BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 8(2018), 135-143

DOI: 10.7251/BIMVI1801135A

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

WEAK SUB SEQUENTIAL CONTINUOUS MAPS IN NON ARCHIMEDEAN MENGER PM SPACE VIA C-CLASS FUNCTIONS

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Abstract. This study deals with an establishment of some common fixed point theorems for weak sub sequential continuous and compatibility of type (E) maps via C-class functions in a non Archimedean Menger Probabilistic Metric space.

1. Introduction

Menger [12] extended the notion of metric space to probabilistic metric space (briefly PM space) by replacing non-negative numbers with random variable that took only non negative real values. One can see the further development in the said field by going through the works of the authors [13, 14, 15] in detail. Istratescu and Crivat [10] introduced the notion of non-Archimedean PM-space and gave some basic topological preliminaries on it. Further, Istrăţescu [8, 9] generalized the results of Sehgal and Bharucha [14] to N.A.Menger PM space where as Achari [1] generalized the results of Istrăţescu [8, 9] by establishing common fixed point theorems for qausi-contraction type of mappings in non-Archimedean PM - space. Chang [6] considered single and multivalued mappings to prove common fixed point theorems in non Archimedean Menger probabilistic metric spaces. Working in the same line , Cho et. al. [7] came out with some commen fixed point results for campatible mappings of type (A) in non-Archimedean Menger PM- spaces. Bouhadjera and Thobie [4] proved common fixed point theorems for pairs of sub compatible maps. Recently, Ansari [2] introduced the concept of C-class functions and established the related fixed point theorems via these special class of functions

2010 *Mathematics Subject Classification.* 47H10; 54H25.

Key words and phrases. fixed point , non Archimedean Menger PM-Space, compatible maps of type (E), weak sub sequential continuous maps (wsc), C-class Function.

whereas Beloul [5] gave some fixed point theorems for two pairs of self mappings satisfying contractive conditions by using the weak sub-sequential mappings with compatibility of type (E) . Motivated from [2] and [5], we established some common fixed point theorems for weak sub sequential continuous and compatibility of type (E) maps via C-class functions in non Archimedean Menger probabilistic metric space.

2. Preliminaries

Definition 2.1. ([10]) Let X be any nonempty set and D be the set of all left-continuous distribution functions. An ordered pair (X, F) is called a non-Archimedean probabilistic metric space (briefly, a N.A. PM-space) if F is a mapping from $X \times X$ into mapping D satisfying the following conditions (we shall denote the distribution function $F(x, y)$ by $F_{x,y}, \forall x, y \in X$) :

$$(\forall t > 0)(F_{x,y}(t) = 1) \Longleftrightarrow x = y; \tag{2.1}$$

$$(\forall x, y \in X)(F_{x,y}(0) = 0); \tag{2.2}$$

$$(\forall x, y \in X)(F_{x,y}(t) = F_{y,x}(t)); \tag{2.3}$$

$$(\forall x, y, z \in X)(F_{x,y}(t_1) = 1 \wedge F_{y,z}(t_2) = 1, \Longrightarrow F_{x,z}\{max(t_1, t_2)\} = 1). \tag{2.4}$$

Definition 2.2. ([12]) A $t-$ norm is a function $\Delta : [0, 1]\times[0, 1] \to [0, 1]$ which is associative, commutative, non-decreasing in each coordinate and $\Delta(a, 1) = a, \forall a \in [0, 1]$.

Definition 2.3. ([10]) A N.A. Menger PM-space is an ordered triplet (X, F, Δ), where Δ is a $t-$ norm and (X, F) is a non-Archimedean PM-space satisfying the following condition:

$$F_{(x,z)}(max\{t_1, t_2\}) \geqslant \Delta(F_{(x,y)}(t_1), F_{(y,z)}(t_2)), \forall x, y, z \in X \, and \, t_1, t_2 \geqslant 0. \tag{2.5}$$

For more details we refer to [10].

Definition 2.4. ([6], [7]) A N.A. Menger PM-space (X, F, Δ), is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F_{(x,z)}(t)) \leqslant g(F_{(x,y)}(t)) + g(F_{(y,z))}(t),$$

$\forall x, y, z \in X$ and $t \geqslant 0$, where $\Omega = \{g|g : [0, 1] \to [0, \infty)$ is continuous, strictly decreasing with $g(1) = 0$ and $g(0) < \infty\}$.

Definition 2.5. ([6], [7]) A N.A. Menger PM-space (X, F, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that

$$g(\Delta(s, t)) \leqslant g(s) + g(t)$$

for all $s, t \in (0, 1)$.

Remark 2.1. ([7])

(2.6)

A N.A. Menger PM-space (X, F, Δ) is of type $(D)_g$, then it is of type $(C)_g$.

(2.7)

If (X, F, Δ) is a N.A.Menger PM-space and $\Delta \geqslant \Delta_m$ where $\Delta_m(s,t) = max\{s+t-1, 1\}$, then (X, F, Δ) is of type $(D)_g$ for $g \in \Omega$ defined by $g(t) = 1 - t$.

Throughout this paper, let (X, F, Δ) be a complete N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing $t-$ norm Δ.

Let $\phi : [0, \infty) \to [0, \infty)$ be a function satisfying the following condition:

(τ) ϕ is a upper semi continuous from the right and $\phi(t) < t$ for all $t > 0$.

Definition 2.6. ([6], [7]) A sequence $\{x_n\}$ in a N.A.Menger PM space (X, F, Δ) converges to a point x if and only if for each $\epsilon > 0$, $\lambda > 0$ there exists an integer $M(\epsilon, \lambda)$ such that $g(F(x_n, x; \epsilon) < g(1-\lambda)$ for all $n > M$.

Definition 2.7. ([6], [7]) A sequence $\{x_n\}$ in a N.A.Menger PM space is a Cauchy sequence if and only if for each $\epsilon > 0$, $\lambda > 0$ there exists an integer $M(\epsilon, \lambda)$ such that $g(F(x_n, x_{n+p}; \epsilon)) < g(1-\lambda)$ for all $n > M$ and $p \geqslant 1$.

Lemma 2.1. ([7]) *If a function* $\phi : [0, \infty) \to [0, \infty)$ *satisfies the condition* (τ) , *then we have*

(2.8) $\quad For\, all\, t \geqslant 0,\, lim_{n\to\infty}\phi^n(t) = 0,\, where\, \phi^n(t)\, is\, the\, nth\, iteration\, of\, t.$

(2.9) $If\, \{t_n\}\, is\, a\, non-decreasing\, sequence\, of\, real\, numbers\, and\, \{t_{n+1}\} \leqslant \phi(t_n),$ $n = 1, 2, ...,$ *then* $lim_{n\to\infty}t_n = 0$. *In particular, if* $t \leqslant \phi(t)$ *for all* $t \geqslant 0$, *then* $t = 0$.

Singh et al. [16, 17] introduced the notion of compatibility of type (E), in the setting of the N.A.Menger PM spaces, it becomes

Definition 2.8. Two self maps A and S on a N.A.Menger PM space (X, F, Δ) are said to be compatible of type (E), if

$$lim_{n\to\infty}S^2x_n = lim_{n\to\infty}SAx_n = Az$$

and

$$lim_{n\to\infty}A^2x_n = lim_{n\to\infty}ASx_n = Sz,$$

whenever $\{x_n\}$ is a sequence in X such that

$$lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = z$$

for some $z \in X$.

Definition 2.9. Two self maps A and S on a N.A.Menger PM space (X, F, Δ) are said to be A-compatible of type (E), if

$$lim_{n\to\infty}A^2x_n = lim_{n\to\infty}ASx_n = Sz$$

for some $z \in X$. Pair A and S are said to be S-compatible of type (E), if

$$lim_{n\to\infty}S^2x_n = lim_{n\to\infty}SAx_n = Az$$

for some $z \in X$.

REMARK 2.2. It is also interesting to see that if A and S are compatible of type (E), then they are A-Compatible and S-Compatible of type (E), but the converse is not true (see example 1 in [5]).

Bouhadjera and Thobie [4] introduced the concept of sub-sequential continuity as follows:

DEFINITION 2.10. Two self maps A and S of a metric space (X, d) are said to be sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = t,$$

for some $t \in X$ and $lim_{n\to\infty} ASx_n = At$, and $lim_{n\to\infty} SAx_n = St$.

Motivated by the definition (2.10) and [5], we define the following.

DEFINITION 2.11. The pair $\{A, S\}$ defined on a N.A.Menger PM space (X, F, Δ) is said to be weakly sub-sequentially continuous (in short wsc), if there exists a sequence $\{x_n\}$ such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $lim_{n\to\infty} ASx_n = Az$, or $lim_{n\to\infty} SAx_n = Sz$

DEFINITION 2.12. The pair $\{A, S\}$ defined on a N.A.Menger PM space (X, F, Δ) is said to be S sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$ and $lim_{n\to\infty} SAx_n = Sz$.

DEFINITION 2.13. The pair $\{A, S\}$ defined on a N.A. Menger PM space (X, F, Δ) is said to be A sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z,$$

for some $z \in X$ and $lim_{n\to\infty} ASx_n = Az$.

REMARK 2.3. If the pair $\{A, S\}$ is A-subsequentially continuous (or S-sub sequentially continuous), then it is wsc. (see example 3 in [5])

In 2014 the concept of C-class functions was intorduced by A.H.Ansari [**2**]. By using this concept, we can generalize many fixed point theorems in the literature.

DEFINITION 2.14. ([**2**]) A continuous function $f : [0, \infty)^2 \to \mathbb{R}$ is called C-class function if for any $s, t \in [0, \infty)$, the following conditions hold:
(1) $f(s, t) \leqslant s$;
(2) $f(s, t) = s$ implies that either $s = 0$ or $t = 0$.

Note for some f we have that $f(0, 0) = 0$.

An extra condition on f that $f(0, 0) = 0$ could be imposed in some cases if required. The letter $\mathcal{C}$ will denote the class of all C-class functions.

EXAMPLE 2.1. ([**2**]) The following functions $F : [0, \infty)^2 \to \mathbb{R}$ are elements of $\mathcal{C}$, for all $s, t \in [0, \infty)$:
(1) $f(s, t) = s - t$, $F(s, t) = s \Rightarrow t = 0$;

(2) $F(s,t) = ms$, $0<m<1$, $F(s,t) = s \Rightarrow s = 0$;

(3) $F(s,t) = \frac{s}{(1+t)^r}$; $r \in (0,\infty)$, $F(s,t) = s \Rightarrow s = 0$ or $t = 0$;

(4) $F(s,t) = \log(t + a^s)/(1+t)$, $a > 1$, $F(s,t) = s \Rightarrow s = 0$ or $t = 0$;

(5) $F(s,t) = \ln(1 + a^s)/2$, $a > e$, $F(s,1) = s \Rightarrow s = 0$;

(6) $F(s,t) = (s+l)^{(1/(1+t)^r)} - l$, $l > 1, r \in (0,\infty)$, $F(s,t) = s \Rightarrow t = 0$;

(7) $F(s,t) = s\log_{t+a} a$, $a > 1$, $F(s,t) = s \Rightarrow s = 0$ or $t = 0$;

(8) $F(s,t) = s - (\frac{1+s}{2+s})(\frac{t}{1+t})$, $F(s,t) = s \Rightarrow t = 0$;

(9) $F(s,t) = s\beta(s)$, $\beta : [0,\infty) \to [0,1)$, and is continuous, $F(s,t) = s \Rightarrow s = 0$;

(10) $F(s,t) = s - \frac{t}{k+t}$, $F(s,t) = s \Rightarrow t = 0$;

(11) $F(s,t) = s - \varphi(s)$, $F(s,t) = s \Rightarrow s = 0$, here $\varphi : [0,\infty) \to [0,\infty)$ is a continuous function such that $\varphi(t) = 0 \Leftrightarrow t = 0$;

(12) $F(s,t) = sh(s,t)$, $F(s,t) = s \Rightarrow s = 0$,here $h : [0,\infty) \times [0,\infty) \to [0,\infty)$is a continuous function such that $h(t,s) < 1$ for all $t,s > 0$;

(13) $F(s,t) = s - (\frac{2+t}{1+t})t$, $F(s,t) = s \Rightarrow t = 0$;

(14) $F(s,t) = \sqrt[n]{\ln(1+s^n)}$, $F(s,t) = s \Rightarrow s = 0$;

(15) $f(s,t) = \phi(s)$, $F(s,t) = s \Rightarrow s = 0$, here $\phi : [0,\infty) \to [0,\infty)$ is a upper semicontinuous function such that $\phi(0) = 0$, and $\phi(t) < t$ for $t > 0$;

(16) $f(s,t) = \frac{s}{(1+s)^r}$; $r \in (0,\infty)$, $F(s,t) = s \Rightarrow s = 0$;

(17) $f(s,t) = \vartheta(s)$; $\vartheta : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is a generalized Mizoguchi-Takahashi type function , $f(s,t) = s \Rightarrow s = 0$;

(18) $f(s,t) = \frac{s}{\Gamma(1/2)} \int_0^\infty \frac{e^{-x}}{\sqrt{x}+t}\, dx$, where Γ is the Euler Gamma function.

DEFINITION 2.15. ([**2**]) A function $\varphi : [0,\infty) \to [0,\infty)$ is called an ultra-altering distance function, if φ is continuous and nondecreasing and $\varphi(t) > 0$ if $t > 0$ and $\varphi(0) \geqslant 0$. Denote the class of such functions by Φ_u.

The function $\varphi : [0,\infty) \to [0,\infty)$ is called an altering distance function, if φ is continuous, nondecreasing and $\varphi(t) = 0$ if and only if $t = 0$. For examples of altering distance functions, we refer to [**3, 11**]. We shall denote the class of altering distance functions by Ψ.

DEFINITION 2.16. A tripled (ψ,φ,F) where $\psi \in \Psi$, $\varphi \in \Phi_u$ and $F \in \mathcal{C}$ is said to be monotone if for any $x, y \in [0,\infty)$

$$x \leqslant y \Longrightarrow F(\psi(x),\varphi(x)) \leqslant F(\psi(y),\varphi(y)).$$

EXAMPLE 2.2. Let $F(s,t) = s - t$, $\phi(x) = \sqrt{x}$

$$\psi(x) = \begin{cases} \sqrt{x} & \text{if } 0 \leqslant x \leqslant 1, \\ x^2, & \text{if x} > 1 \end{cases}.$$

Then (ψ,ϕ,F) is monotone.

EXAMPLE 2.3. Let $F(s,t) = s - t$, $\phi(x) = x^2$

$$\psi(x) = \begin{cases} \sqrt{x} & \text{if } 0 \leqslant x \leqslant 1, \\ x^2, & \text{if x} > 1 \end{cases}.$$

Then (ψ, ϕ, F) is not monotone.

3. Main Results

THEOREM 3.1. *Let A, B, S and T be four self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$\psi\big(g(F(Ax, By, t))\big) \leqslant f(\psi\big(M(x, y, t)\big), \varphi\big(M(x, y, t)\big)), \tag{3.1}$$

$$M(x,y,t) = max\{g(F(Sx,Ty,t)), g(F(Ax,Sx,t)), g(F(By,Ty,t)), \\ g(F(Sx,By,t)), g(F(Ty,Ax,t))\}] \tag{3.2}$$

where $\psi \in \Psi$,$\varphi \in \Phi_u$ and $f \in \mathcal{C}$ such that (ψ, φ, f) is monotone. If the pairs $\{A, S\}$ and $\{B, T\}$ are weakly sub sequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point in X.

PROOF. Since the pair $\{A, S\}$ is wsc (Suppose that it is A-sub-sequentially continuous) and compatible of type (E), therefore there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = z$, for some $z \in X$ and $lim_{n\to\infty} ASx_n = Az$. The compatibility of type (E) implies that $lim_{n\to\infty} A^2x_n = lim_{n\to\infty} ASx_n = Sz$ and $lim_{n\to\infty} S^2x_n = lim_{n\to\infty} SAx_n = Az$. Therefore $Az = Sz$, whereas in respect of the pair $\{B, T\}$ (Suppose that it is B-sub-sequentially continuous), there exists a sequence $\{y_n\}$ in X such that $lim_{n\to\infty} By_n = lim_{n\to\infty} Ty_n = w$, for some $w \in X$ and $lim_{n\to\infty} BTy_n = Bw$. The pair $\{B, T\}$ is compatible of type (E) ,then so $lim_{n\to\infty} B^2y_n = lim_{n\to\infty} BTy_n = Tw$ and $lim_{n\to\infty} T^2y_n = lim_{n\to\infty} TBy_n = Bw$, for some $w \in X$, then $Bw = Tw$. Hence z is a coincidence point of the pair $\{A, S\}$ whereas w is a coincidence point of the pair $\{B, T\}$. Now we prove that $z = w$. By putting $x = x_n$ and $y = y_n$ in inequality (3.1), we have

$$\begin{aligned} \psi\big(g(F(Ax_n, By_n, t))\big) &\leqslant f(\psi\big(max\{g(F(Sx_n, Ty_n, t)), g(F(Ax_n, Sx_n, t)), \\ &g(F(By_n, Ty_n, t)), g(F(Sx_n, By_n, t)), g(F(Ty_n, Ax_n, t))\}\big), \\ &\varphi\big(max\{g(F(Sx_n, Ty_n, t)), g(F(Ax_n, Sx_n, t)), \\ &g(F(By_n, Ty_n, t)), g(F(Sx_n, By_n, t)), g(F(Ty_n, Ax_n, t))\}\big)), \end{aligned} \tag{3.3}$$

Taking the limit as $n \to \infty$, we get

$$\begin{aligned} \psi\big(g(F(z, w, t))\big) &\leqslant f(\psi\big(\max\{g(F(z, w, t)), g(F(z, z, t)), g(F(w, w, t)), \\ &g(F(z, w, t)), g(F(z, w, t))\}\big), \\ &\varphi\big(\max\{g(F(z, w, t)), g(F(z, z, t)), g(F(w, w, t)), \\ &g(F(z, w, t)), g(F(z, w, t))\}\big)), \\ &\leqslant f(\psi\big(\max\{g(F(z, w, t)), 0, 0, g(F(z, w, t)), g(F(w, z, t))\big), \\ &\varphi\big(\max\{g(F(z, w, t)), 0, 0, g(F(z, w, t)), g(F(w, z, t))\big)) \\ &\leqslant f(\psi\big(g(F(z, w, t))\big), \varphi\big(g(F(z, w, t))\big)), \end{aligned} \tag{3.4}$$

so, $\psi\big(g(F(z,w,t))\big) = 0$ or $\varphi\big(g(F(z,w,t))\big) = 0$ i.e. $g(F(z,w,t)) = 0$. Thus, we have $z = w$. Now we prove that $Az = z$. By putting $x = z$ and $y = y_n$ in the inequality (3.1), we get

$$\begin{aligned} \psi\big(g(F(Az,By_n,t))\big) \leqslant f(\psi\big(&\max\{g(F(Sz,Ty_n,t)), g(F(Az,Sz,,t)), \\ &g(F(By_n,Ty_n,t)), g(F(Sz,By_n,t)), g(F(Ty_n,Az,t))\}\big), \\ &\varphi\big(\max\{g(F(Sz,Ty_n,t)), g(F(Az,Sz,,t)), \\ &g(F(By_n,Ty_n,t)), g(F(Sz,By_n,t)), \\ &g(F(Ty_n,Az,t))\}\big)), \end{aligned} \tag{3.5}$$

Taking the limit as $n \to \infty$, we get

(3.6)

$$\begin{aligned} \psi\big(g(F(Az,w,t))\big) &\leqslant f(\psi\big(\max\{g(F(Sz,w,t)), g(F(Az,Sz,t)), g(F(w,w,t)), \\ &\quad g(F(Sz,w,t)), g(F(w,Az,t))\}\big), \\ &\quad \varphi\big(\max\{g(F(Sz,w,t)), g(F(Az,Sz,t)), g(F(w,w,t)), \\ &\quad g(F(Sz,w,t)), g(F(w,Az,t))\}\big)) \\ &\leqslant f(\psi\big(\max\{g(F(Sz,w,t)), 0, 0, g(F(Sz,w,t)), g(F(w,Az,t))\}\big), \\ &\quad \varphi\big(\max\{g(F(Sz,w,t)), 0, 0, g(F(Sz,w,t)), g(F(w,Az,t))\}\big)) \\ &\leqslant f(\psi\big(g(F(w,Az,t))\big), \psi\big(g(F(w,Az,t))\big)), \end{aligned}$$

so, $\psi\big(g(F(Az,w,t))\big) = 0$ or $\varphi\big(g(F(Az,w,t))\big) = 0$ i.e. $g(F(Az,w,t)) = 0$, which yields $Az = w$. Since $Az = Sz$. Therefore $Az = Sz = w = z$.
Now we prove that $Bz = z$. By putting $x = \{x_n\}$ and $y = z$ in the inequality (3.1), we get

$$\begin{aligned} \psi\big(g(F(Ax_n,Bz,t))\big) \leqslant f(\psi\big(&\max\{g(F(Sx_n,Tz,t)), g(F(Ax_n,Sx_n,t)), \\ &g(F(Bz,Tz,t)), g(F(Sx_n,Bz,t)), g(F(Tz,Ax_n,t))\}\big), \\ &\varphi\big(\max\{g(F(Sx_n,Tz,t)), g(F(Ax_n,Sx_n,t)), g(F(Bz,Tz,t)), \\ &g(F(Sx_n,Bz,t)), g(F(Tz,Ax_n,t))\}\big)), \end{aligned} \tag{3.7}$$

Taking the limit as $n \to \infty$, we get

(3.8)

$$\begin{aligned} \psi\big(g(F(z,Bz,t))\big) &\leqslant f(\psi\big(\max\{g(F(z,Tz,t)), g(F(z,z,t)), g(F(Bz,Tz,t)), \\ &\quad g(F(z,Bz,t)), g(F(Tz,z,t))\}\big), \\ &\quad \varphi\big(\max\{g(F(z,Tz,t)), g(F(z,z,t)), g(F(Bz,Tz,t)), \\ &\quad g(F(z,Bz,t)), g(F(Tz,z,t))\}\big)), \\ &\leqslant f(\psi\big(\max\{g(F(z,Tz,t)), 0, 0, g(F(z,Bz,t)), g(F(Tz,z,t))\}\big), \\ &\quad \varphi\big(\max\{g(F(z,Tz,t)), 0, 0, g(F(z,Bz,t)), g(F(Tz,z,t))\}\big)) \\ &\leqslant f(\psi\big(g(F(z,Bz,t))\big), \varphi\big(g(F(z,Bz,t))\big)) \end{aligned}$$

so, $\psi\big(g(F(z,Bz,t))\big) = 0$ or $\varphi\big(g(F(z,Bz,t))\big) = 0$ i.e. $g(F(z,Bz,t)) = 0$, which yields $Bz = z$. Since $Bz = Tz$. Therefore, $Bz = Tz = z$. Therefore in all $z = Az = Bz = Sz = Tz$, i.e. z is a common fixed point of A, B, S and T. The uniqueness of common fixed point is an easy consequence of inequality (3.1). □

If we put $A = B$ in Theorem 3.1 we have the following corollary for three mappings:

Corollary 3.1. *Let A, S and T be three self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$\psi\big(g(F(Ax, Ay, t))\big) \leqslant f(\psi\big(M(x, y, t)\big), \varphi\big(M(x, y, t)\big)), \tag{3.9}$$

$$\begin{aligned} M(x, y, t) = max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(Ay, Ty, t)),\\ g(F(Sx, Ay, t)), g(F(Ty, Ax, t))\}] \end{aligned} \tag{3.10}$$

where $\psi \in \Psi$,$\varphi \in \Phi_u$ and $f \in \mathcal{C}$ such that (ψ, φ, f) is monotone. If the pairs $\{A, S\}$ and $\{A, T\}$ are weakly sub sequentially continuous and compatible of type (E), then A, S and T have a unique common fixed point in X.

Alternatively, if we set $S = T$ in Theorem 3.1, we'll have the following corollary for three self mappings:

Corollary 3.2. *Let A, B and S be three self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$\psi\big(g(F(Ax, By, t))\big) \leqslant f(\psi\big(M(x, y, t)\big), \varphi\big(M(x, y, t)\big)), \tag{3.11}$$

$$\begin{aligned} M(x, y, t) = max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(By, Sy, t)),\\ g(F(Sx, By, t)), g(F(Sy, Ax, t))\}] \end{aligned} \tag{3.12}$$

where $\psi \in \Psi$,$\varphi \in \Phi_u$ and $f \in \mathcal{C}$ such that (ψ, φ, f) is monotone. If the pairs $\{A, S\}$ and $\{B, S\}$ are weakly sub sequentially continuous and compatible of type (E), then A, B and S have a unique common fixed point in X.

If we put $S = T$ in corollary 3.1, we have the following result for two self mappings:

Corollary 3.3. *Let A and S be two self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$\psi\big(g(F(Ax, Ay, t))\big) \leqslant f(\psi\big(M(x, y, t)\big), \varphi\big(M(x, y, t)\big)), \tag{3.13}$$

$$\begin{aligned} M(x, y, t) = max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(Ay, Sy, t)),\\ g(F(Sx, Ay, t)), g(F(Sy, Ax, t))\}] \end{aligned} \tag{3.14}$$

where $\psi \in \Psi$,$\varphi \in \Phi_u$ and $f \in \mathcal{C}$ such that (ψ, φ, f) is monotone. If the pair $\{A, S\}$ is weakly sub sequentially continuous and compatible of type (E), then A and S have a unique common fixed point in X.

Theorem 3.2. *Let A, B,S and T be four self maps of a N. A. Menger PM-space (X, F, Δ) satisfying (3.1.). where $\psi \in \Psi$,$\varphi \in \Phi_u$ and $f \in \mathcal{C}$ such that (ψ, φ, f) is monotone. Assume that*
(i) *the pair $\{A, S\}$ is A-compatible of type (E) and A-sub sequentially continuous.*
(ii) *the pair $\{B, T\}$ is B-compatible of type (E) and B-sub sequentially continuous. Then A, B, S and T have a unique common fixed point in X.*

Proof. The proof is obvious as on the lines of theorem 3.1. □

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Received by editors 04.03.2017; Revised version 23.08.2017; Accepted 28.08.2017; Available online 04.09.2017.

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