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EDGE PAIR SUM LABELING OF SOME SUBDIVISION OF GRAPHS

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Abstract. An injective map $f : E(G) \rightarrow \{\pm 1, \pm 2, \cdots, \pm q\}$ is said to be an edge pair sum labeling of a graph $G(p, q)$ if the induced vertex function $f^* : V(G) \rightarrow Z - \{0\}$ defined by $f^*(v) = \sum_{e \in E_v} f(e)$ is one-one, where E_v denotes the set of edges in G that are incident with a vetex v and $f^*(V(G))$ is either of the form

$$\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p}{2}}\right\} \text{ or } \left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p-1}{2}}\right\} \bigcup \left\{\pm k_{\frac{p+1}{2}}\right\}$$

according as p is even or odd. A graph which admits edge pair sum labeling is called an edge pair sum graph. In this paper, we prove that the subdivision of graph such as bistar $S(B_{m,n})$, $P_n \odot k_1$, triangular snake $S(T_n)$ if n is odd, double triangular snake $D(T_n)$, double quadrilateral snake $D(Q_n)$, double alternative triangular snake $DA(T_n)$ and double alternative quadrilateral snake $DA(Q_n)$ are edge pair sum graph.

1. preliminaries

A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and for a dynamic survey of various graph labeling problems along with extensive bibliography we refer to Gallian [1]. The concept of edge pair sum labeling has been introduced in [3] and further studied in [4-12]. This is the further extension work on edge pair sum labeling. Through out this paper we consider finite, simple and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. G is also called a (p, q)

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graph. Terms and notations not defined here are used in the sense of Harary [2]. We give the basic definitions relevant to this paper.

DEFINITION 1.1. *The double triangular snake $D(T_n)$ is a graph obtained from a path P_n with vertices $v_1, v_2, ..., v_n$ by joining v_i and v_{i+1} to the new vertices w_i and u_i for $i = 1, 2, ..., n-1$.*

DEFINITION 1.2. *The double quadrilateral snake $D(Q_n)$ is a graph obtained from a path P_n with vertices $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to the new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i for $i = 1, 2, ..., n-1$.*

DEFINITION 1.3. *A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to the two new vertices v_i and w_i for $i = 1, 2, ..., n-1$.*

DEFINITION 1.4. *A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} (alternatively) to the new vertices v_i, x_i and w_i, y_i respectively and adding the edes v_iw_i and x_iy_i for $i = 1, 2, ..., n-1$.*

DEFINITION 1.5. *Let G be a graph. The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.*

2. Main Results

In this section, we prove that subdivision of bistar $S(B_{m,n})$, $P_n \odot k_1$, triangular snake $S(T_n)$ if n is odd, double triangular snake $D(T_n)$, double quadrilateral snake $D(Q_n)$, double alternative triangular snake $DA(T_n)$ and double alternative quadrilateral snake $DA(Q_n)$ are edge pair sum graph.

THEOREM 2.1. *The subdivision of bistar graph $S(B_{m,n})$ is an edge pair sum graph.*

PROOF. Let

$$V(S(B_{m,n})) = \{u, v, w, u_i : 1 \leqslant i \leqslant 2m, v_i : 1 \leqslant i \leqslant 2n\}$$

and

$$E(S(B_{m,n})) = \{e_1^{''} = uw, e_2^{''} = wv, e_{2i-1} = uu_{2i-1}, e_{2i} = u_{2i-1}u_{2i} : 1 \leqslant i \leqslant m, e_{2i-1}^{'} = vv_{2i-1}, e_{2i}^{'} = v_{2i-1}v_{2i} : 1 \leqslant i \leqslant n\}$$

are the vertices and edges of the graph $S(B_{m,n})$.

Define

$$f : E(S(B_{m,n})) \to \{\pm 1, \pm 2, \pm 3, ..., \pm 2(m + n + 1)\}$$

by considering the following three cases:

Case (i). m and n are even.

$$f(e_1^{''}) = -2, f(e_2^{''}) = 1,$$

for $1 \leqslant i \leqslant \frac{m}{2}$

$$f(e_{2i-1}) = (2i+1) = -f(e_{m-1+2i}),\ f(e_{2i}) = (m+1+2i) = -f(e_{m+2i});$$

for $1 \leqslant i \leqslant \frac{n}{2}$

$$f(e_{2i-1}^{'}) = (2m+1+2i) = -f(e_{n-1+2i}^{'}),\ f(e_{2i}^{'}) = (2m+n+1+2i) = -f(e_{n+2i}^{'}).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u) = -2,\ \ f^*(w) = -1 = -f^*(v),$$

for $1 \leqslant i \leqslant \frac{m}{2}$

$$f^*(u_{2i-1}) = (m+2+4i) = -f^*(u_{m-1+2i}),$$
$$f^*(u_{2i}) = (m+1+2i) = -f^*(u_{m+2i}),$$

for $1 \leqslant i \leqslant \frac{n}{2}$

$$f^*(v_{2i-1}) = (4m+n+2+4i) = -f^*(v_{n-1+2i}),$$
$$f^*(v_{2i}) = (2m+n+1+2i) = -f^*(v_{n+2i}).$$

Then

$f^*(V(B_{m,n})) =$
$\{\pm 1, \pm(m+6), \pm(m+10), \pm(m+14), ..., \pm(3m+2), \pm(m+3), \pm(m+5), \pm(m+7), ..., \pm(2m+1), \pm(4m+n+6), \pm(4m+n+10), \pm(4m+n+14), ..., \pm(4m+3n+2), \pm(2m+n+3), \pm(2m+n+5), \pm(2m+n+7), ..., \pm(2m+2n+1)\} \bigcup \{-2\}$.

It can be verified that f is an edge pair sum labeling of $S(B_{m,n})$ if m and n are even. Hence $S(B_{m,n})$ is an edge pair sum graph if m and n are even.

Case (ii). m and n are odd.

$$f(e_1) = -3 = -f(e_1^{'}), f(e_2) = -5 = -f(e_2^{'}), f(e_1^{''}) = 2, f(e_2^{''}) = -1,$$

for $1 \leqslant i \leqslant \frac{m-1}{2}$

$$f(e_{2i+1}) = (2i+5) = -f(e_{m+2i}),$$
$$f(e_{2i+2}) = (m+4+2i) = -f(e_{m+1+2i}),$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f(e_{2i+1}^{'}) = (2m+3+2i) = -f(e_{n+2i}^{'}),$$
$$f(e_{2i+2}^{'}) = (2m+n+2+2i) = -f(e_{n+1+2i}^{'}).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(v) = 2,\ f^*(w) = 1 = -f^*(u),$$
$$f^*(u_1) = -8 = -f^*(v_1),\ f^*(u_2) = -5 = -f^*(v_2),$$

for $1 \leqslant i \leqslant \frac{m-1}{2}$

$$f^*(u_{2i+1}) = (m+9+4i) = -f^*(u_{m+2i}),$$
$$f^*(u_{2i+2}) = (m+4+2i) = -f^*(u_{m+1+2i}),$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f^*(v_{2i+1}) = (4m+n+5+4i) = -f^*(v_{n+2i}),$$
$$f^*(v_{2i+2}) = (2m+n+2+2i) = -f^*(v_{n+1+2i}).$$

Then

$f^*(V(B_{m,n})) =$

$\{\pm 1, \pm 5, \pm 8, \pm(m+13), \pm(m+17), \pm(m+21), ..., \pm(3m+7), \pm(m+6), \pm(m+8), \pm(m+10), ..., \pm(2m+3), \pm(4m+n+9), \pm(4m+n+13), \pm(4m+n+17), ..., \pm(4m+3n+3), \pm(2m+n+4), \pm(2m+n+6), \pm(2m+n+8), ..., \pm(2m+2n+1)\} \bigcup \{2\}$.

It can be verified that f is an edge pair sum labeling of $S(B_{m,n})$ if m and n are odd. Hence $S(B_{m,n})$ is an edge pair sum graph if m and n are odd.

Case (iii). m is odd and n is even.

$$f(e_1) = -1 = -f(e_2^{''}), f(e_2) = 3, f(e_1^{''}) = -2,$$

for $1 \leqslant i \leqslant \frac{m-1}{2}$

$$f(e_{2i+1}) = (2i+3) = -f(e_{m+2i}),$$
$$f(e_{2i+2}) = (m+2+2i) = -f(e_{m+1+2i}),$$

for $1 \leqslant i \leqslant \frac{n}{2}$

$$f(e_{2i-1}^{'}) = (2m+1+2i) = -f(e_{n-1+2i}^{'}),$$
$$f(e_{2i}^{'}) = (2m+n+1+2i) = -f(e_{n+2i}^{'}).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = 2,\ f^*(w) = -1 = -f^*(v),\ f^*(u_2) = 3 = -f^*(u),$$

for $1 \leqslant i \leqslant \frac{m-1}{2}$

$$f^*(u_{2i+1}) = (m+5+4i) = -f^*(u_{m+2i}),$$
$$f^*(u_{2i+2}) = (m+2+2i) = -f^*(u_{m+1+2i}),$$

for $1 \leqslant i \leqslant \frac{n}{2}$

$$f^*(v_{2i-1}) = (4m+n+2+4i) = -f^*(v_{n-1+2i}),$$
$$f^*(v_{2i}) = (2m+n+1+2i) = -f^*(v_{n+2i}).$$

Then

$f^*(V(B_{m,n})) =$

$\{\pm 1, \pm 3, \pm(m+9), \pm(m+13), \pm(m+17), ..., \pm(3m+3), \pm(m+4), \pm(m+6), \pm(m+8), ..., \pm(2m+1), \pm(4m+n+6), \pm(4m+n+10), \pm(4m+n+14), ..., \pm(4m+3n+2), \pm(2m+n+3), \pm(2m+n+5), \pm(2m+n+7), ..., \pm(2m+2n+1)\} \bigcup \{2\}$.

It can be verified that f is an edge pair sum labeling of $S(B_{m,n})$ if m is odd and n is even. Hence $S(B_{m,n})$ is an edge pair sum graph if m is odd and n is even. □

An example for the edge pair sum labeling of $S(B_{3,4})$ is given in Figure 1.

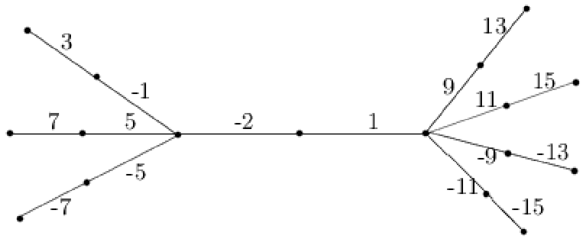

Figure 1:

FIGURE 1. Edge pair sum labeling of $S(B_{3,4})$

THEOREM 2.2. *The subdivision of $(P_n \odot K_1)$ graph is an edge pair sum graph.*

PROOF. Let

$$V(S(P_n \odot K_1)) = \{v_i, w_i : 1 \leqslant i \leqslant n, u_i : 1 \leqslant i \leqslant 2n-1\}$$

and

$$E(S(P_n \odot K_1)) = \{e_i^{'} = v_i u_{2i-1}, e_i^{''} = v_i w_i : 1 \leqslant i \leqslant n, e_i = u_i u_{1+i} : 1 \leqslant i \leqslant 2n-2\}$$

are the vertices and edges of the graph $(S(P_n \odot K_1))$.

Define an edge labeling $f : E((P_n \odot K_1)) \to \{\pm 1, \pm 2, \pm 3, ..., \pm(4n-2)\}$ by considering the following two cases:

Case (i). n is odd.

$$f(e^{'}_{\frac{n+1}{2}}) = 2,\ f(e^{''}_{\frac{n+1}{2}}) = -4,$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f(e_i^{'}) = (2i-1),\ f(e^{'}_{\frac{n+1}{2}+i}) = -(n-2i),\ f(e_i^{''}) = (4+2i)$$

and

$$f(e^{''}_{\frac{n+1}{2}+i}) = -(n+5-2i),$$

for $1 \leqslant i \leqslant (n-1)$

$$f(e_i) = (n+2+2i) \text{ and } f(e_{n-1+i}) = -(3n+2-2i).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(w_{\frac{n+1}{2}}) = -4,\ f^*(v_{\frac{n+1}{2}}) = -2 = -f^*(u_n),\ f^*(u_1) = (n+5) = -f^*(u_{2n-1}),$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f^*(w_i) = (4+2i),\ f^*(w_{\frac{n+1}{2}+i}) = -(n+5-2i),\ f^*(v_i) = (4i+3),$$
$$f^*(v_{\frac{n+1}{2}+i}) = -(2n+5-4i),\ f^*(u_{2i}) = (2n+2+8i) \text{ and}$$
$$f^*(u_{n-1+2i}) = -(6n+6-8i),$$

for $1 \leqslant i \leqslant \frac{n-3}{2}$

$$f^*(u_{2i+1}) = (2n+7+10i) \text{ and } f^*(u_{n+2i}) = -(7n+2-10i).$$

Hence we get
$f^*(V(P_n \odot K_1)) = \{\pm 2, \pm 6, \pm 8, \pm 10, ..., \pm(n+3), \pm 7, \pm 11, \pm 15, ..., \pm(2n+1), \pm(2n+17), \pm(2n+27), \pm(2n+37), ..., \pm(7n-8), \pm(2n+10), \pm(2n+18), \pm(2n+26), ..., \pm(6n-2), \pm(n+5)\} \bigcup \{-4\}$.

It can be verified that f is an edge pair sum labeling of $(P_n \odot K_1)$ if n is odd. Hence $(P_n \odot K_1)$ is an edge pair sum graph if n is odd.

Case (ii). n is even.
Subcase (a). $n = 2$.

$$f(e_1^{'}) = 4 = -f(e_2^{''}),\ f(e_2^{'}) = 2 = -f(e_1),\ f(e_1^{''}) = -3 \text{ and } f(e_2) = 1.$$

Then the induced vertex labeling is as follows:

$$f^*(u_1) = 2 = -f^*(v_2),\ f^*(u_2) = -1 = -f^*(v_1),\ f^*(u_3) = 3 = -f^*(w_1) \text{ and}$$
$$f^*(w_2) = -4.$$

Then $f^*(V(P_n \odot K_1)) = \{\pm 1, \pm 2, \pm 3\} \bigcup \{-4\}$.

Hence f is an edge pair sum labeling if $n = 2$.

Subcase (b). $n \geqslant 4$.

$$f(e_{\frac{n}{2}+1}) = 3n,\ f(e_{\frac{n}{2}+2}) = -(3n-3),$$

for $1 \leqslant i \leqslant n-2$

$$f(e_i) = -(2n+1-2i) \text{ and } f(e_{\frac{n+4}{2}+i}) = (3+2i),$$

for $1 \leqslant i \leqslant \frac{n}{2}$

$$f(e_i^{'}) = -(2n-2+2i),\ f(e_{\frac{n}{2}+i}) = (3n-2i),\ f(e_i^{''}) = -(3n-3+2i) \text{ and}$$
$$f(e^{''}_{\frac{n}{2}+i}) = (4n-1-2i).$$

For each edge label f the induced vertex label f^* is defined as follows:
for $1 \leqslant i \leqslant \frac{n}{2}$

$$f^*(w_i) = -(3n-3+2i),\ f^*(w_{\frac{n}{2}+i}) = (4n-1-2i),\ f^*(v_i) = -(5n-5+4i),$$
$$f^*(v_{\frac{n}{2}+i}) = (7n-1-4i),\ f^*(u_1) = -(4n-1) = -f^*(u_n),$$
$$f^*(u_n) = 3 = -f^*(u_{n-1}),$$

for $1 \leqslant i \leqslant \frac{n-2}{2}$

$$f^*(u_{2i}) = -(4n+4-8i) \text{ and } f^*(u_{n+2i}) = (4+8i),$$
$$\text{for } 1 \leqslant i \leqslant \tfrac{n-4}{2}\ f^*(u_{2i+1}) = -6(n-8) \text{ and } f^*(u_{n+2+i}) = (3n+6+6i) \text{ and}$$
$$f^*(u_{n+1}) = 6.$$

We get
$f^*(V(P_n \odot K_1)) = \{\pm3, \pm(3n-1), \pm(3n+1), \pm(3n+3), ..., \pm(4n-3), \pm(5n-1), \pm(5n+3), \pm(5n+7), ..., \pm(7n-5), \pm(4n-1), \pm12, \pm20, \pm28, ..., \pm(4n-4), \pm(3n+12), \pm(3n+18), \pm(3n+24), ..., \pm(6n-6)\} \bigcup \{6\}$.

It can be verified that f is an edge pair sum labeling of $(P_n \odot K_1)$ if $n \geqslant 4$. Hence $(P_n \odot K_1)$ is an edge pair sum graph if $n \geqslant 4$. □

The example for the edge pair sum labeling of $(P_3 \odot K_1)$ and $(P_2 \odot K_1)$ are shown in Figure 2.

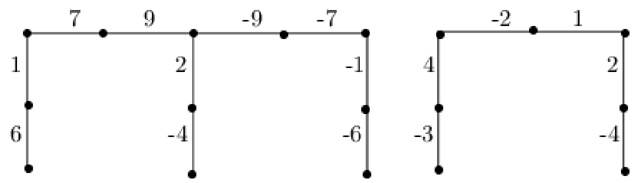

Figure 2:

FIGURE 2. Edge pair sum labeling of $(P_3 \odot K_1)$ and $(P_2 \odot K_1)$

THEOREM 2.3. *The subdivision of triangular snake graph $S(T_n)$ is an edge pair sum graph if n is odd.*

PROOF. Let

$$V(S(T_n)) = \{w_i : 1 \leqslant i \leqslant (n-1), v_i : 1 \leqslant i \leqslant 2(n-1), u_i : 1 \leqslant i \leqslant (2n-1)\}$$

and

$$E(S(T_n)) = \{e''_{2i-1} = v_{2i-1}w_i, e''_{2i} = v_{2i}w_i, e'_{2i-1} = u_{2i-1}v_{2i-1}, e'_{2i} = u_{2i+1}v_{2i} : 1 \leqslant i \leqslant n-1, e_i = u_iu_{1+i} : 1 \leqslant i \leqslant 2(n-1)\}$$

are the vertices and edges of the graph $S(T_n)$.

Define an edge labeling $f : E(S(T_n)) \to \{\pm1, \pm2, \pm3, ..., \pm6(n-1)\}$.

$$f(e'_{n-2}) = -6 = -f(e'_{n+1}),\ f(e'_{n-1}) = 1,\ f(e'_n) = 2,\ f(e''_{n-2}) = -5 = -f(e''_{n+1}),$$
$$f(e''_{n-1}) = -4 = -f(e''_n),\ f(e_{n-2}) = -7 = -f(e_{n+1}),\ f(e_{n-1}) = -8 = -f(e_n),$$

for $1 \leqslant i \leqslant \frac{n-3}{2}$

$$f(e'_{2i-1}) = -(3n+3-6i),\ f(e'_{2i}) = -(3n-6i),\ f(e'_{n+2i}) = (3+6i),$$
$$f(e'_{n+1+2i}) = (6+6i),\ f(e''_{2i-1}) = -(3n+2-6i),\ f(e''_{2i}) = -(3n+1-6i),$$
$$f(e''_{n+2i}) = (4+6i),\ f(e''_{n+1+2i}) = (5+6i),\ f(e_{2i-1}) = -(3n+4-6i),$$
$$f(e_{2i}) = -(3n+5-6i),\ f(e_{n+2i}) = (8+6i) \text{ and } f(e_{n+1+2i}) = (7+6i).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(w_{\frac{n-1}{2}}) = -9 = -f^*(w_{\frac{n+1}{2}}),\ f^*(v_{n-2}) = -11 = -f^*(v_{n+1}),$$
$$f^*(v_{n-1}) = -3 = -f^*(u_n),\ f^*(v_n) = 6,\ f^*(u_1) = -(6n-5) = -f^*(u_{2n-1}),$$
$$f^*(u_{2n-2}) = (6n-3),$$

for $1 \leqslant i \leqslant \frac{n-3}{2}$

$$f^*(w_i) = -(6n+3-12i),\ f^*(w_{\frac{n+1}{2}+i}) = (9+12i),\ f^*(v_{2i-1}) = -(6n+5-12i),$$
$$f^*(v_{2i}) = -(6n+1-12i),\ f^*(v_{n+2i}) = (7+12i),\ f^*(v_{n+1+2i}) = (11+12i),$$
$$f^*(u_{2i+1}) = -(12n-24i),\ f^*(u_{n-1+2i}) = (3+12i) \text{ and } f^*(u_{n+2i}) = (12+24i),$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f^*(u_{2i}) = -(6n+9-12i).$$

Then
$f^*(V(S(T_n)) = \{\pm 3, \pm 11, \pm 9, \pm 21, \pm 33, \pm 45, ..., \pm(6n-9), \pm 23, \pm 35, \pm 47, ...,$
$\pm(6n-7), \pm 19, \pm 31, \pm 43, ..., \pm(6n-11), \pm(6n-5), \pm 15, \pm 27, \pm 39, ..., \pm(6n-15),$
$\pm 36, \pm 60, \pm 84, ..., \pm(12n-24), \pm(6n-3)\} \bigcup \{6\}$.

It can be verified that f is an edge pair sum labeling of $S(T_n)$ if n is odd. Hence $S(T_n)$ is an edge pair sum graph if n is odd. □

THEOREM 2.4. *The subdivision of double triangular snake graph $S(D(T_n))$ is an edge pair sum graph.*

PROOF. Let

$$V(S(D(T_n))) =$$
$$\{u_i : 1 \leqslant i \leqslant (2n-1), v_i, v'_i : 1 \leqslant i \leqslant 2(n-1), w_i w'_i : 1 \leqslant i \leqslant (n-1)\}$$

and

$$E(S(D(T_n))) = \{e_i = u_i u_{i+1} : 1 \leqslant i \leqslant 2(n-1), e'_{4i-3} = u_{2i-1}v_{2i-1}, e'_{4i-2} =$$
$$v_{2i-1}w_i, e'_{4i-1} = w_i v_{2i}, e'_{4i} = v_{2i}u_{2i+1}, e''_{4i-3} = u_{2i-1}v'_{2i-1}, e''_{4i-2} = v'_{2i-1}w'_i, e''_{4i-1} =$$
$$w'_i v'_{2i}, e''_{4i} = v_{2i}u'_{2i+1} : 1 \leqslant i \leqslant (n-1)\}$$

are the vertices and edges of the graph $S(D(T_n))$.

Define $f : E(S(D(T_n))) \to \{\pm 1, \pm 2, \pm 3, ..., \pm 10(n-1)\}$ by considering the following two cases:

Case (i). $n = 2$.

$$f(e_1) = -2,\ f(e_2) = 1,\ f(e'_1) = 4 = -f(e''_1),\ f(e'_2) = 5 = -f(e''_2),$$
$$f(e'_3) = 6 = -f(e''_3) \text{ and } f(e'_4) = 7 = -f(e''_4).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = -2,\ f^*(u_2) = -1 = -f^*(u_3),\ f^*(v_1) = 9 = -f^*(v_1^{'}),$$
$$f^*(v_2) = 13 = -f^*(v_2^{'}) \text{ and } f^*(w_1) = 11 = -f^*(w_1^{'}).$$

Then

$$f^*(V(S(D(T_n)))) = \{\pm1, \pm9, \pm11, \pm13\} \bigcup \{-2\}.$$

Case (ii). $n \geqslant 3$.
for $1 \leqslant i \leqslant (n-2)$

$$f(e_i) = -(2n+1-2i) \text{ and } f(e_{n+i}) = (3+2i),\ f(e_{n-1}) = 2,\ f(e_n) = 1,$$
$$\text{for } 1 \leqslant i \leqslant (n-1)\ f(e_{4i-3}^{'}) = (2n-4+4i) = -f(e_{4i-3}^{''}),$$
$$f(e_{4i-2}^{'}) = (2n-3+4i) = -f(e_{4i-2}^{''}),\ f(e_{4i-1}^{'}) = (2n-2+4i) = -f(e_{4i-1}^{''}) \text{ and}$$
$$f(e_{4i}^{'}) = (2n-1+4i) = -f(e_{4i}^{''}).$$

For each edge label f the induced vertex label f^* is defined as follows:
for $1 \leqslant i \leqslant (n-1)$

$$f^*(w_i) = (4n-5+8i) = -f^*(w_i^{'}),\ f^*(v_{2i-1}) = (4n-7+8i) = -f^*(v_{2i-1}^{'}) \text{ and}$$
$$f^*(v_{2i}) = (4n-3+8i) = -f^*(v_{2i}^{'}),$$

for $1 \leqslant i \leqslant (n-3)$

$$f^*(u_{1+i}) = -4(n-i) \text{ and } f^*(u_{n+1+i}) = (8+4i),$$
$$f^*(u_1) = -(2n-1) = -f^*(u_{2n-1}),\ f^*(u_{n-1}) = -3 = -f^*(u_n) \text{ and } f^*(u_{n+1}) = 6.$$

Then
$f^*(V(S(D(T_n)))) = \{\pm3, \pm12, \pm16, \pm20, ..., \pm(4n-4), \pm(2n-1), \pm(4n+5), \pm(4n+13), \pm(4n+21), ..., \pm(12n-11), \pm(4n+1), \pm(4n+9), \pm(4n+17), ..., \pm(12n-15), \pm(4n+3), \pm(4n+11), \pm(4n+19), ..., \pm(12n-13)\} \bigcup \{6\}$.

It can be verified that f is an edge pair sum labeling of $S(D(T_n))$. Hence $S(D(T_n))$ is an edge pair sum graph. □

An example for the edge pair sum labeling of subdivision of double triangular snake graph $S(D(T_5))$ is shown in Figure 3.

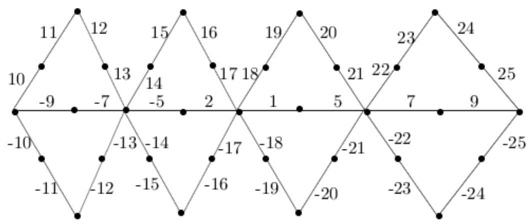

FIGURE 3. Edge pair sum labeling of $S(D(T_5))$

Corollary 2.1. *The subdivision of double alternative triangular snake graph $S(DA(T_n))$ is an edge pair sum graph.*

Proof. The proof follows from the Theorem 2.4. □

Theorem 2.5. *The subdivision of double quadrilateral snake graph $S(D(Q_n))$ is an edge pair sum graph.*

Proof. Let

$$V(S(D(Q_n))) = \{u_i : 1 \leqslant i \leqslant (2n-1), v_{ij}, v_{ij}^{'} : 1 \leqslant i \leqslant (n-1), 1 \leqslant j \leqslant 5\}$$

and

$$E(S(D(Q_n))) = \{e_i = u_i u_{i+1} : 1 \leqslant i \leqslant (2n-2), e_{i1} = u_{2i-1}v_{i1}, e_{i1}^{'} = u_{2i-1}v_{i1}^{'}, e_{i6} = v_{i5}u_{2i+1}, e_{i6}^{'} = v_{i5}^{'}u_{2i+1} e_{i1+j} = v_{ij}v_{i1+j}, e_{i1+j}^{'} = v_{ij}^{'}v_{i1+j}^{'} : 1 \leqslant i \leqslant (n-1), 1 \leqslant j \leqslant 4\}$$

are the vertices and edges of the graph $S(D(Q_n))$.

Define

$$f : E(S(D(Q_n))) \to \{\pm 1, \pm 2, \pm 3, ..., \pm 14(n-1)\}$$

by considering the following two cases:

Case (i). $n = 2$.

$$f(e_1) = -2,\ f(e_2) = 1,\ f(e_{11}) = 3 = -f(e_{11}^{'}),\ f(e_{16}) = 8 = -f(e_{16}^{'}),$$

for $1 \leqslant j \leqslant 4$

$$f(e_{11+j}) = (3+j) = -f(e_{11+j}^{'})$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = -2,\ f^*(u_2) = -1 = -f^*(u_3)$$

for $1 \leqslant j \leqslant 5$

$$f^*(v_{1j}) = (5+2j) = -f^*(v_{1j}^{'}).$$

Then

$$f^*(V(S(D(Q_n))) = \{\pm 1, \pm 7, \pm 9, \pm 11, \pm 13, \pm 15\} \bigcup \{-2\}.$$

Case (ii). $n \geqslant 3$.

$$f(e_{n-1}) = 2,\ f(e_n) = 1,$$

for $1 \leqslant i \leqslant (n-2)$

$$f(e_i) = -(2n-2i+1) and f(e_{n+i}) = (3+2i),$$

for $1 \leqslant i \leqslant (n-1)$,

$$1 \leqslant j \leqslant 6,\ f(e_{ij}) = (2n-7+6i+j) = -f(e_{ij}^{'}).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = -(2n-1) = -f^*(u_{2n-1}),\ f^*(u_{n-1}) = -3 = -f^*(u_n),\ f^*(u_{n+1}) = 6,$$

for $1 \leqslant i \leqslant (n-3)$,

$$f^*(u_{1+i}) = -2(2n-2i) and f^*(u_{n+1+i}) = (8+4i),$$

for $1 \leqslant i \leqslant (n-1)$ and $1 \leqslant j \leqslant 5$,

$$f^*(v_{ij}) = (4n-13+12i+2j) = -f^*(v'_{ij}).$$

Then $f^*(V(S(D(Q_n)))) = \{\pm 3, \pm(2n-1), \pm 12, \pm 16, \pm 20, ..., \pm 4(n-1), \pm(4n-13+12i+2j) : 1 \leqslant i \leqslant (n-1), 1 \leqslant j \leqslant 5\} \bigcup \{6\}$.

It can be verified that f is an edge pair sum labeling of $S(D(Q_n))$. Hence $S(D(Q_n))$ is an edge pair sum graph. □

An example for the edge pair sum labeling of subdivision of double quadrilateral snake graph of $S(D(Q_3))$ is shown in Figure 4.

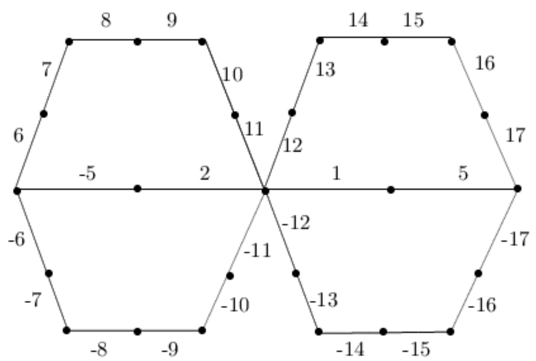

Figure 4:

FIGURE 4. Edge pair sum labeling of $S(D(Q_3))$

COROLLARY 2.2. *The subdivision of double alternative quadrilateral snake graph $S(DA(Q_n))$ is an edge pair sum graph.*

PROOF. The proof follows from the Theorem 2.5. □

THEOREM 2.6. *The subdivision of slanting graph $S(Sl_n)$ is an edge pair sum graph.*

PROOF. Let

$$V(S(Sl_n)) = \{u_i, w_i : 1 \leqslant i \leqslant (2n-1), v_i : 1 \leqslant i \leqslant (n-1)\}$$

and

$$E(S(Sl_n)) = \{e_i = u_iu_{i+1}, e'''_i = w_iw_{i+1} : 1 \leqslant i \leqslant (2n-2), e'_i = u_{2i-1}v_i, e''_i = w_{2i+1}v_i : 1 \leqslant i \leqslant (n-1)\}$$

are the vertices and edges of the graph $S(Sl_n)$.

Define $f : E(S(Sl_n)) \to \{\pm 1, \pm 2, \pm 3, ..., \pm(6n-6)\}$ by considering the following two cases:

Case (i). n is odd.

for $1 \leqslant i \leqslant (2n-2)$

$$f(e_i) = (2i-1),\ f(e_i^{'''}) = -(4n-3-2i),$$

for $1 \leqslant i \leqslant (n-1)$

$$f(e_i^{'}) = 4n+2i-5,\ f(e_i^{''}) = -(6n-5-2i).$$

For each edge label f the induced vertex label f^* is defined as follows::

$$f^*(u_1) = (4n-2) = -f^*(w_{2n-1}),$$

for $1 \leqslant i \leqslant (n-1)$

$$f^*(u_{2i}) = (8i-4),$$

for $1 \leqslant i \leqslant (n-2)$

$$f^*(u_{2i+1}) = (4n-3+10i),$$

for $1 \leqslant i \leqslant \frac{n-1}{2}$

$$f^*(v_{\frac{n-1}{2}+i}) = (4i-2) \text{ and } f^*(v_i) = -(2n-4i),\ f^*(u_{2n-1}) = (4n-5) = -f^*(w_1),$$

for $1 \leqslant i \leqslant (n-1)$

$$f^*(w_{2i}) = -(8n-4-8i)$$

and for $1 \leqslant i \leqslant (n-2)$

$$f^*(w_{2i+1}) = -(14n-13-10i).$$

Then

$f^*(V(S(Sl_n))) = \{\pm 2, \pm 6, \pm 10, ..., \pm(2n-4), \pm 4, \pm 12, \pm 20, ..., \pm(8n-12), \pm(4n-2), \pm(4n-5), \pm(4n+7), \pm(4n+17), ..., \pm(14n-23)\}$.

It can be verified that f is an edge pair sum labeling of $S(Sl_n)$ if n is odd. Hence $S(Sl_n)$ is an edge pair sum graph if n is odd.

Case (ii). n is even.

Subcase (a). $n = 2$.

$$f(e_1) = 3 = -f(e_2^{'''}),\ f(e_2) = 4 = -f(e_1^{'''}),\ f(e_1^{''}) = 1 \text{ and } f(e_1^{'}) = -2.$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = 1 = -f^*(v_1),\ f^*(u_3) = 4 = -f^*(w_1),\ f^*(u_2) = 7 = -f^*(w_2) \text{ and } f^*(w_3) = -2.$$

Then

$$f^*(V(S(D(Q_n))) = \{\pm 1, \pm 4, \pm 7\} \bigcup \{-2\}.$$

Hence f is an edge pair sum labeling if $n = 2$.

Subcase (b). $n \geqslant 4$.

for $1 \leqslant i \leqslant (2n-2)$

$f(e_i) = (2i-1)$ and $f(e_i^{'''}) = -(4n-3-2i)$, $f(e^{'}_{\frac{n}{2}}) = -(4n-3)$, $f(e^{''}_{\frac{n}{2}}) = (4n+2)$,

for $1 \leqslant i \leqslant \frac{n-2}{2}$

$$f(e_i^{'}) = (4n-3+2i),\ f(e^{'}_{\frac{n}{2}+i}) = (5n-5+2i),\ f(e_i^{''}) = -(6n-5-2i),$$
$$f(e^{''}_{\frac{n}{2}+i}) = -(5n-3-2i).$$

For each edge label f the induced vertex label f^* is defined as follows:

$$f^*(u_1) = 4n = -f^*(w_{2n-1}),\ f^*(u_{2n-1}) = (4n-5) = -f^*(w_1),$$

for $1 \leqslant i \leqslant n-1$

$$f^*(u_{2i}) = (8i-4),\ \ f^*(u_{n-1}) = -5 = -f^*(v_{\frac{n}{2}}),$$

for $1 \leqslant i \leqslant \frac{n-4}{2}$

$$f^*(u_{2i+1}) = (4n-1+10i),$$

for $1 \leqslant i \leqslant \frac{n-2}{2}$

$$f^*(u_{n-1+2i}) = (9n-13+10i),\ f^*(v_i) = -(2n-2-4i) \text{ and } f^*(v_{\frac{n}{2}+i}) = (4i-2),$$

for $1 \leqslant i \leqslant n-1$

$$f^*(w_{2i}) = -(8n-4-8i),$$

for $1 \leqslant i \leqslant \frac{n}{2}-1$

$$f^*(w_{2i+1}) = (-14n+13+10i),$$

for $1 \leqslant i \leqslant \frac{n}{2}-2$

$$f^*(w_{n+1+2i}) = (-9n+11+10i),\ \ f^*(w_{n+1}) = 10.$$

Then

$f^*(V(S(Sl_n))) =$

$\{\pm2, \pm6, \pm10, ..., \pm(2n-6), \pm4, \pm12, \pm20, ..., \pm(8n-12), \pm5, \pm4n, \pm(4n-5), \pm(4n+9), \pm(4n+19), \pm(4n+29), ..., \pm(9n-21), \pm(9n-3), \pm(9n+7), \pm(9n+17), ..., \pm(14n-23)\} \bigcup \{10\}$.

It can be verified that f is an edge pair sum labeling of $S(Sl_n)$ if n is even. Hence $S(Sl_n)$ is an edge pair sum graph if n is even. □

The subdivision of slanting of $S(Sl_4)$ and $S(Sl_3)$ are shown in Figure 5.

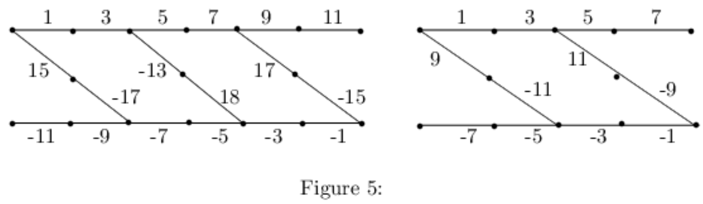

FIGURE 5. Edge pair sum labeling of $S(Sl_4)$ and $S(Sl_3)$

References

[1] J.A.Gallian. A dynamic survey of graph labeling. *The Elec. J. Combin.*, **19**, #DS6 (2016).

[2] F.Harary. *Graph theory*. Addison Wesley, Massachusetts, 1972.

[3] P. Jeyanthi and T.Saratha Devi. Edge pair sum labeling. *J. Sci. Res.*, **5**(3)(2013), 457-467.

[4] P. Jeyanthi and T.Saratha Devi. On edge pair sum labeling of graphs. *Int. J. Math. Trends Tech*, **7**(2)(2014), 106-113.

[5] P. Jeyanthi and T.Saratha Devi. Edge pair sum labeling of spider graph. *J. Algorithms Comp.*, **45**(1)(2014), 25 - 34.

[6] P. Jeyanthi and T.Saratha Devi. Some edge pair sum graphs. *J. Discrete Math. Sci. and Crypt.*, **18**(5)(2015), 481-493.

[7] P. Jeyanthi, T.Saratha Devi and Gee-Choon Lau. Some results of edge pair sum labeling. *Elec. Notes Discrete Math.*, **48**(2015), 169-173.

[8] P. Jeyanthi, T.Saratha Devi and Gee-Choon Lau. Edge pair sum labeling of $WT(n:k)$ Tree, *Global J. Pure Appl. Math.*, **11**(3)(2015), 1523-1539.

[9] P. Jeyanthi and T.Saratha Devi. New results on edge pair sum graphs. *Int. J. Math. Appl.*, **4**(1-B)(2016), 57-65.

[10] P. Jeyanthi and T.Saratha Devi. Edge pair sum labeling of butterfly graph with shell order. *Malaya J. Math.*, **4**(2)(2016), 205-210.

[11] P. Jeyanthi and T.Saratha Devi. Edge Pair Sum labeling of some Cartesian product of graphs. *Discrete Math., Algor. Appl.*, **8**(2)(2016) 1650024.

[12] P. Jeyanthi and T.Saratha Devi. Edge Pair Sum labeling of some cycle related graphs. *J. Algor. Comp.*, **47**(2016), 57-68.

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