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Hydromagnetic Nanofluid Flow over a Rotating Porous Disk with Prescribed Heat Flux

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ABSTRACT

Owing to the abundant applications of nanofluids, this study is mainly concerned with the steady, laminar, nonlinear MHD flow of nanofluids over a rotating porous disk subjected to prescribed heat flux and suction. Two types of nanofluids such as copper - water nanofluid and silver - water nanofluid are considered. Governing equations are system of nonlinear partial differential equations and they are converted to a system of nonlinear ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved numerically using Nachtsheim-Swigert iteration shooting technique for satisfaction of asymptotic boundary conditions and employing Runge - Kutta Fehlberg Method. The features of the flow and heat transfer characteristics are analysed and discussed. The radial, tangential and axial velocities are obtained numerically and are represented graphically. Numerical computations for dimensionless temperature, the radial skin friction coefficient and the tangential skin friction coefficient for both the types of nanofluid flows are obtained for various values of physical parameters.

Keywords: MHD, Nanofluid, Rotating Disk, Suction, Heat flux

INTRODUCTION

The study of hydromagnetic (MHD) boundary layer flow on a rotating disk has attracted considerable attention during the last few decades due to its numerous applications in rotating machinery, turbine rotors, medical equipment's, thermal-power generating systems, computer storage devices, crystal growth processes, aero dynamics air cleaning, rotating heat exchangers, rotating disk reactors for bio-fuels production and in many other geothermal and geophysical applications.

In recent days nanofluids have great potential in heat transfer applications, because of its increased thermal conductivity and increased Nusselt numberdue to higher thermal conductivity and other various effects on heat transfer phenomenon. All these applications motivated the authors to deal with the problem involving hydromagnetic flow of nanofluids over a rotating disk subjected to suction in the presence of prescribed heat flux. Pao [1] was one of the first to investigate the flow of an incompressible viscous conducting fluid over a rotating disk when a circular magnetic field is imposed. He concluded that the presence of the magnetic field thickens the flow boundary layer and also reduces the strength of the axial flow field. Furthermore, for sufficiently large values of the applied magnetic field, the boundary layer separates from the surface of the disk. Many researchers (Kuiken [2], Ockendon [3], Owen and Rogers [4], El-Mistikawy and Attia [5] have studied and reported results on disk-shaped bodies with or without heat transfer.

Attia and Aboul-Hassan [6] studied steady hydromagnetic flow due to an infinite disk rotating with uniform angular velocity in the presence of an axial magnetic field. In their analysis they neglected the induced magnetic field but considered Hall current. Attia [7] has studied the effect of suction and injection on the transient flow of a conducting fluid due to a rotating disk in the presence of an external uniform magnetic field neglecting the Hall current and ion slip.The effect of slip on entropy generation in MHD flow over a rotating disk was investigated by Arikoglu *et al* [8] via the differential transform method. Devi and Ganga [9] scrutinized the nonlinear hydromagnetic flow and heat transfer over a stretching porous surface in the presence of constant heat flux. Devi and Devi [10], Bhattacharyya

[11], Hakeem *et al* [12] and Das [13] have studied the disguised flow conditions in various geometries with the magnetic field effects.

Convective heat transfer in nanofluids is a topic of major contemporary interest both in sciences and engineering. The term 'nanofluid' which was first proposed by Choi [14] to indicate engineered colloids composed of nanoparticles dispersed in a base fluid. Various benefits of the application of nanofluids include improved heat transfer, heat transfer system size reduction, minimal clogging, microchannel cooling and miniaturization of the system. Therefore, research is underway to apply nanofluids in environments where higher heat flux is encountered and the convectional fluid is not capable of achieving the desired heat transfer rate. In the past few years, many experimental investigations on the thermal conductivity of nanofluids have been studied. Recently several authors investigated about natural convection and effect of adding nanoparticle in the base fluid on flow and heat transfer (Eastman *et al* [15], Xuan *et al* [16]). Buongiorno [17] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (that he calls the slip velocity). He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity settling. The nanofluid mathematical model proposed by Buongiorno was very recently used by several researchers such as, Nield and Kuznetsov [18], Kuznetsov and Nield [19], Khan and Pop [20], Khan and Aziz [21] etc.

Vajravelu *et al* [22] discussed the convective heat transfer in a nanofluid flow over a stretching surface using Ag-water or Cu-water nanofluid. Magnetic effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate have been discussed by Hamad *et al* [23]. Sheikholeslami *et al* [24] used heat line analysis to simulate two phase simulation of nanofluid flow and heat transfer. Their results indicated that the average Nusselt number decreases as buoyancy ratio number increases until it reaches a minimum value and then starts increasing. Furthermore, Julie Andrews and Anjali Devi [25] studied the steady flow and heat transfer over a rotating disk with a prescribed heat flux in Copper water nanofluid. Rashidi *et al* [26] considered the analysis of the second law of thermodynamics applied to an electrically conducting incompressible nanofluid flowing over a porous rotating disk. Sandeep Naramgari and Sulochana [27] investigated the heat transfer behavior of hydromagnetic nanofluid embedded with conducting dust particles past a stretching surface. Devi and Priya [28] presented the hydromagnetic slip flow of nanofluids over a vertical stretching surface subjected to injection.

Keeping the above curious findings in mind, the present work is devoted to examine the MHD steady, laminar, axisymmetric flow of an incompressible, electrically conducting, viscous nanofluid over a rotating disk subjected to prescribed heat flux and suction. In the present study water is assumed as a base fluid and the effects of nanoparticle volume fraction are considered for two types of nanofluids. Introducing the similarity transformations, the momentum and energy equations, under boundary layer assumptions reduce the equations to the set of nonlinear ordinary differential equations. Solutions of reduced nonlinear ordinary differential equations are solved numerically using Nachtsheim - Swigert iteration shooting technique for satisfaction of asymptotic boundary conditions along with Runge - Kutta Fehlberg Method. Numerical results are obtained for various physical parameters and displayed graphically. Physical behaviour of skin-friction coefficient is also discussed through tables.

FORMULATION OF THE PROBLEM

The problem of steady, laminar, three dimensional, axisymmetric, hydromagnetic flow of an incompressible, electrically conducting, viscous, nanofluid on a rotating disk in the presence of suction and prescribed heat flux is considered. Two types of nanofluids such as copper-water nanofluid and silver-water nanofluid are considered. Cylindrical polar coordinates (r, ϕ, z), where z is the vertical axis in the cylindrical polar coordinate system with r and ϕ as the radial and tangential axes are chosen. The electrically conducting fluid occupies the region $z > 0$ with the rotating disk placed at $z = 0$ as indicated in fig. 1 and rotating with constant angular velocity Ω. The components of the flow velocity are (u, v, w) in the directions of increasing (r, ϕ, z) respectively, the pressure is P, the density of the fluid ρ and T is the fluid temperature.

The surface of the rotating disk is subjected to suction and it is maintained at a uniform temperature T_w. For away from the wall, the free stream is kept at a constant temperature T_∞ and at a constant pressure P_∞. An uniform magnetic field of strength H_0 is applied in the direction normal to the surface of the disk. The induced magnetic field due to the motion of the electrically conducting fluid is assumed to be negligible by taking small magnetic Reynolds number (Rm<<1). Since the induced magnetic field is neglected and $\overrightarrow{B_0}$ is independent of time, $\mathrm{curl}\,\vec{E} = 0$. Also, $\mathrm{div}\,\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$. The governing equations of the Nanofluid flow and energy in cylindrical polar co-ordinates are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}+\frac{1}{\rho_{nf}}\frac{\partial P}{\partial r}= \nu_{nf}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho_{nf}}u \quad (2)$$

$$u\frac{\partial v}{\partial r}+\frac{uv}{r}+w\frac{\partial v}{\partial z}= \nu_{nf}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho_{nf}}v \quad (3)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}+\frac{1}{\rho_{nf}}\frac{\partial P}{\partial z}=\nu_{nf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \quad (4)$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \quad (5)$$

accompanied by the boundary conditions

$$\begin{gathered} u=0,\quad v=\Omega r,\quad w=-W_0,\quad q_w=-k_{nf}\left(\frac{\partial T}{\partial z}\right)\quad at\quad z=0 \\ u\to 0,\quad v\to 0,\quad T\to T_\infty,\quad P\to P_\infty\quad as\quad z\to\infty \end{gathered} \quad (6)$$

where P is the Pressure, T is the temperature, with T_w and T_∞ denoting wall and ambient conditions. Moreover, ρ_{nf} is the density of the nanofluid, ν_{nf} is the kinematic viscosity of the nanofluid, μ_{nf} is the dynamic viscosity of the nanofluid and α_{nf} is the thermal diffusivity of the nanofluid, which are respectively given by

$$\begin{gathered} \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\qquad \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}},\quad \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)} \\ \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s\ ,\quad (\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_s \end{gathered} \quad (7)$$

here, ϕ is the nanoparticle volume fraction , $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the base fluid and the solid particles (Cu, Ag) respectively, and ρ_f and ρ_s are the densities of the base fluid and of the solid particles, respectively. It should be mentioned that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles. Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman as viscosity of a base fluid μ_f contains dilute suspension of the spherical particle.

The following dimensional variables are introduced

$$\begin{gathered} \eta=\left(\Omega/\nu_f\right)^{1/2}z,\ \ u=\Omega rF(\eta),\ \ v=\Omega rG(\eta),\ \ w=(\Omega\nu_f)^{1/2}H(\eta), \\ P-P_\infty=-\mu_f\Omega P(\eta),\qquad T-T_\infty=\frac{q_w}{k_{nf}}\left(\frac{\nu_f}{\Omega}\right)^{1/2}\theta(\eta) \end{gathered} \quad (8)$$

Substituting Equation (8) in (1) – (5), it is obtained as

$$2F+H'=0 \quad (9)$$

$$F''-(1-\varphi)^{2.5}\left[A\left(F^2+HF'-G^2\right)+M^2F\right]=0 \quad (10)$$

$$G''-(1-\varphi)^{2.5}\left[A\left(HG'+2FG\right)+M^2G\right]=0 \quad (11)$$

$$H''-(1-\varphi)^{2.5}\left[AHH'+2P'\right]=0 \quad (12)$$

$$\frac{1}{\Pr}\frac{k_{nf}}{k_f}\theta''-BH\theta'=0 \quad (13)$$

where the prime denotes differentiation with respect to η and the dimensionless quantities are given by

$M^2=\dfrac{\sigma B_0^2}{\Omega\rho_f}$ [here M is the magnetic interaction parameter]

where $\Pr=\dfrac{\alpha_f}{\nu_f}$ is the Prandtl number and $A=(1-\phi+\phi\,\rho_S/\rho_f)$, $B=(1-\phi+\phi\left(\rho C_p\right)_S/\left(\rho C_p\right)_f)$

and the boundary conditions are given by

$$F(0)=0, \quad G(0)=1, \quad H(0)=-W_s, \quad \theta'(0)=-1$$
$$F(\eta)\to 0, \quad G(\eta)\to 0, \quad P(\eta)\to 0, \quad \theta(\eta)\to 0 \quad as \quad \eta\to\infty \tag{14}$$

where, $W_s = \dfrac{W_0}{(\Omega \nu_f)^{1/2}}$ is the uniform suction parameter when $W_s > 0$.

The tangential shear stress and radial shear stress are given by the Newtonian formulae:

$$\tau_t = \left[\mu_{nf}\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right)\right]_{z=0} \quad \text{and} \quad \tau_r = \left[\mu_{nf}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right]_{z=0}$$

Hence the local tangential and radial skin-friction coefficient for prescribed heat flux case are respectively given by

$$(\mathrm{Re})^{1/2} C_{ft} = \frac{1}{(1-\phi)^{2.5}} G'(0) \quad \text{and} \quad (\mathrm{Re})^{1/2} C_{fr} = \frac{1}{(1-\phi)^{2.5}} F'(0)$$

where $\mathrm{Re} = \left(\Omega r^2 / \nu_f\right)$ is the local rotational Reynolds number.

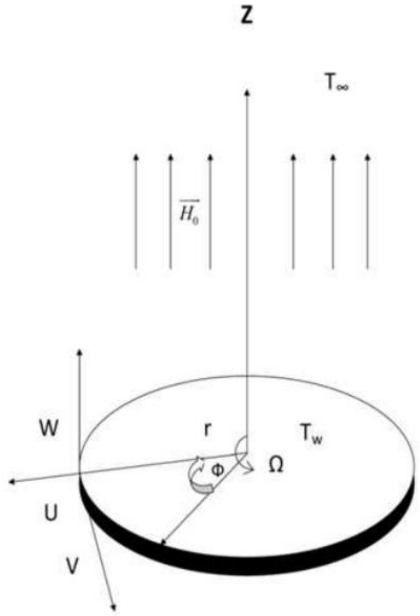

Fig. 1 Schematic diagram of the problem

NUMERICAL SOLUTION OF THE PROBLEM

Numerical solution of the MHD flow of nanofluids over a rotating disk in the presence of prescribed heat flux and suction has been obtained. Two cases of nanoparticles such as copper and silver are considered for the present study. In doing do, since the non-linear equations constitute a non-linear boundary value problem it is highly ridiculous to obtain the solution straight away. Hence the boundary value problem constituted by equations (9) to (11) along with (13) has to be converted into initial value problem. Here the problem is solved using Nachtsheim- Swigert shooting iteration technique for satisfaction of asymptotic boundary conditions along with Runge - Kutta Fehlberg Method. In this study, a uniform grid of $\Delta\eta = 0.01$ has been used so that the results are mesh independent and the convergence criterion was set to 10^{-5}, which gives accuracy upto four decimal places. The thermophysical properties of the fluid and the nanoparticles (Cu and Ag) used in the simulation is presented in Table 1.

Fig. 2 to Fig. 4 are the graphical representations which portray the effect of magnetic interaction parameter on the radial, tangential and axial velocities for both copper- water nanofluid and silver-water nanofluid. It is found that the effect of Magnetic field is to reduce the radial, tangential and axial velocities. This reveals that the effect of magnetic field is to decrease the velocity, which is due to the fact that increase of magnetic interaction parameter signifies the increase of Lorentz force, which opposes the flow in the reverse direction. The effect of magnetic field over temperature is shown through Fig. 5 for both types of nanofluids. As magnetic interaction parameter increases, the temperature increases elucidating the fact that the effect of magnetic field is to increase the thickness of the thermo-magnetic layer.

Table - 1 Thermophysical Properties of Base Fluid Water, Copper and Silver at 25° C

	ρ (Kg/m^3)	c_p (J/Kg.K)	k (W/m.K)
Water	997.1	4179	0.613
Copper	8933	385	400
Sliver	10500	235	429

Table - 2 Comparison of the results F′(0), -G′(0) and θ(0) when M^2 = 0 and W$_s$ = 0 (PHF case)

	Pr	F′(0)	- G′(0)	θ(0)
Author's Result	7.02	0.5102326	0.6159220	1.017305
Devi and Devi [10]	7.02	0.5102326	0.6159220	1.017310

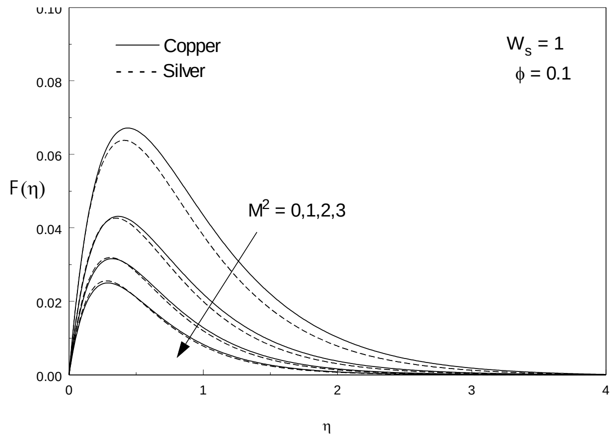

Fig.2 Effect of Magnetic field on radial velocity profiles for copper-water nanofluid and silver-water nanofluid

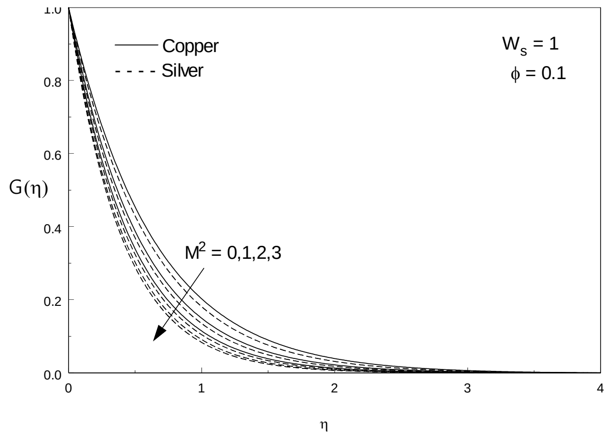

Fig.3 Tangential velocity profiles for various values of magnetic interaction parameter for copper-water nanofluid and silver-water nanofluid

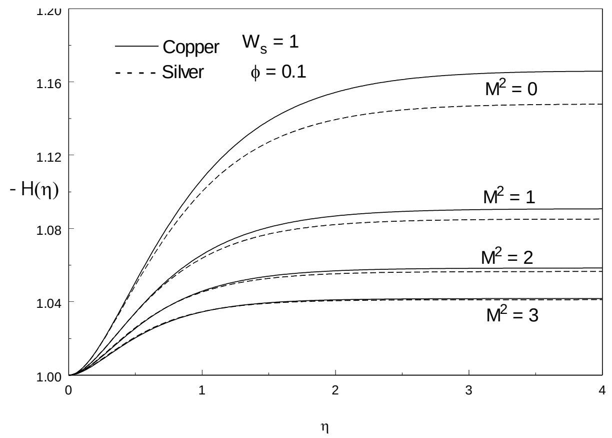

Fig. 4 Effect of Magnetic field on axial velocity profiles for copper-water nanofluid and silver-water nanofluid

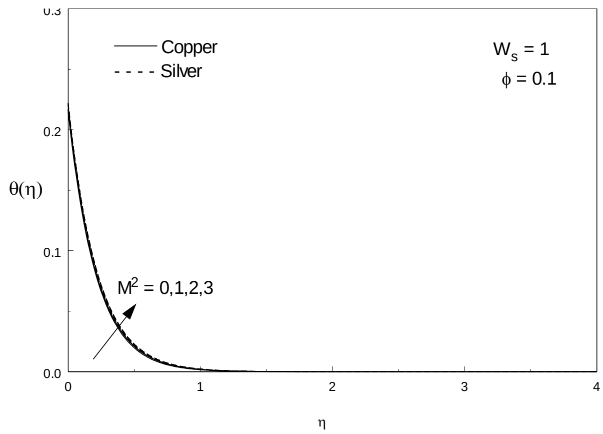

Fig. 5 Temperature distribution for various values of magnetic interaction parameter for copper – water nanofluid and silver-water nanofluid

RESULTS AND DISCUSSION

In order to have a clear physical insight of the problem, numerical solutions are obtained for various values of physical parameters and their effects over velocity and temperature are analysed in detail. In the absence of magnetic interaction parameter, radiation parameter, heat absorption parameter and suction Parameter, results are identical to that of Devi and Devi [10] for the base fluid (ϕ= 0) and these results are shown in Table -2.

Figs. 6 to 8 illustrate the radial, tangential and axial velocities for different values of suction parameter for both the nanofluids with nanoparticles, say Cu and Ag. For strong suction, the radial velocity is small. The fact that suction stabilizes the boundary layer is also apparent from Fig. 6 and 7. As suction increases, radial and tangential velocities decrease. As the wall suction increases, escape through the wall becomes easier and easier. Therefore, as suction parameter increases, axial velocity increases in magnitude for both the types of nanofluids. Further the value of axial velocity remains uniform with respect to η when W_s takes values 4, 3, 2 and 1. All these are noted in Fig. 8.

The effect of suction parameter on temperature for both types of nanofluids is depicted in Fig. 9. Imposition of wall suction, leads to the reduction in the temperature and as well as in the thermal boundary layer thickness.

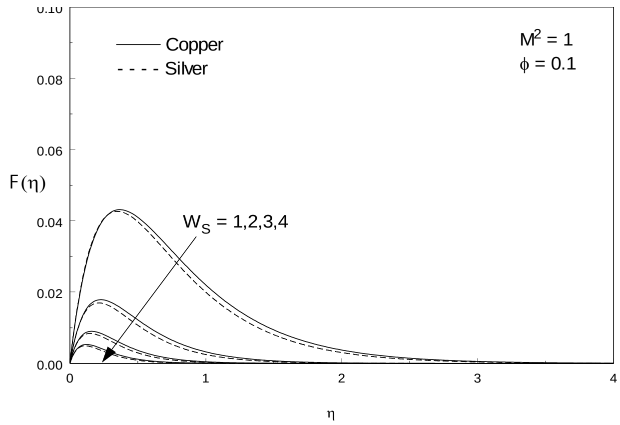

Fig. 6 Radial velocity profiles for different values of suction parameter for copper-water nanofluid and silver -water nanofluid

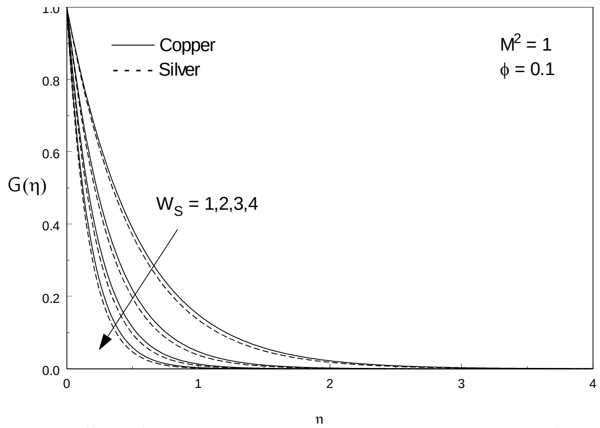

Fig. 7 Effect of suction parameter on tangential velocity profiles for copper-water nanofluid and silver -water nanofluids

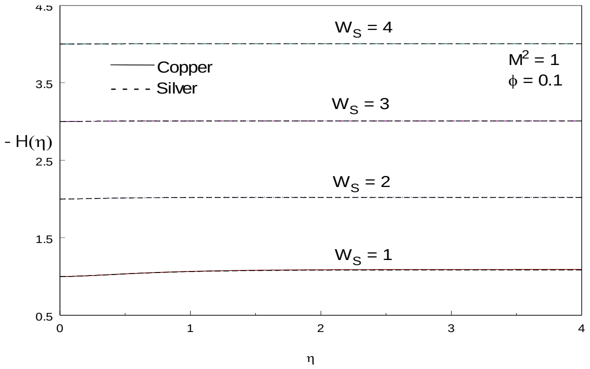

Fig. 8 Axial velocity profiles for different values of suction parameter for copper-water nanofluid and silver -water nanofluid

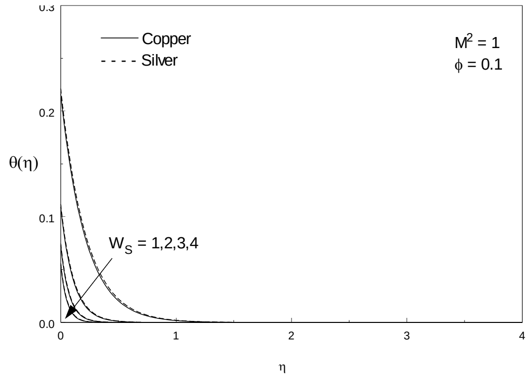

Fig. 9 Influence of suction parameter on temperature distribution for copper-water nanofluid and silver -water nanofluid

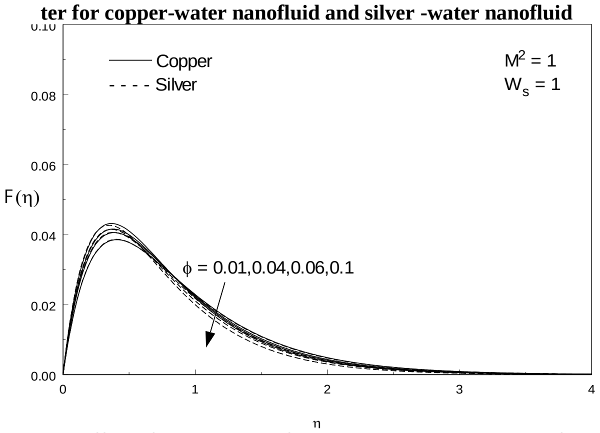

Fig. 10 Effect of Solid volume fraction on radial velocity profiles for copper-water nanofluid and Silver -water nanofluid

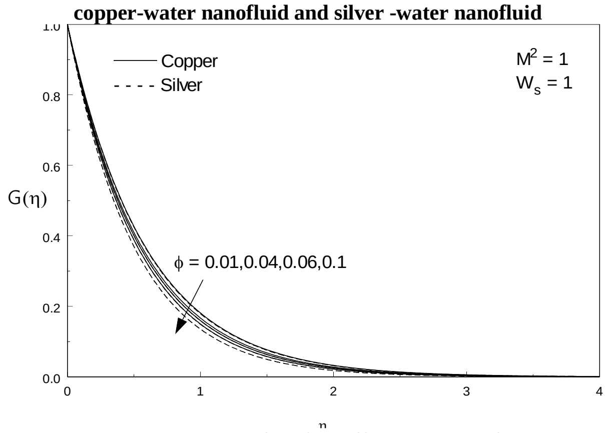

Fig. 11 Tangential velocity profiles for different values of Solid volume fraction for copper-water nanofluid and silver -water nanofluid

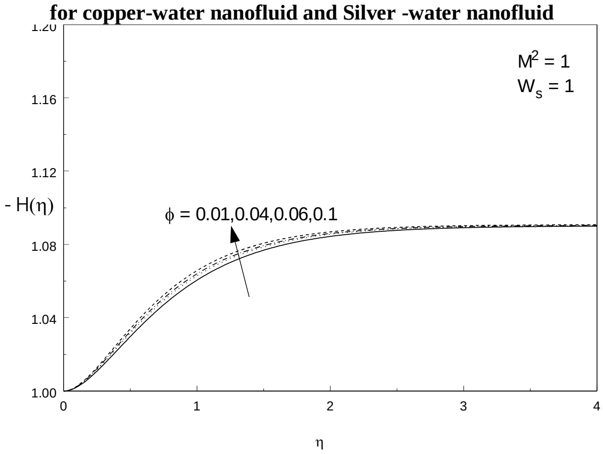

Fig. 12 Variation of the Solid volume fraction on axial velocity profiles for copper-water nanofluid

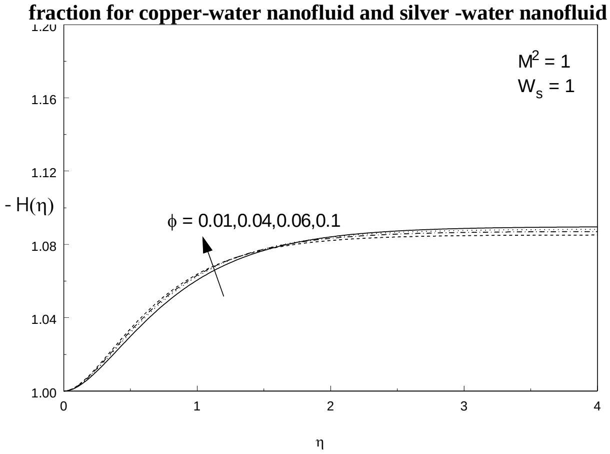

Fig. 13 Variation of the Solid volume fraction on axial velocity profiles for silver -water nanofluid

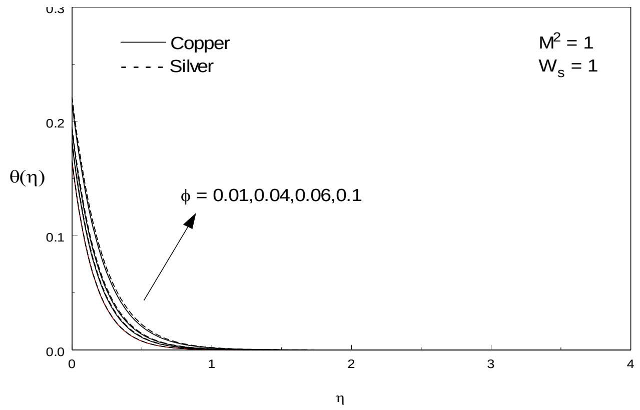

Fig. 14 Effect of Solid volume fraction on temperature distribution for copper-water nanofluid and silver -water nanofluid

The variations in velocities (F(η), G(η) and -H(η)) and temperature with respect to nanoparticle volume fraction parameter ϕ on the flow field for both copper-water nanofluid and silver-water nanofluid are presented in Figs. 10 – 14. It is noted from Fig. 10 and 11 that the momentum boundary layer thickness decrease with increasing solid volume fraction ϕ and the existence of nanoparticles lead to more thinning of the boundary layer. Fig. 12 and 13 elucidate the variation in velocity due to the hype in solid volume fraction. It is noted that the velocity got promoted for the effect of solid volume fraction. The dimensionless temperature distribution for both types of nanofluids for different ϕ is shown in Fig. 14. The thermal conductivity of the nanofluid increases as the nanoparticle volume fraction increases, leading to a disseminating of the thermal boundary layer thickness. This agrees with the physical behaviour that due to the addition of nanoparticles the thermal conductivity of the fluid increases and consequently the thickness of the thermal boundary layer also increases.

Table 3 displays the effect of M^2, W_s and ϕ over radial and tangential skin friction coefficients respectively, for both copper-water nanofluid and silver-water nanofluid for the prescribed heat flux case. It is noted that the effect of M^2 and W_s have similar trend over radial skin friction coefficient so as to decrease it. Similarly increasing values of the magnetic interaction parameter and W_s have increasing effect over the tangential skin friction coefficient in magnitude. The growing effects of radial skin friction coefficient and tangential skin friction coefficient in magnitude are observed due to the effect of solid volume fraction for both types of nanofluids which are shown in table 3. It is also noted from table 3 that the temperature at the wall for both types of nanofluid rises higher with the growing effect of magnetic interaction parameter and solid volume fraction. But the opposite effect is observed for the effect of suction parameter.

Table - 3 Variation of $\frac{1}{(1-\phi)^{2.5}}F'(0)$, $\frac{-1}{(1-\phi)^{2.5}}G'(0)$ and θ(0) for different values of M^2, W_s and ϕ

M^2	W_s	ϕ	Copper-Water Nanofluid			Silver-Water Nanofluid		
			$\frac{1}{(1-\phi)^{2.5}}F'(0)$	$\frac{-1}{(1-\phi)^{2.5}}G'(0)$	θ(0)	$\frac{1}{(1-\phi)^{2.5}}F'(0)$	$\frac{-1}{(1-\phi)^{2.5}}G'(0)$	θ(0)
0	1	0.1	0.54815	2.00137	0.21496	0.55853	2.15055	0.22019
1			0.42293	2.43269	0.21607	0.44075	2.56497	0.22126
2			0.35458	2.79124	0.21666	0.37363	2.91338	0.22186
3			0.31230	3.09642	0.21700	0.33014	3.21453	0.22223
1	1	0.1	0.42293	2.43269	0.21607	0.44075	2.56497	0.22126
	2		0.28290	3.03435	0.10917	0.28820	4.23950	0.11183
	3		0.20321	5.62872	0.07285	0.20513	6.08237	0.07462
	4		0.15672	7.65930	0.05464	0.15759	7.97942	0.05598
1	1	0.01	0.26844	1.73069	0.16532	0.27096	1.74291	0.16570
		0.04	0.31950	1.95709	0.18124	0.32830	2.00770	0.18296
		0.06	0.35350	2.11249	0.19248	0.36570	2.18973	0.19523
		0.1	0.42293	2.43269	0.21607	0.44075	2.56497	0.22126

CONCLUSION

The nonlinear hydromagnetic flow of nanofluids over a rotating porous disk with prescribed heat flux has been investigated numerically. A detailed study on dimensionless velocities, temperature, radial and tangential skin friction coefficients are carried out. In general, the velocity and temperature distribution are dominantly influenced by the physical parameters involved in the problem. In the absence of magnetic interaction parameter and suction parameter, the numerical values of $F'(0)$ and $-G'(0)$ are found to be in excellent agreement with that of Devi and Devi [10].

The significant findings drawn from the present investigation are listed as follows:

- The effect of magnetic field decelerates the components of velocity, reduces radial skin friction coefficient for both the nanofluids, whereas the temperature distribution and tangential skin friction coefficient in magnitude enhance in both cases.
- The radial and tangential velocities, temperature and radial skin friction coefficient are seen to remit due to the increasing values of suction parameter for both the copper-water nanofluid and silver-water nanofluid. Also Suction parameter shoots up the axial velocity and tangential skin friction in magnitude.
- The radial and tangential velocities get suppressed due to an increase in solid volume fraction for both the types of nanofluid and augment the axial velocity, radial skin friction coefficient and tangential skin friction coefficient in magnitude.

The experimental result shows that applying the template of half eye gives a satisfying result in the frontal face image. Furthermore, applying composite template produces better results of eye detection through all orientations. In order to detect head poses in the face image, three templates were generated to represent the entire face, in frontal case and both profile cases. Finally, a detection rate of 98.83% was reported by applying the proposed method on PICS Database while it is 93.87% for the templates which have entire eye image.

REFERENCES

[1] HP Pao, Magneto Hydrodynamic Flows Over a Rotating Disk, *American Institute of Aeronautics and Astronautics*, **1968**, 6,1285-1291.
[2] HK Kuiken, The Effect of Normal Blowing on the Flow Near a Rotating Disk of Infinite Extent, *Journal of Fluid Mechanics,* **1971**, 47(4), 789-798.
[3] H Ockendon, An Asymptotic Solution for Steady Flow Above an Infinite Rotating Disk with Suction, *The Quarterly Journal of Mechanics and Applied Mathematics*, **1972**, 25, 291-301.
[4] JM Owen and RH Rogers, *Flow and Heat Transfer in Rotating Disc System, Rotor-Stator Systems*, Research Studies Press, Taunton, UK and John Wiley, NY, **1989**.
[5] El-Mistikawy, HA Attia and AA Megahed, The Rotating Disk Flow in The Presence of Weak Magnetic Field, *Proceedings of 4th Conference on Theoretical and Applied Mechanics*, Cairo, Egypt, **1991**.
[6] AL Aboul-Hassan and HA Attia, Flow Due to a Rotating Disk with Hall Effects, *Physics Letters A*, **1997,** 22, 286-290.
[7] HA Attia and AL Aboul-Hassan, Effect of Hall Current on the Unsteady MHD Flow Due to a Rotating Disk with Uniform Suction or Injection, *Applied Mathematical Modeling*, **2001,** 25(12), 1089-1098.
[8] A Arikoglu, I Ozkol and G Komurgoz, Effect of Slip on Entropy Generation in a Single Rotating Disk in MHD Flow, *Applied Energy*, **2008,** 85(12), 1225-1236.
[9] SP Anjali Devi and B Ganga, MHD Nonlinear Flow and Heat Transfer Over a Stretching Porous Surface of Constant Heat Flux, *Computer Assisted Mechanics and Engineering Sciences,* **2008,** 15(1), 15-22.
[10] SP Anjali Devi and R Uma Devi, On Hydromagnetic Flow Due to a Rotating Disk with Radiation Effects, *Nonlinear Analysis: Modelling and Control,* **2011,** 16(1), 17-29.
[11] AK Abdul Hakeem, R Kalaivanan, N Vishnu Ganesh and B Ganga, Effect of Partial Slip on Hydromagnetic Flow Over a Porous Stretching Sheet with Non-Uniform Heat Source/Sink, Thermal Radiation and Wall Mass Transfer, *Ain Shams Engineering Journal,* **2014,** 5, 913-922.
[12] Krishnendu Bhattacharyya, Heat Transfer Analysis in Unsteady Boundary Layer Stagnation-Point Flow Towards a Shrinking/Stretching Sheet, *Ain Shams Engineering Journal,* **2013,** 4, 259-264.
[13] Kalidas Das, Radiation and Melting Effects on MHD Boundary Layer Flow Over a Moving Surface, *Ain Shams Engineering Journal*, **2014,** 5, 1207-1214.
[14] SUS Choi, Enhancing Thermal Conductivity of Fluids with Nanoparticles in Developments and Applications of Non-Newtonian Flow, *The American Society of Mechanical Engineers*, **1995,** 66, 99–105.
[15] JA Eastman, SUS Choi, S Li, LJ Thompson and Lee, Enhanced Thermal Conductivity Through the Development of Nanofluids, *Nanophase and Nanocomposite Materials II*, **1997,** 3-11.
[16] Y Xuan and Q Li, Heat Transfer Enhancement of Nanofluids, *International Journal of heat and fluid flow*, **2000,** 21, 58-64.

[17] J Buongiorno, Convective Transport in Nanofluids, *The American Society of Mechanical Engineers - Heat Transfer*, **2006,** 128, 240-250.
[18] DA Nield and AV Kuznetsov, The Cheng-Minkowycz Problem for Natural Convective Boundary-Layer Flow in a Porous Medium Saturated by a Nanofluid, *International Journal of Heat and Mass Transfer*, **2009,** 52, 5792-5795.
[19] AV Kuznetsov and DA Nield, Natural Convective Boundary-Layer Flow of a Nanofluid Past a Vertical Plate, *International Journal of Thermal Sciences*, **2010,** 49, 243-247.
[20] AV Khan and I Pop, Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet, *International Journal of Heat and Mass Transfer*, **2010,** 53, 2477-2483.
[21] WA Khan and A Aziz, Natural Convection Flow of a Nanofluid Over a Vertical Plate with Uniform Surface Heat Flux, *International Journal of Thermal Sciences*, **2011,** 50, 1207-1214.
[22] K Vajravelu, KV Prasad, L Jinho, L Changhoon, I Pop and A Robert, Convective Heat Transfer in the Flow of Viscous Ag-Water and Cu-Water Nanofluids Over a Stretching Surface, *International Journal of Thermal Sciences*, **2011,** 50, 843-51.
[23] A Hamad, Pop and Ismali, Magnetic Field Effects on Free Convection Flow of a Nanofluid Past a Vertical Semi-Infinite Flat Plate, *Nonlinear Analysis: Real World Applications*, **2011,** 12, 1338-46.
[24] Julie Andrews, SP Anjali Devi, Nanofluid Flow Over a Rotating Disk with Prescribed Heat Flux, *Mapana Journal of Sciences*, **2012,**11(2), 76-90.
[25] M Sheikholeslami, M Gorji-Bandpy and Soheil Soleimani, Two Phase Simulation of Nanofluid Flow and Heat Transfer using Heat Line Analysis, *International Communications in Heat and Mass Transfer*, **2013,** 47, 73-81.
[26] MM Rashidi, S Abelman and N Freidooni Mehr, Entropy Generation in Steady MHD Flow Due to a Rotating Porous Disk in a Nanofluid, *International Journal of Heat and Mass Transfer*, **2013,** 62, 515-525.
[27] Sandeep Naramgari and CSulochana, MHD Flow of Dusty Nanofluid Over a Stretching Surface with Volume Fraction of Dust Particles, *Ain Shams Engineering Journal*, **2016,** 7(2), 709-716.
[28] SP Anjali Devi and T Elakkiya Priya, MHD Slip Flow and Convective Heat Transfer of Nanofluids over a Permeable Stretching Surface, *International Journal of Science and Research*, **2015,** 4(4), 138-147.