Available online www.ejaet.com

European Journal of Advances in Engineering and Technology, 2017, 4 (4): 255-267

Research Article ISSN: 2394 - 658X

Deadbeat Control of Dynamics of Inverted Pendulum using Signal Correction Technique

Soumen Paul[1], Asim Halder[2] and Asoke Kumar Nath[3]

[1]Department of Information Technology, Haldia Institute of Technology, Haldia, India
[2]Department of Applied Electronics Engineering, Haldia Institute of Technology, Haldia, India
[3]Department of Electronics and Tele Communication Engineering, Jadavpur University, Kolkata, India
s.paul.it@gmail.com

ABSTRACT

This work presents the deadbeat control technique to mitigate transient oscillations of dynamics of Inverted pendulum or 1-link robot arm based on Signal Correction Technique (SCT). In SCT a suitable additional or corrected signal is generated and incorporated along with a reference input to the system through the feedback loop. It is pointed out that in some applications, such as biological control systems, it may not always be possible to incorporate a controller inside the system. This technique may be very much useful in realizing the transient performance of a system with SCT based deadbeat control where either incorporation of any controller within the system or processing of the input command is not permitted. The deadbeat representation with state space equations is demonstrated with the reference input as step. The SCT technique, which is considered in this work, is applied to nonlinear system for deadbeat realization without restriction of system parameters and experimental data as long as system is stable. In this work, the model of Inverted Pendulum is selected in the aspect of nonlinear control theory, with an emphasis on feedback linearization. Then the SCT based deadbeat control is implemented after ensuring the stability of the system.

Key words: Signal Correction Technique, Deadbeat, Feedback Linearization

INTRODUCTION

Control systems are dynamic systems with an input and at least one output. The input and output of the control system obeys the cause and effect relationships of the physical world. The outputs are the entities of the physical world (like voltage, temperature, pressure, flow rate, position of a gun turret etc) which the control engineer is interested to control to a desirable level using suitable control actions (inputs). The measures of satisfaction are, often quantified by a mathematical expression known as performance index. The dynamics of a control system can be best studied by preparing a mathematical model and the classical differential equation representation of the system dynamics is the oldest mathematical model. A system is broadly classified as linear and nonlinear depending on the nature of differential equation model. If the principle of homogeneity together with the principle of superposition holds good for a certain range of inputs the system is linear in that range of inputs. The systems which are not linear are referred to as nonlinear. There are various other types of classifications of dynamic systems [1] like continuous and discrete, stationary and time varying, deterministic and nondeterministic systems for convenience of their representation and analysis. Of all these systems, the analyses of linear deterministic continuous systems are the simplest.

Unfortunately, most of the real world control systems encountered in everyday life are nonlinear in nature. These nonlinear systems are generally approximated to the linear systems for the sake of simplicity and to utilize the benefits of mathematical advantages of linear system analysis. These techniques of converting nonlinear systems to linear systems are known as linearization process. Some literature [2-6] shows that most of the process control systems are inherently nonlinear. Though, there are some applications such as bang-bang or relay control where nonlinearity is deliberately introduced. Therefore, the nonlinearity may be present either in process or in the controller itself. Nonlinear plants arise naturally in numerous engineering and natural systems, including mechanical and biological systems, aerospace and automotive control, industrial process control, and many others. The theory of nonlinear control is normally concerned with the analysis and design of nonlinear control systems. It is closely related to nonlinear systems theory in general, which provides its basic analytical tools. In practical sense, the computational complexity of the control systems may be very high due to the partial availability of the information about system parameters as well as the issues of robustness and stability are also crucial. Thus, the main challenge of researchers and control system engineers are the design of feedback control systems with the consideration of those practical constraints.

The concept of deadbeat control is very old in control engineering. The behaviour of the output of a control system in the transient state as well as in the steady state following the application of an input or sudden change of set point is a very important performance measure of any system whether linear or nonlinear. In many applications, the overshoot and undershoot or ringing behaviour in the output cannot be tolerated. In such a system one prefers what is known as dead beat response of the output. The deadbeat response has the following desirable characteristics:

i) Zero steady-state error. ii) Minimum rise time. iii) Minimum settling time. iv) Less than 2% overshoot/undershoot. v) Very high control signal output.

Lot of interest was shown in this field and lot of research produced many interesting results in linear and discrete control. In recent times, the deadbeat control is being used in many practical engineering and scientific applications such as Industrial plants, Flight Control System, UPS Inverter, Rocket and Missile, Balancing Robots, etc.

In the past many approaches were taken for realizing deadbeat control. A new approach, called signal correction technique (SCT) for the deadbeat realization is being researched by many workers. In the publications including the recent ones, show that though the signal correction technique has many advantages, not much work of SCT designed deadbeat control been reported so far. These led the authors to be interested in the application of SCT in the deadbeat realization of linear and nonlinear systems. Signal Correction Technique (SCT) is a technique of injecting a suitable additional signal to the system along with the reference input to get deadbeat response. This technique has also been used for removal of system instability and modification of the system nonlinearity. In this work, SCT is used for deadbeat realization of linear systems and nonlinear systems. The required suitable signal may be generated by using states of the system. Figure 1 represents the corrected signal in control system. Various techniques [7-14] have been suggested for realizing deadbeat transient response of linear control systems. In those methods, a deadbeat controller is designed and put in the forward path of the control loop.

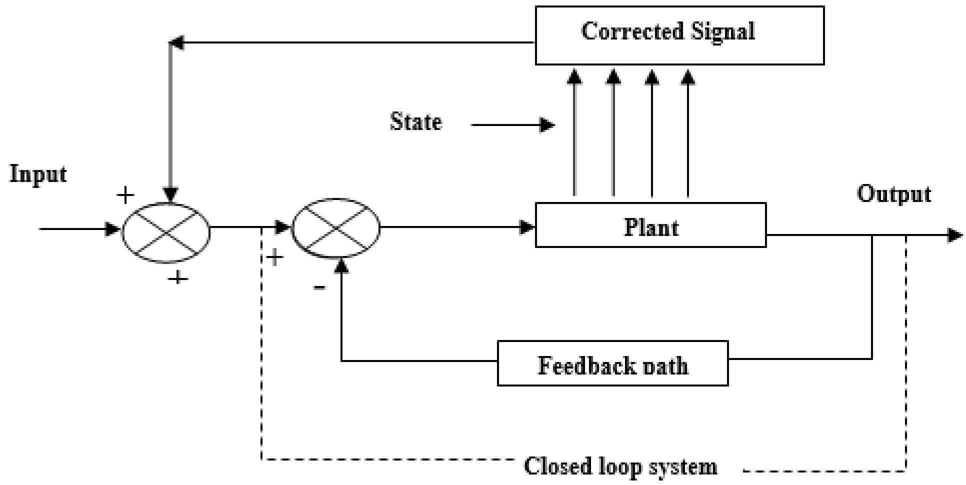

Fig.1 Signal Correction Technique

This work presents the techniques of deadbeat transient performance of higher order linear system based on SCT scheme using pulse signal to the linear continuous system. Some works [15-21] other than SCT had been done for deadbeat control of discrete time linear control system with the representation of state space equations. In some applications, such as biological control systems, it has been observed that there is very low possibility of incorporation of a controller inside the system. Numerous research papers had been published on deadbeat control of discrete time linear control system represented in the state variable form. The parameterization of deadbeat controller [15] by adopting polynomial methods is achieved. It guarantees the deadbeat behaviour of the output of a periodic plant. In [16], deadbeat control is extended to linear multivariable discrete-time generalized state-space systems using algebraic methods. Modification of digital controller algorithms is achieved in [17] using phase variables as state variables. The existence and construction of deadbeat control laws in discrete time using the state-space techniques is described in [18], a general minimum-time state deadbeat controller is presented for a class of simple Hammerstein systems. Dahlin's control algorithm [19] is presented for improved identification and control of discrete time nonlinear system. By Selection of the weight on the states [20], new algorithm is presented to compute output deadbeat controls for linear multivariable systems.

In view of the above discussion the background of this research work is described in the next section. Then a study of formulation of SCT based deadbeat in state space description is exclusively elaborated. The feedback linearization is applied to make the dynamic system of the inverted pendulum, a globally asymptotically stable. SCT based deadbeat realization of linearized model of Inverted Pendulum is implemented for step input. Conclusion and future scope of the work is described at the end of the work.

BACKGROUND AND MOTIVATION

The signal correction technique (SCT) is a method where a suitable signal is generated by an algorithm using the states of the system and added with the command signal to implement the deadbeat response. No controller has to be incorporated in the control loop nor is any signal-shaping required. In references [5, 21] a general formulation for SCT for deadbeat response of linear systems of any order has been suggested, but that algorithm had been implemented only for second order systems with some restrictions on parameter. It is noticed that such a signal would be difficult to find by adopting the conventional continuous data control techniques and has the implementation problem. This motivated the present work.

During the last few decades, there has been extensive research on RF-DB (Ripple free Deadbeat) control systems [7, 9, 11, 22-25] and various schemes have been proposed, aiming at the application of such modern techniques to the control of widely diverse plants. The present work suggests the approach based on the SCT concept that does not require any restrictions on the system parameters. The signal for the SCT is chosen as a representation of state variables within the system. In the present approach, it is decided that the deadbeat response must be obtained by adding a signal to the reference input. This corrected signal can be generated through the proper simulation by using the relation between state space variables within the system and the desired output.

FORMULATION OF SCT BASED DEADBEAT REALIZATION

In this work multiple numbers of deadbeat transient responses $y(t)$ may be considered with different rising time. Let $y(t)$ is the desired output and a polynomial type of function in time domain.

$Y(s)$: *output signal*

$E(s)$: *actuating signal*

$U(s)$: *reference input*

$G(s) = \frac{Y(s)}{E(s)} =$ *forward path transfer function*

$H(s)$: *transfer functionat of the feedback elements. Here* $H(s) = 1$ *as no transfer function in feedback*

$$G(s) = \frac{Y(s)}{E(s)} = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} + \dots\dots\dots + a_2 s^2 + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + .b_{n-3}s^{n-3} + \dots\dots\dots + b_2 s^2 + b_1 s + b_0} \tag{1}$$

Let us consider the open loop transfer function G(s) of a n[th] order linear continuous system in the form:
If the loop is closed around the forward path transfer function G(s) with unity feedback, we have E(s) = U(s)-Y(s).
Therefore, the closed loop transfer function $\frac{\mathrm{Y(s)}}{\mathrm{U(s)}}$ will be given by

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1+G(s)} \tag{2}$$

$$= \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} + \dots\dots\dots + a_1 s + a_0}{s^n + \left(a_{n-1} + b_{n-1}\right)s^{n-1} + \left(a_{n-2} + b_{n-2}\right)s^{n-2} + \dots\dots\dots + \left(a_1 + b_1\right)s + \left(a_0 + b_0\right)} \tag{3}$$

$$\text{Let, } X(s) = \frac{1}{s^n + \left(a_{n-1} + b_{n-1}\right)s^{n-1} + \left(a_{n-2} + b_{n-2}\right)s^{n-2} + \dots\dots\dots + \left(a_1 + b_1\right)s + \left(a_0 + b_0\right)} U(s) \tag{4}$$

$$Y(s) = \frac{a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} + \dots\dots\dots + a_1 s + a_0}{s^n + \left(a_{n-1} + b_{n-1}\right)s^{n-1} + \left(a_{n-2} + b_{n-2}\right)s^{n-2} + \dots\dots\dots + \left(a_1 + b_1\right)s + \left(a_0 + b_0\right)} U(s) \tag{5}$$

So, Y(s) is given by, $Y(s) = \left(a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + a_{n-3}s^{n-3} + \dots\dots\dots + a_1 s + a_0\right)X(s)$ (6)

Hence Y(s) can be written as

$$or\ Y(s) = \left(\sum_{i=0}^{n-1} a_i s^i\right) X(s) \tag{7}$$

From (4) to (7) with choosing the state x = x_1 as the first state and its derivatives as other states it is reduced to a set of n first-order differential equations given below.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ &\ \ . \\ &\ \ . \\ &\ \ . \\ \dot{x}_{n-2} &= x_{n-1} \\ \dot{x}_{n-1} &= x_n \end{aligned} \tag{8}$$

$$\dot{x}_n = -\left(a_0 + b_0\right)x_1 - \left(a_1 + b_1\right)x_2 - \left(a_2 + b_2\right)x_3 - \dots\dots - \left(a_{n-2} + b_{n-2}\right)x_{n-1} - \left(a_{n-1} + b_{n-1}\right)x_n + u(t) \tag{9}$$

$$or\ \dot{x}_n = u(t) - \sum_{i=0}^{n-1}\left(a_i + b_i\right)x_{i+1} \tag{10}$$

From (5), the output response *y(t)* in time domain

$$y(t) = a_0 x_1 + a_1 x_2 + a_2 x_3 + \dots\dots\dots\dots\dots\dots\dots + a_{n-2} x_{n-1} + x_n \tag{11}$$

$$or\ y(t) = \sum_{i=0}^{n-2} a_i x_{i+1} + x_n \tag{12}$$

Thus, considering phase variables as the state variables, the dynamics of the system can be readily written in the general state variable in form of (13) and (14).

$$\dot{X} = AX + BU \tag{13}$$

$$Y = CX + DU \tag{14}$$

where *X* is the state vector, *U* and *Y* are the input and output vectors respectively.

For a single input single output (SISO) system, *u(t)*, and *y(t)* are scalars. Further, if *u(t)* is unit step and *y(t)* is not deadbeat, then have to add another corrected signal f (x_1, x_2, x_3,.....x_n, *t*) through another feedback loop to ensure deadbeat response. Thus, for all $t > t_0$, *y(t)* will not have any overshoot or undershoot and *y(t)* will be equal to *u(t)*, where t_0 is the time when *y(t)* attains the steady state condition for the first time.

For the time, $0 \le t \le t_0$ *y(t)* is strictly increasing.

Thus $y(t)$ will give the deadbeat response if and only if the following two conditions hold:

i. $y(t)$ is strictly increasing for $0 \le t \le t_0$ i.e. $y'(t) > 0\ for\ 0 \le t \le t_0$ (15)

ii. $y(t) = u(t)\ for\ t > t_0$ (16)

Then the state and output equation of the deadbeat system can be written as follows.

As $f(x_1, x_2, x_3, \dots\dots\dots x_n, t)$ is added to the system along with reference input u(t) for the deadbeat, then using (10)

$$\dot{x}_n = -(a_0 + b_0)x_1 - (a_1 + b_1)x_2 - (a_2 + b_2)x_3 - ....-(a_{n-2} + b_{n-2})x_{n-1} - (a_{n-1} + b_{n-1})x_n + u(t)$$
$$+ f(x_1, x_2, x_3, ...x_n, t)$$

$$or\ \dot{x}_n = u(t) + f(x_1, x_2, x_3, ...x_n, t) - \sum_{i=0}^{n-1}(a_i + b_i)x_{i+1} \quad (17)$$

$$or\ \dot{x}_n = u_1(t) - \sum_{i=0}^{n-1}(a_i + b_i)x_{i+1} \quad (18)$$

$$where\ u_1(t) = u_1 = u(t) + f(x_1, x_2, x_3, ...x_n, t)$$

$$\dot{x} = Ax + Bu_1 \quad (19)$$
$$y = Cx + Du_1 \quad (20)$$

where D=null matrix

$$A_{nxn} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \cdots & \cdots & & & & \\ \cdots & \cdots & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ -(a_0 + b_0) & -(a_1 + b_1) & -(a_2 + b_2)\cdots\cdots & -(a_{n-3} + b_{n-3}) & -(a_{n-2} + b_{n-2}) & -(a_{n-1} + b_{n-1}) \end{bmatrix} \quad (21)$$

$$x_{nx1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}, \quad (22)$$

$$B_{nx1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix} \quad (23)$$

$$c_{1xn} = [a_0, a_1, a_2, ..., a_{n-2}, 1] \quad (24)$$

$$u_1 = u_1(t) = u(t) + f(x_1, x_2, .x_3 ....x_n, t) \quad (25)$$

Now the objective is to find $f(x_1, x_2 . \ . \ . \ . , x_n, t)$ so that conditions (15) and (16) are satisfied.

$$y(t) = a_0 x_1 + a_1 x_2 + a_2 x_3 + ...+ a_{n-2} x_{n-1} + x_n \quad (26)$$

Taking the derivative with respect to time, we have

$$\dot{y}(t) = a_0 \dot{x}_1 + a_1 \dot{x}_2 + a_2 \dot{x}_3 + ...+ a_{n-2} \dot{x}_{n-1} + \dot{x}_n \quad (27)$$

Substituting the values from the state space equations (8), (9), (10) and new input $u_1(t)$ in (25), we get the first-order differential equation given below.

(28)

$$\dot{y}(t) = a_0 x_2 + a_1 x_3 + a_2 x_4 + \ldots\ldots + a_{n-2} x_n - (a_0 + b_0)x_1 - (a_1 + b_1)x_2 - (a_2 + b_2)x_3 - (a_3 + b_3)x_4 - \ldots - (a_{n-1} + b_{n-1})x_n + u(t) + f(x_1, x_2, x_3, \ldots\ldots x_n, t) \quad (29)$$

$$\text{or} \quad f(x_1, x_2, x_3, \ldots\ldots x_n, t) = \dot{y}(t) - a_0 x_2 - a_1 x_3 - a_2 x_4 - \ldots\ldots - a_{n-2} x_n + (a_0 + b_0)x_1 + (a_1 + b_1)x_2 + (a_2 + b_2)x_3 + (a_3 + b_3)x_4 + \ldots + (a_{n-1} + b_{n-1})x_n - u(t) \quad (30)$$

DYNAMICS OF INVERTED PENDULUM

In this section the dynamics of inverted pendulum along with feedback linearization is elaborated. The model of Inverted Pendulum is selected in the aspect of nonlinear control theory, with an emphasis on feedback design. As it is seen, feedback is central to control systems, and techniques from differential geometry and dynamic optimization play leading roles. Feedback is used to stabilize and regulate a system in the presence of disturbances and uncertainty, and the main problem of control engineers is to design feedback controllers. In this section, feedback linearization is applied to make the dynamic system of the inverted pendulum, a globally asymptotically stable. The deadbeat realization technique, discussed in earlier section is implemented to the linearized model of inverted pendulum.

MODELLING OF INVERTED PENDULUM

Lot of research works [26-28] had been done over feedback linearization over the dynamic system of inverted pendulum [6]. To illustrate some of the benefits of feedback, the model of an inverted pendulum, or a one-link robot manipulator. In figure 2, the angle θ is measured from the vertical ($\theta = 0$ corresponds to the vertical equilibrium), θ = angular displacement from vertical position, g= gravitational acceleration, l = length of the pendulum and τ = the torque applied by a motor to the revolute joint attaching the pendulum to a frame. The motor is the active component, and the pendulum is controlled by adjusting the motor torque appropriately. Thus the input is u = τ. If the joint angle is measured, then the output is y = θ. If the angular velocity is also measured, then $y = (\theta, \dot{\theta})$. The pendulum has length 1 m, mass m kg, and g is acceleration due to gravity. The target is to restore the pendulum bob vertically with the application of motor torque (τ). Figure 2 represents the inverted pendulum or one-link robot arm. From figure 3 it is clear that there are two force (component of force) acting perpendicularly on the bob of the pendulum

- Torque due to gravity.
- Torque due to generated acceleration in the bob.

Only perpendicular component of force can produce a torque on a body.

Hence, $\tau = I\ddot{\theta} - mg\sin\theta \times l = ml^2\ddot{\theta} - mgl\sin\theta$ (30)

where I is the moment of inertia.

Now if l =1, then (30) becomes $m\ddot{\theta} - mg\sin\theta = \tau$ (31)

The model neglects effects such as friction, motor dynamics, etc.

We begin by analyzing the stability of the equilibrium θ = 0 of the homogeneous system

$$m\ddot{\theta} - mg\sin\theta = 0 \quad (32)$$

corresponding to zero motor torque (no control action). The linearization of (32) at θ = 0 of the homogeneous system is

$$m\ddot{\theta} - mg\theta = 0 \quad (33)$$

Since sin θ ≈ θ for small θ, this equation has general solution

$$\alpha e^{t\sqrt{g}} + \beta e^{-t\sqrt{g}}$$

and, because of the first term (exponential growth), θ = 0 is not a stable equilibrium for (32). This means that if the pendulum is initially set-up in the vertical position, then a small disturbance can cause the pendulum to fall. We would like to design a control system to prevent this from happening, i.e. to restore the pendulum to its vertical position in case a disturbance causes it to fall. One could design a stabilizing feedback controller for the linearized system

$$m\ddot{\theta} - mg\theta = \tau \quad (34)$$

and use it to control the nonlinear system (31). This will result in a locally asymptotically stable system, which may be satisfactory for small deviations θ from 0. For globally asymptotically stable an approach is adopted, which is called the computed torque method in robotics, or feedback linearization. This method has the potential to yield a globally stabilizing controller. The nonlinearity in (31) is not neglected as in the linearization method just mentioned, but rather it is cancelled. This is achieved by applying the following feedback control law.

$$\tau = -mg\sin\theta + \tau' \quad (35)$$

Where τ' is a new torque input, and results in the closed loop system is

$$m\ddot{\theta} = \tau^{'} \tag{36}$$

A linear system described in (36) is a linear system with input $\tau^{'}$. By the idea of feedback linearization of a non-linear system, the new input has to be given in such a way that the system is stabilized, i.e. the pendulum is stabilized.

Indeed, let's set $\tau^{'} = -k_1\dot{\theta} - k_2\theta.$, where k_1 *and* k_2 two arbitrarily taken values.

Thus (36) becomes $m\ddot{\theta} + k_1\dot{\theta} + k_2\theta = 0$ (37)

If the feedback gains are selected as $k_1 = 3m$, $k_2 = 2m$ (for example), then the system (37) has general solution

$\alpha e^{-t} + \beta e^{-2t}$,

So θ = 0 is now globally asymptotically stable. Thus the effect of any disturbance will decay, and the pendulum will be restored to its vertical position. The feedback controller just designed and applied to (31). So the ultimate feedback controller, which is designed is given by the reference input,

$$\tau = -mg\sin\theta + \tau$$
$$or \quad \tau = -mg\sin\theta - 3m\dot{\theta} - 2m\theta. \tag{38}$$

From (38), it is clear that the system becomes an autonomous system after feedback linearization and stabilization. The torque τ is an explicit function of the angle θ and its derivative, the angular velocity θ. The controller is a feedback controller. Because it takes measurements of θ and $\dot{\theta}$ and uses this information to adjust the motor torque in a way which is stabilizing, In Figure 4, for instance, if θ is non-zero, then the last term (proportional term) in (38) has the effect of forcing the motor to act in a direction opposite to the natural tendency to fall. The second term (derivative term) in (38) responds to the speed of the pendulum. It is worth noting that the feedback controller (38) has fundamentally altered the dynamics of the pendulum. With the controller in place, it is no longer an unstable nonlinear system. Indeed, in a different context, it was reported in the article [29] that it is possible to remove the effects of chaos with a suitable controller, even using a simple proportional control method. It is noted that this design procedure requires explicit knowledge of the model parameters (length, mass, etc), and if they are not known exactly, performance may be degraded. Similarly, unmodelled influences (such as friction, motor dynamics) also impose significant practical limitations. Ideally, one would like a design which is robust and tolerates these negative effects. The objective of this section is to make the deadbeat realization after the feedback linearization and stabilization. In this context the derivation of transfer function is essential to get the linear time invariant transient response of the system.

Transfer Function after Linearization

By feedback linearization, it is obvious that the dotted part has transfer function

$$T_1(s) = \frac{1}{ms^2} \quad as\ \tau^{'} = m\ddot{\theta} \tag{39}$$

and transfer function of the rest part is $T_2(s) = \frac{\tau^{'}}{\theta} = -(3ms + 2m)$ (40)

If it is assumed that that some disturbance force acting on the pendulum, then the linear transfer function with stability consideration

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{1}{ms^2}}{1 + \frac{1}{ms^2}m(3s+2)} = \frac{1}{ms^2 + 3ms + 2m} \tag{41}$$

$$so\ the\ characteristic\ equation\ ms^2 + 3ms + 2m = 0 \tag{42}$$

Figure 4 represents Feedback stabilization of the pendulum using feedback linearization with stability consideration. The transient response of (41) is checked with the block diagram given in figure 5. The objective is to make the deadbeat realization of this transient response using SCT. In the earlier section, an additional signal has been used to compensate the output as a deadbeat form. If m=1/30 gm =0.0333 gm is considered with the unit step input, then using (41) as a transfer function (after feedback linearization) with closed loop system the transient response of the whole system must be linear. The linearized transfer function of the dynamic system of inverted pendulum is

$$T(s) = \frac{30}{s^2 + 3s + 2} \tag{43}$$

The closed loop transfer function of the dynamics of inverted pendulum is

$$\theta(s) = \frac{T(s)}{1+T(s)H(s)} = \frac{30}{s^2+3s+32} \tag{44}$$

Let us assume step input as disturbance force acting on inverted pendulum. The transient response of system (44) is given in figure 6.

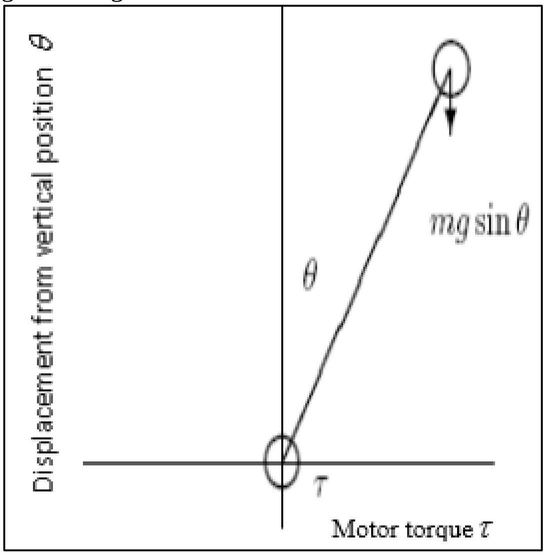

Fig.2 Inverted pendulum or 1-link robot arm

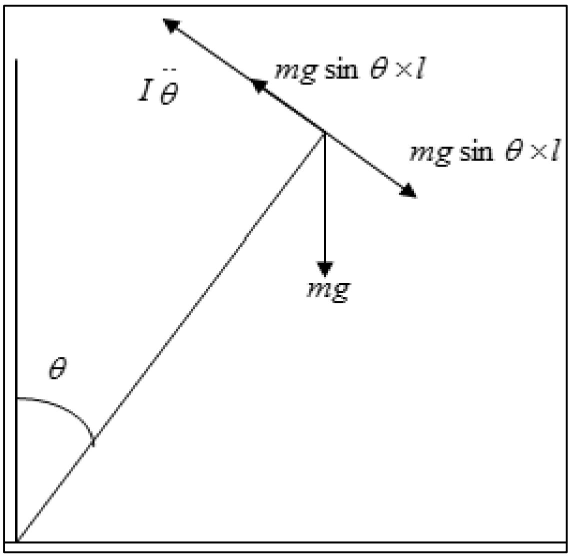

Fig.3 Torque calculation of inverted pendulum

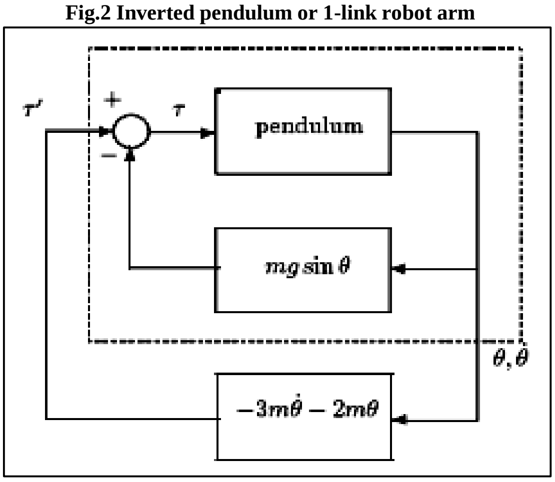

Fig.4 Feedback stabilization of pendulum

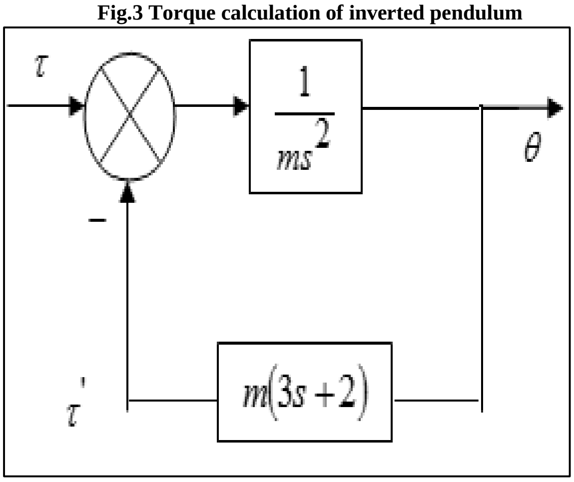

Fig.5 Transfer function with stability consideration

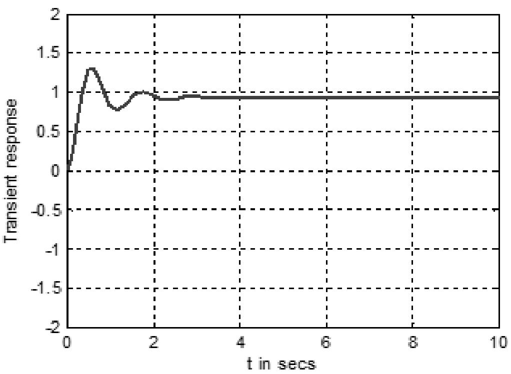

Fig.6 Transient Response of Linearized Model Inverted Pendulum

IMPLEMENTATION OF DEADBEAT REALIZATION

If *u(t)* is the unit step and *y(t)* is not deadbeat, a corrected signal $f(x_1, x_2, t)$ will be added through a feedback loop making *y(t)* a deadbeat response i.e. for all $t>t_0$, y(t) will not have any overshoot or undershoot and y(t) will be equal to u(t), where t_0 is the time when y(t) attains the steady state condition for the first time. For the time $0 \le t \le t_0$, *y (t)* is strictly increasing function of *t*.

From (15) and (16) $y(t)$ will give the deadbeat response if and only if the following two conditions hold.

i $y(t)$ is strictly increasing for $0 \le t \le t_0$ i.e. $y'(t) > 0 \ for \ 0 \le t \le t_0$ (45)

ii $y(t) = u(t) \ for \ t > t_0$ (46)

From (8), the state and output equation of the deadbeat system are given by

Now the objective is to find $f(x_1, x_2, t)$ so that (45) and (46) are satisfied.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(a_0 + b_0) & -(a_1 + b_1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ u(t) + \mathrm{f}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{t}) \end{bmatrix} \tag{47}$$

$$y(t) = a_0 x_1 + x_2 \tag{48}$$

$$or \quad \dot{y}(t) = a_0 \dot{x}_1 + \dot{x}_2 \tag{49}$$

$$from \ (28) \quad \dot{y}(t) = a_0 x_2 - (a_0 + b_0) x_1 - (a_1 + b_1) x_2 + u(t) + f(x_1, x_2, t) \tag{50}$$

$$or \quad f(x_1, x_2, t) = \dot{y}(t) + (a_0 + b_0) x_1 + (a_1 + b_1 - a_0) x_2 - u(t) \tag{51}$$

y(t) is the deadbeat transient response after the additional or corrected signal *f* (x_1, x_2, t) into the system and this *y(t)* must hold the above conditions (45) and (46). In this technique there is no restriction of rise time of the system output for steady state condition. Hence some arbitrary parabolic equation or linear equation *y(t),* which is strictly increasing for $t<t_0$ may be considered as deadbeat response from start time to rise time t_0 (time to reach the steady state condition for the first time) of the controller.

Thus $y'(t) > 0 \ for \ 0 \le t \le t_0$

Figure 6 represents the transient response of closed loop system of (44).

Case I: For the deadbeat realization of the system given in (44), let us consider $y(t)= Y_1(t)=1.5t - 0.5t^2$, selected arbitrarily. Since unit step input is given, the output at steady state condition should also be unity. The polynomial $Y_1(t)$ intersects the line *y(t)*=1 (as unit step input) at the point *t=1.0*. So the rise time of output transient response is t_0=1.0

$\dot{y}(t) = 1.5 - t > 0 \ for \ 0 \le t \le 1.0$

Hence *y(t)* is strictly increasing in $0 \le t \le t_0$

$$y(t) = Y_1(t) = 1.5t - 0.5t^2, \ for \ 0 \le t \le t_0, \ where \ t_0 = 1.0 \tag{52}$$

$$y(t) = u(t) = 1, \ for \ t > 1.0 \tag{53}$$

y(t) reaches the steady state condition of the system at time t_0=1.0. The additional or corrected signal $f(x_1, x_2, t)$ and deadbeat response $y(t)$ are depicted in figure 8and figure 9 respectively.

Now $\dot{y}(t) = \dot{Y}_1(t) = 1.5 - t$ (54)

In figure 7, using (54), 1.5 is added as gain value in the simulation block of G5 and t is given as a ramp input. In the simulation diagram, step time is given as 1.0 (since t_0=1.0) in the source block, step 2 of figure 7. Initial value of step input in step 1 block of figure 7 is zero and final value of step input is 1. These are adjusted in the MATLAB simulink to ensure the additional or corrected signal $f(x_1, x_2, t)$ in the feedback loop holding the above two conditions i.e. (52) and (53). Using (50) and (51)

$a_0 = 30$, $a_1 = 0$, $b_0 = 2$, $b_1 = 3$, $(a_0 + b_0) = 32$ and $(a_1 + b_1 - a_0) = -27$

The above values) are imposed in the simulation block of G1, G3, G2 and G4 respectively in the figure 7.

Case II: If we choose $y(t) = Y_2(t) = 1.7679t - 0.5t^2$,
the polynomial $Y_2(t)$ intersects the line *y(t)*=1 (as unit step input) at the point $t_0=0.707$ So the rise time of output transient response is t_0=0.707. So the rise time of the deadbeat system will improve. In this case rise t_0 is 0.707, i.e. at t_0 = 0.707 the output of the system *y(t)* reaches to steady sate condition.

$$y(t) = Y_2(t) = 1.7679t - 0.5t^2, \ for\ 0 \le t \le t_0, \ where\ t_0 = 0.707 \quad (55)$$

$$y(t) = u(t) = 1, \ for\ t > 0.707 \quad (56)$$

*y(t)*reaches the steady state condition of the deadbeat system at time t_0=0.707. In this case even the rise time of deadbeat system improves but the additional or corrected input signal given to the system is more oscillatory in nature compare to case I with the high amplitude. The additional or corrected signal $f(x_1, x_2, t)$to the feedback loop with $y(t) = 1.7679t - 0.5t^2$ and the deadbeat response of linearized inverted pendulum system are given in figure 10 and figure 11 respectively.

Case III: If we choose $y(t) = Y_3(t) = 4.0833t - 2.5t^2$, (57)
the polynomial $Y_2(t)$ intersects the line *y(t)*=1 (as unit step input) at the point $t_0=0.30$ So the rise time of output transient response is $t_0=0.30$. So the rise time of the deadbeat system will improve further. In this case rise t_0 is *0.30* i.e. at t_0 = *0.30*the output of the system *y(t)* reaches to steady state condition.

$$y(t) = Y_3(t) = 4.0833t - 2.5t^2, \ for\ 0 \le t \le t_0, \ where\ t_0 = 0.30 \quad (58)$$

$$y(t) = u(t) = 1, \ for\ t > 0.30 \quad (59)$$

Y (t) reaches the steady state condition of the deadbeat system at time t_0=0.30. In this case even the rise time of deadbeat system improves further but the additional or corrected input signal given to the system is more oscillatory in nature compare to case I and case II with the high amplitude. The additional or corrected signal $f(x_1, x_2, t)$to the feedback loop with $y(t) = 4.0833t - 2.5t^2$ and the deadbeat response of linearized inverted pendulum system are given in figure 12 and figure 13 respectively. The polynomial curve $Y_1(t)$, $Y_2(t)$, *and* $Y_3(t)$ considered for inverted pendulum system are presented in figure 14, figure 15, and figure 16 respectively. Table 1 represents the selected curve and rise time of deadbeat controller with step input in the above three cases for the linearized model of Inverted Pendulum.

Table -1 Selected Polynomial Curve and Rise Time of Deadbeat Controller

System	Reference Input Pattern	Selected curve for Dead beat Response *y(t)*	Rise time t_0
		$Y_1(t) = 1.5.t\text{-}0.5t.^2$	1
Dynamics of Inverted Pendulum	Step	$Y_2(t) = 1.7679t - 0.5t.^2$	0.707
		$Y_3(t) = 4.0833t - 2.5t.^2$	0.3

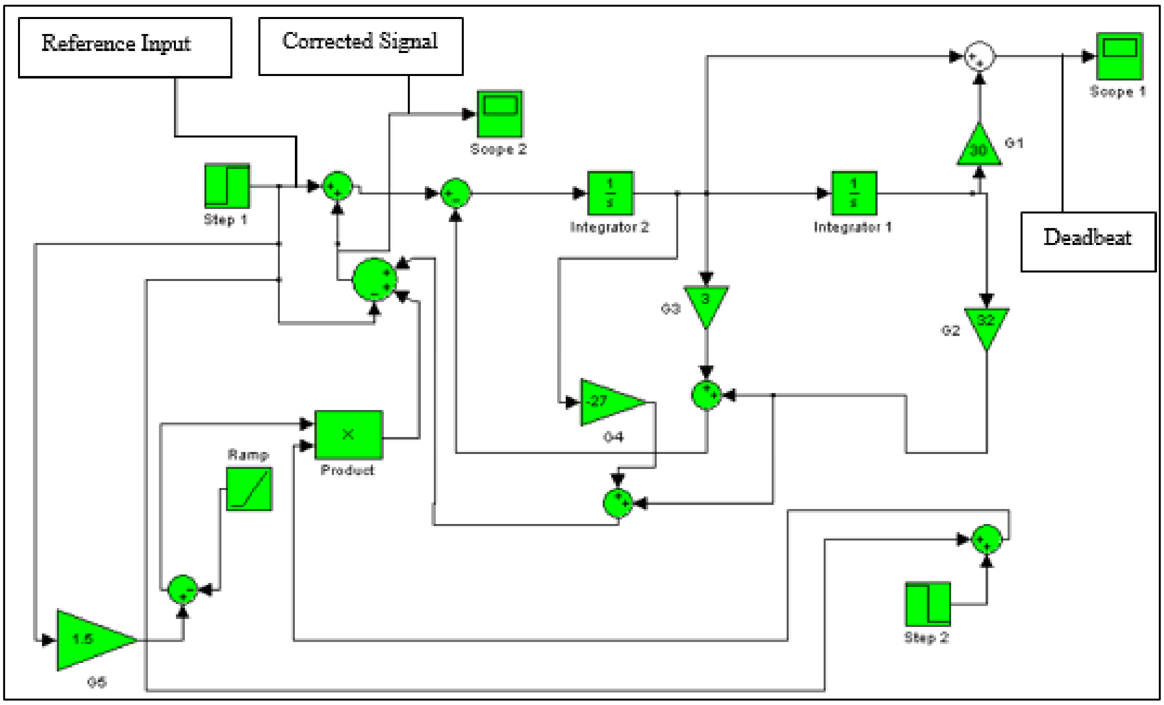

Fig.7 The simulation diagram of Deadbeat realization in case I

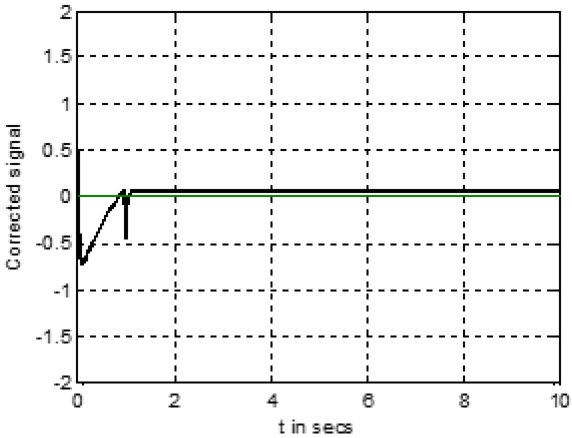

Fig.8 The corrected signal in case I

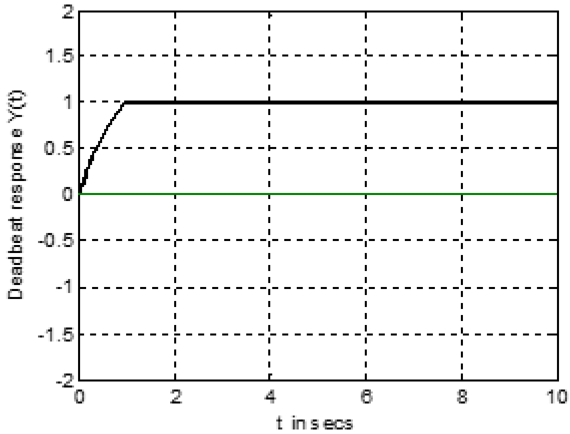

Fig.9 Deadbeat response in case I

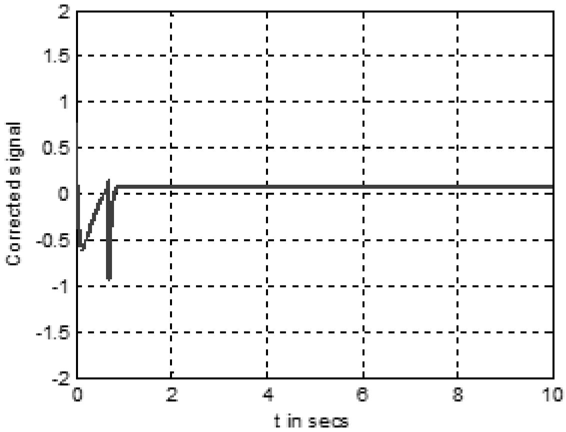

Fig.10 Corrected signal in case II

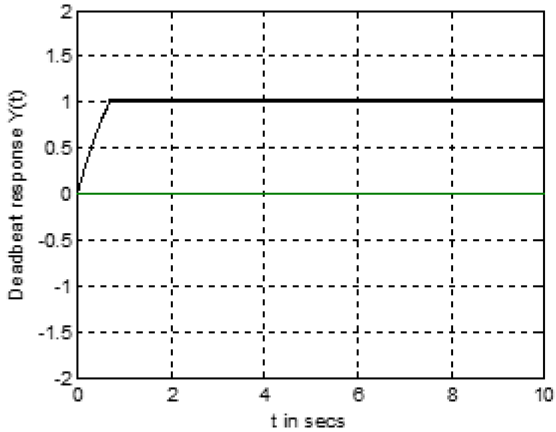

Fig.11 Deadbeat response in case II

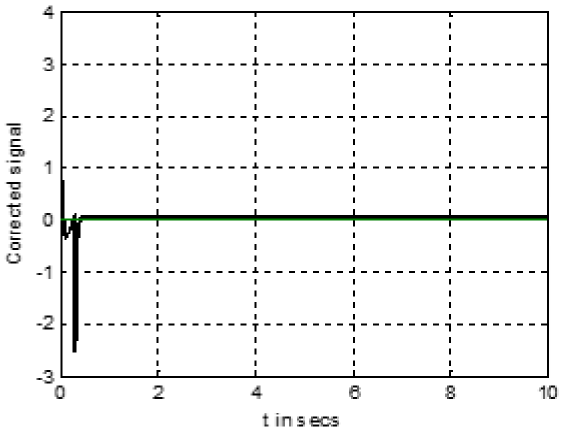

Fig.12 Corrected signal in case III

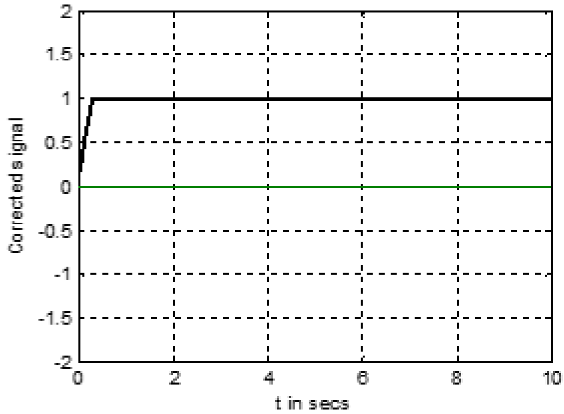

Fig.13 Deadbeat response in case III

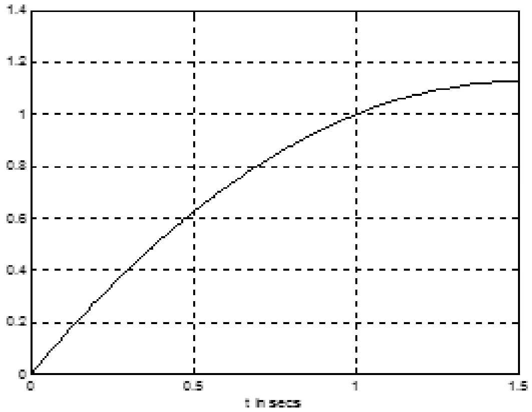

Fig.14 polynomial curve Y1(t)

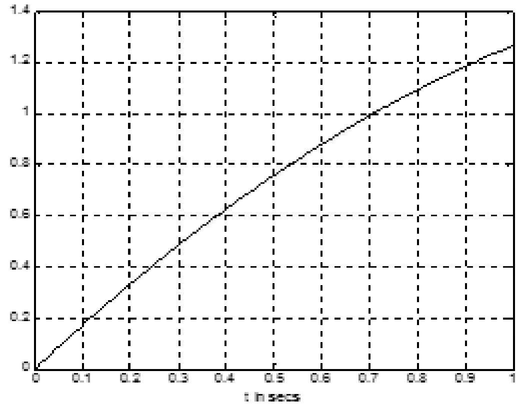

Fig.15 Polynomial curve Y2(t)

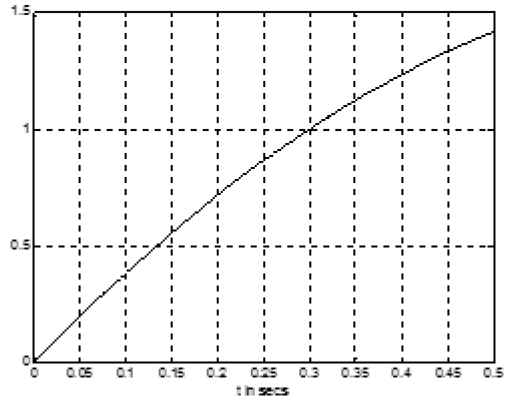

Fig.16 Polynomial curve Y3(t)

CONCLUSION AND FUTURE SCOPE

The work reported in this paper is mainly concerned with the realization of dead beat transient response of linear and nonlinear system using SCT scheme. It starts with the introduction of the concept of deadbeat response for linear, non-linear systems. The review of past research works along with the objective and scope of the present work has been presented in this paper. Here the dead-beat control has been achieved by signal correction technique, which does not require any restriction on the system parameters as long as system is stable. A signal is generated through the state space equation and applied to the system through the feedback loop. Feedback is used to stabilize and regulate a system in the presence of disturbances and uncertainty. The main problem of control engineers is to design feedback controllers. Feedback linearization is applied to make the dynamic system of the inverted pendulum, a globally asymptotically stable. The deadbeat realization technique using SCT scheme, discussed in this work, is implemented to the linearized model of inverted pendulum. This technique also can be applied over linearized model of Inverted Pendulum with Cart, where SCT based deadbeat control in state space can be applied in the same way as described in this work. However there is a huge scope also to explore the SCT based deadbeat control scheme to other nonlinear and time varying system, such as Industrial plants, Flight Control System, UPS Inverter, Rocket and Missile, Balancing Robots, Pitch control of an aircraft, etc..As deadbeat realization is applicable only for the linearized system, the nonlinear systems are approximated by linear models using suitable techniques known as feedback linearization and other linearization technique, which can introduce the global stability and robustness of the system.

REFERENCES

[1] AK Mandal, *Introduction to Control Engineering- Modelling, Analysis and Design*, 2nd ed, New Age International, New Delhi, **2012.**
[2] BDO Anderson and JB Moore, *Optimal Control: Linear Quadratic Methods*, 12th ed, Prentice Hall, Englewood Cliffs, New Jersey.**2007.**
[3] T Kailath, AH Sayed and B Hassibi, *Linear Estimation*, 2nd ed, Prentice Hall,The University of California,**2000.**
[4] RA Bergen and RJ Ragazzini, Sampled Data Processing Techniques for Feedback Control Systems, *Transactions of the American Institute of Electrical Engineers*,**1954**, 73(5), 236-247.
[5] A Das, R Bag and NG Nath, A Modification to Realize Dead-Beat Performance of Control Systems Signal Correction Technique, *IEEE Transactions on Instrumentation and Measurement*, **2006**, 55(5), 1546–1550.
[6] K Ogata, *Modern Control Engineering*, 4th ed, Prentice-Hall, Upper Saddle River, New Jersey, **2002**
[7] S Urikura and A Nagata, Ripple-Free Deadbeat Control for Sampled-Data Systems, *IEEE Transaction on Automation Control*, **1987**, 32(6), 474-482.
[8] E Zafiriou and M Morari, Digital Controllers for SISO Systems: A Review and a New Algorithm, *International Journal of Control*, **1985**,42(4), 855-876.
[9] SH Zak and EE Blouin, Ripple-Free Deadbeat Control for Sampled-Data Systems, *IEEE Transactions on Automatic Control*, **1985**, 32(6), 474-482.
[10] CA Barbargires, Study of Discrete-Time Control Systems with Dead-Beat to Polynomial Inputs, Ph.D. Dissertation, Aristotle University of Thessaloniki, **1994.**
[11] CA Barbargires and CA Karybakas, Ripple Free Dead-Beat Control of DC Servo Motors, *2ndIEEE Mediterranean Symposium on New Directions in Control and Automation*, Crete-Greece, **1994**, 469-476.
[12] C Fielding, The Design of Fly-By-Wire Flight Control Systems, Flight Control Systems *Technologist BAE Systems, Aerodynamics (W427D)*, Warton Aerodrome, Preston PR4 1AX, **2000.**
[13] D Nesic and IMY Mareels, Dead-Beat Control of Simple Harmonic Models, *IEEE Transaction on Automatic Control*, **1998**, 43(8), 1184-1188.

[14] M Iwashiro, M Yatsu and H Suzuki, Time Optimal Track-to Track Seek Control by Model Following Deadbeat Control, *IEEE Transactions On Magnetics*, **1999**, 35(7), 904-909.
[15] L Jetto and S Longhi, Parameterization of Periodic Dead-beat Controlers, *European Control Conference (ECC97)*, Brussels, Belgium, **1997**, 3806-3811.
[16] AE Naeini, Deadbeat Control of Linear Multivariable Generalized State-Space Systems, *IEEE Transactions on Automatic Control*, **1992**, 31(5), 648-652.
[17] KB Janiszowski, Control Error Dynamic Modification as an Efficient Tool for Reduction of Effects Introduced by Actuator Constraints, *International Journal of Applied Mathematics* and *Computer Science*, **2009**, 19(2), 271–279.
[18] V Kucera, Deadbeat Control, Pole Placement and LQ Regulation, *Kybernetika*, **1999**, 35(6), 681-692.
[19] N Selvaganesan and S Renganathan, Identification and Dahlin's Control for Nonlinear Discrete Time Output Feedback Systems, *Journal of Electrical Engineering*, **2006**, 57(6), 329–337.
[20] MR Marrari, AENaeini and GF Franklin, Output Deadbeat Control of Discrete-Time Multivariable Systems, *IEEE Transactions on Automatic Control*, **1989**, 34(6), 644-648.
[21] R Bag, A Das and DN Tibarewala, Signal Correction Technique (SCT) to Realize Dead Beat Performance of Control System, *International Journal of Advanced Engineering & Application*, **2011**, 1(1), 47-50.
[22] CA Karybakas and CA Barbargires, Explicit conditions for Ripple-Free Dead Beat Control, *Kybernetika*, **1996**, 32(6), 601–614.
[23] R Paz and H Elaydi, Optimal Ripple-Free Deadbeat Controllers, *International Journal of Control*, **1998**, 71(6), 1087-1104.
[24] SH Zak and EE Blouin, Ripple-Free Deadbeat Control, *IEEE Control System Magazine*, **1993**, 13(4), 51-56.
[25] L JETTO, Deadbeat Controllers with Ripple-Free Requirement for SISO Discrete Systems, *IEE Proceedings D - Control Theory and Applications*,**1990**, 137(5), 323-328.
[26] JK Hedrick and A Girar, Feedback Linearization, *Control of Nonlinear Dynamic Systems: Theory and Applications*, **2005**, 133-160.
[27] E Petlenkov, J Belikov, S Nõmm and M Wyrwas, Dynamic Output Feedback Linearization Based Adaptive Control of Nonlinear MIMO Systems, *American Control Conference*, Westin Seattle Hotel, Seattle, Washington, USA, **2008**, 3446-3451.
[28] KG Libbrecht and VDO Sannibale, The Inverted Pendulum, Freshman Physics Laboratory, *California Institute of Technology, Physics, Mathematics and Astronomy Division*, USA, **2010**, 27-44.
[29] ER Hunt and G Johnson, Keeping Chaos at Bay, *IEEE Spectrum*, **1993**, 30(11), 32-36.