_{z \in f^{-1}(f(x))} \{\mu_w(z)\} \geq \mu_w(x).$$

Thus  $w \subset f^{-1}(f(w))$  and  $2 \Rightarrow w \subset f^{-1}(f(w)) \subset f^{-1}(v)$ . Finally  $f^{-1}(v)$  is a neighbourhood of  $w$ .  $\square$

**THEOREM 3.7.** *Let  $(X, \tau)$  and  $(Y, \sigma)$  be a FSTS on SU-Algebra  $X$  and  $Y$  respectively and  $f$  be a function from  $X$  to  $Y$ . If the function  $f$  is  $F$ -continuous then for each fuzzy set  $A$  in  $X$ ; the inverse of every neighbourhood of  $f(A)$  is a neighbourhood of  $A$ :*

**PROOF.** Let  $\mathcal{A}$  be the fuzzy sets of  $X$ . Let  $\mathcal{I}$  be the family of neighbourhood of fuzzy set on  $\mathcal{A}$ . Let  $A \in \mathcal{A}$  and  $v \in \mathcal{I}$ . Then  $A$  is a fuzzy set in  $X$  and  $V$  is a neighbourhood of  $f(A)$ . There is an open neighbourhood  $w$  of  $f(A)$  such that  $f(A) \subset w \subset v$  and  $f^{-1}(f(A)) \subset f^{-1}(w) \subset f^{-1}(v) \rightarrow 1$ . Since  $f$  is  $F$ -continuous,  $f^{-1}(w)$  is open. Further on

$$\mu_{f^{-1}(f(A))}(x) = \mu_{f(A)}(f(x)) = \sup_{z \in f^{-1}(f(x))} \{\mu_A(z)\} \geq \mu_A(x)$$

for all  $x \in X$ . Therefore  $A \subset f^{-1}(f(A))$ . Thus  $1 \Rightarrow A \subset f^{-1}(f(A)) \subset f^{-1}(w) \subset f^{-1}(v)$  and  $A \subset f^{-1}(w) \subset f^{-1}(v)$ . Finally  $f^{-1}(v)$  is a neighbourhood of  $A$ .  $\square$

**THEOREM 3.8.** *Let  $(X, \tau)$  and  $(Y, \sigma)$  be a FSTS on SU-Algebra  $X$  and  $Y$  respectively. If for each fuzzy set  $A$  in  $X$  and each neighbourhood  $v$  of  $f(A)$ , there is neighbourhood  $w$  of  $A$  such that  $f(w) \subset v$ , then for each sequence of fuzzy sets  $\{A_n, n = 1, 2, 3, \dots\}$  in  $X$  which converges to a fuzzy set  $A$  in  $X$ , the sequence  $\{f(A_n), n = 1, 2, 3, \dots\}$  converges to  $f(A)$ .*

PROOF. Since  $v$  is a neighbourhood of  $f(A)$ , there is a neighbourhood  $w$  of  $A$  such that  $f(w) \subset v$ . Since  $\{A_n, n = 1, 2, 3, \dots\}$  in  $X$  which converges to  $A$  in  $X$  by definition of converges,  $\{A_n, n = 1, 2, 3, \dots\}$  is eventually contained in  $w$ . Then there is an  $m$  such that for  $n \geq m$ ,  $A_n \subset w$ . Thus  $f(A_n) \subset f(w)$  and  $f(A_n) \subset f(w) \subset v$  for  $n \geq m$  since  $v$  is a neighbourhood of  $f(A)$ . By definition of converges,  $\{f(A_n), n = 1, 2, 3, \dots\}$  converges to  $f(A)$ .  $\square$

#### 4. Conclusion

This paper dealt several interesting results by connecting the notions of fuzzy SU-ideal and fuzzy topology. It can be further extended to intuitionistic fuzzy and interval valued fuzzy sub structures on SU-algebras by connecting with respective fuzzy topological settings.

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