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Finite Element Method Analysis of Hydrodynamic Journal Bearing

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ABSTRACT

The analysis and design of hydrodynamic journal bearings has a great attention to the engineers. Emphasis has been given to design those bearings so as to avoid metal-to-metal contact .To design these elements, few important characteristics, like load-carrying capacity, maximum pressure and their location lubricant flow requirement between mating surfaces and so on are to be predicted accurately. These parameters can be determined if the pressure within the clearance space between contact surfaces is known. To study and find this pressure one has to solve in general the Reynolds's equation that governs the flow of lubricant inside the clearance space. Moreover the accuracy of the results obtained by FEM is mainly depends on the proper finite element modeling that includes proper selection of the type and size of finite elements and solution algorithm. As computing cost and time increases the size (i.e. the number) of elements increases, one has to determine the minimum number of elements those can be used to obtain the accurate solution. Therefore this work is aimed to study the effect of number of finite elements on the computed result of hydrodynamic journal bearing and to determine minimum number of finite elements to be used in the analysis of the journal bearing.

Key words: Hydrodynamic journal bearing, Reynold's equation and FEM modeling.

INTRODUCTION

Hydrodynamic bearing is an important member of rotating machines and it comes under the sliding family of bearings with the simplest construction. A large number of rotating machines use this class of bearing and operate continuously at high speed and heavy loads. A circular shaft, called the journal is made to rotate in a fixed sleeve or bush called bearing. The bearing may form a complete circle (360) or part of it (<360) in which the former is called full journal bearing and the latter one is called partial journal bearing. The bearing and the journal operate with a small clearance of the order of 1/1000 of the journal radius. Fig. 1.1 shows the principle of operation of a typical hydrodynamic journal bearing. When the bearing at rest Fig.1 (a), the radial load W squeezes out the lubricating oil from the clearance space and metal-to-metal contact is established at A. When the journal begins to rotate, it climbs up the bearing wall due to friction up to point B. The minimum fluid film thickness still remains zero at point B as shown in Fig. 1(b). As the journal attains higher speed, it will build up a wedge shaped film at B and more oil is drawn towards the converging wedge which begins to build up pressure with increasing journal speed as shown in Fig 1.2. At particular speed, the pressure developed becomes enough to support the load.

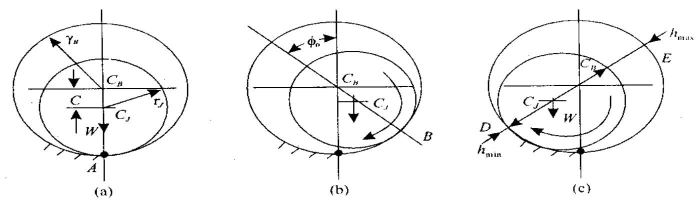

Fig. 1 working principle of hydrodynamic journal bearing

W and the journal is thrown to the other side of the vertical load line as shown in Fig.1(c). The minimum film thickness 'h_{min}' between the bearing and journal surfaces is found to occur at point D. The value of film thickness 'h', the angle of line of centres with the vertical load line, called attitude angle ø and location of maximum fluid film pressure P_{max} are the important considerations in the analysis of hydrodynamic journal bearing.

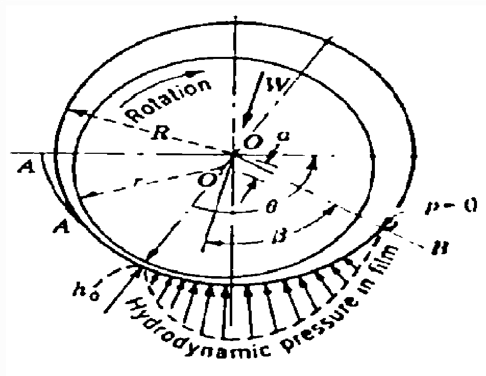

Fig. 2 Pressure profile in hydrodynamic

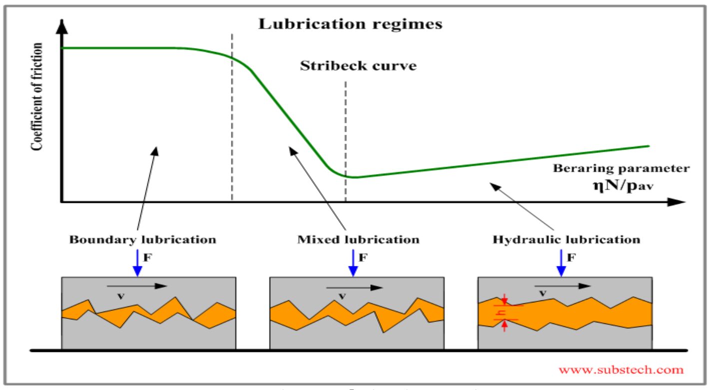

Fig. 3 Lubrication Regimes

As discussed above the hydrodynamic journal bearing develops positive pressure by virtue of relative motion of two mating surfaces separated by a fluid film and hence this class of bearing is called self-acting bearing in which the load is supported by the fluid film pressure developed due wedging action of lubricant caused by the relative motion between two surfaces. Terms used in hydrodynamic journal bearing: Diametric clearance: It is the difference between the diameter of journal or shaft and the inside diameter of bearing in which it rotates. [C=D-d, here, C=Diametric clearance, D=Diameter of bearing and d=Diameter of journal or shaft] Radial clearance: It is one half of the diametric clearances, i.e., it is the difference between the radii of the bearing and the journal. Diametric clearance ratio: It is the ratio of the diametric clearance to the diameter of journal. Eccentricity: It is the radial distance between the centre of the bearing and the displaced centre of the journal under the load (Minimum oil film thickness). It is the ratio of the eccentricity to the radial clearance.

$$\varepsilon = \frac{e}{c} = 1 - \frac{h_{min}}{c} \quad (1)$$

LUBRICATION REGIMES

Sliding friction is significantly reduced by an addition of a lubricant between the rubbing surfaces. Engine bearings are lubricated by Engine oils constantly supplied in sufficient amounts to the bearings surfaces. Lubricated friction is characterized by the presence of a thin film of the pressurized lubricant (**squeeze film**) between the surfaces of the bearing and the journal. The ratio of the squeeze film (oil film) thickness **h** to the surface roughness **Ra** determines the type of the lubrication regime:

Boundary lubrication (h<Ra)

A constant contact between the friction surfaces at high surface points (micro asperities) occurs at boundary lubrication. This regime is the most undesirable since it is characterized by high coefficient of friction (energy loss), increased wear, possibility of seizure between the bearing and journal materials , non-uniform distribution of the bearing load (localized pressure peaks). Very severe engine bearing failures are caused by boundary lubrication. Conditions for boundary lubrication are realized mainly at low speed friction (engine start and shutdown) and high loads. Extreme pressure (EP) additives in the lubricant prevent seizure conditions caused by direct metal-to-metal contact between the parts in the boundary lubrication regime.

Mixed lubrication (h~Ra)

An intermittent contact between the friction surfaces at few high surface points (micro asperities) occurs at mixed lubrication. Mixed lubrication is the intermediate regime between boundary lubrication and hydrodynamic friction.

Hydrodynamic lubrication (h»Ra)

High rotation speed at relatively low bearing loads results in hydrodynamic friction, which is characterized by stable squeeze film (oil film) between the rubbing surfaces. No contact between the surfaces occurs in hydrodynamic lubrication. The squeeze film keeps the surfaces of the bearing and the shaft apart due to the force called hydrodynamic lift generated by the lubricant squeezed through the convergent gap between the eccentric journal and bearing. Bearings working under the conditions of hydrodynamic lubrication are called hydrodynamic journal bearings. The three lubrication regimes are clearly distinguished in the Steinbeck curve (see the Fig. above), which demonstrates the relationship between the coefficient of friction and the bearing parameter η*N/pav (η - dynamic viscosity of the lubricant, N - rotation speed, pav - average bearing pressure).Stability of different lubrication regimes may be explained by means of the Striebeck curve :Temperature increase due to heat generated by friction causes drop of the lubricant viscosity and the bearing parameter. According to the Striebeck curve decrease of the bearing parameter in mixed regime causes increase of the coefficient of friction followed by further temperature rise and consequent increase of the coefficient of friction. Thus mixed lubrication is unstable Increase of the bearing parameter due to temperature rise (lower viscosity) in hydrodynamic regime of lubrication causes the coefficient of friction to drop with consequent decrease of the temperature. The system corrects itself. Thus hydrodynamic lubrication is stable.

HYDRODYNAMIC JOURNAL BEARING ANALYSIS USING FEM

Wear is the major cause of material wastage and loss of mechanical performance of machine elements, and any reduction in wear can result in considerable savings which can be made by improved friction control. Lubrication is an effective means of controlling wear and reducing friction, and it has wide applications in the operation of machine elements such as bearings. The principles of hydrodynamic lubrication were first established by a well-known scientist Osborne Reynolds and he explained the mechanism of hydrodynamic lubrication through the generation of a viscous liquid film between the moving surfaces. The journal bearing design parameter such as load capacity can be determined from Reynolds equation both analytically and numerically.

Reynolds Equation

Lubricant pressure distribution as a function of journal speed, bearing geometry, oil clearance and lubricant viscosity is described by Reynolds equation: Close form solution of Reynolds equation cannot be obtained therefore finite elements method is used to solve it.

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta U\frac{\partial h}{\partial x} \quad (2)$$

Where:

h – local oil film thickness, **η** – dynamic viscosity of oil, **p** – local oil film pressure, **U** – linear velocity of journal, **x** - circumferential direction and **z** - longitudinal direction.

Analytical Solutions of Reynolds Equation Exist only for Certain Assumptions

Somerfield Solution: The equation is solved with the assumption that there is no lubricant flow in the axial direction (**infinitely long bearing assumption**).

$$p=\frac{\eta Ur}{C_r^2}\left(\frac{6\varepsilon(2+\varepsilon\cos\varphi)\sin\varphi}{(2+\varepsilon^2)(1+\varepsilon\cos\varphi)^2}\right)+p_0 \quad (3)$$

Ocvirk Solution: Ocvirk solution for **infinitely short bearing assumption** neglects circumferential pressure gradients (first term of Reynolds equation).

$$p=\frac{\eta U}{rC_r^2}\left(\frac{B^2}{4}-z^2\right)\frac{3\varepsilon\sin\varphi}{(1+\varepsilon\cos\varphi)^3}+p_0$$

or

$$p=\frac{p_{av}}{D^2So}\left(B^2-4z^2\right)\frac{3\varepsilon\sin\varphi}{(1+\varepsilon\cos\varphi)^3}+p_0 \quad (4)$$

Where, Cr - radial clearance **Cr = (D-D_j)/2**, **r** - bearing radius, **D_j** - journal diameter, **e** - absolute bearing eccentricity, **B** - bearing length, **ε** - Eccentricity ratio **ε = e/Cr,** , **Po** - cavitations pressure, **So** - Somerfield number Somerfield and Ocvirk solutions are applicable only in the region of positive pressure Numerical analysis has allowed models of hydrodynamic lubrication to include closer approximations to the characteristics of real bearings.. Numerical solutions to hydrodynamic lubrication problems can now satisfy most engineering requirements for prediction of bearing characteristics. To analyze the bearing design parameters, several approximate numerical methods have evolved over the years such as the finite difference method and the finite element method. The finite difference method is difficult to use when irregular geometries are to be solved. Nowadays finite element method becomes more popular and can overcome these difficulties.

OBJECTIVES OF THE PRESENT WORK

To develop mathematical modelling of the bearing, to study the effect of number of finite elements on the computed results of hydrodynamic journal bearing, to develop a computer code to generate finite element mesh input data for the different discretization, to estimate the error in the bearing performance results obtained from the various numbers of elements and to determine the minimum number of finite elements to be used to get accurate results.

METHODOLOGY

Fluid film bearings have been defined as bearings in which the opposing or mating surfaces are completely separated from each other by a layer of fluid lubricant. Thus it is essential to understand the fundamental process by which the fluid is maintained while supporting the load. This chapter deals with the basic principles of fluid film bearings, viscosity of fluids, the Reynolds equation and the finite element interpretation of the Reynolds equation. The basic principle can be understood by considering high-speed fluid flow between two plane surfaces with a converging wedge and lubricant between the surfaces. The convergence coupled with high-speed fluid flow and fluid viscosity generates a high-pressure fluid film that supports the load. In the case of a plain journal bearing, the converging wedge is formed at the bottom of the bearing due to the weight of the shaft and any other external applied load. The fluid is pulled into the region under the shaft due to the shear forces generated by the shaft rotation. The fluid is forced into the converging film thickness at the bottom of the shaft producing a high-pressure film. This film supports the weight of the rotating machine components and prevents the shaft from touching the bushing surface. For a given eccentricity the fluid film has converging and diverging geometry, such that cavitations may occur in the diverging portion. It is thus very important to be able to predict the pressure distribution and load capacity of the bearing.

Average Fluid-Film Thickness ($\bar{\mathbf{h}}_{\mathbf{T}}$)

The fluid film thickness in non-dimensional form is expressed as

$$\bar{h}=1-\bar{X}_j\cos\alpha - \bar{Z}_j \sin\alpha \tag{5}$$

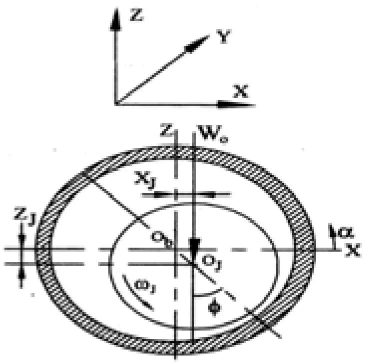

Fig.4 Geometry and Coordinate System

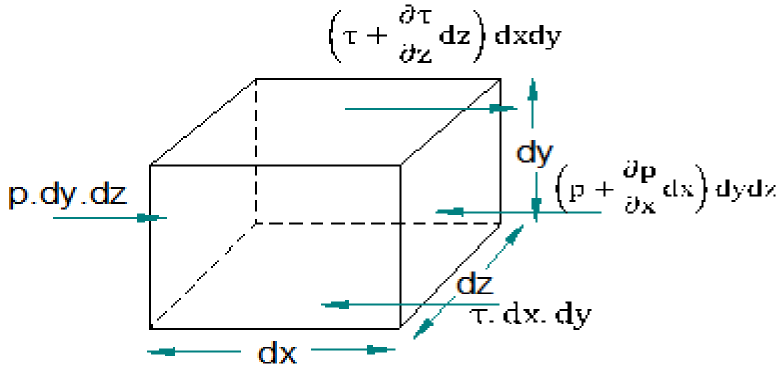

Fig. 5

Reynold's Equation Derivation

Body forces are neglected. This means that there are no inertial or other distributed forces acting on the fluid. The effect of gravity of the fluid is neglected. Pressure is constant through the thickness of film. Curvature of surface is large compared with fluid film thickness. No slip at boundaries. The velocity of the oil layer adjacent to the boundary is the same as that of the boundary. The fluid attached to the bearing surface is stationary while the fluid near the rotor or shaft has the same angular velocity as the shaft itself. For simplifying the analysis, the following assumptions are added. (i) Flow is laminar (ii) Fluid inertia is neglected.

Force balance of the element considered gives

$$\mathrm{p.dy.dz} - \left(\mathrm{p} + \frac{\partial \mathrm{p}}{\partial \mathrm{x}}\mathrm{dx}\right)\mathrm{.dy.dz} + \left(\tau + \frac{\partial \tau}{\partial \mathrm{z}}\mathrm{dz}\right)\mathrm{dx.dy} - \tau\mathrm{.dx.dy} = 0 \tag{6}$$

Therefore $$\frac{\partial \tau}{\partial z} = \frac{\partial p}{\partial x} \tag{7}$$

But $$\tau = \mu \times {}^{\partial u}/_{\partial x} \tag{8}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) \tag{9}$$

Similarly $${}^{\partial p}/_{\partial x} = {}^{\partial}/_{\partial z} \times (\mu \times \partial v/\partial z) \tag{10}$$

So the momentum equation can be written as

$$\frac{\partial}{\partial z}(\mu \times \partial u/\partial z) = \partial p/\partial x \tag{11}$$

$$\frac{\partial}{\partial z}(\mu.\partial v/\partial z) = \partial p/\partial y \tag{12}$$

Integrating $$\mu \times \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} \tag{13}$$

$$\mu.\frac{\partial v}{\partial z} = (\partial p/\partial y)\, z + c3 \quad \text{or} \quad \frac{\partial v}{\partial z} = \frac{1}{\mu}.\frac{\partial p}{\partial y}.z + c3/\mu$$

(14)

Integrating the above equation again

$$u = \int \frac{1}{\mu}\frac{\partial p}{\partial x} z.dz + c1 \int \frac{\partial z}{\mu} + c2 \tag{15}$$

$$v = \int \frac{1}{\mu} \times \frac{\partial p}{\partial y} z dz + c3 \int \frac{\partial z}{\mu} + c4 \tag{16}$$

Boundary Conditions: at z=0 u=u1, v=v1, w=w1

z=h u=u2, v=v2, w=w2 (17)

Substituting in eq 2, we get

c2=u1 and

$$c1 = \left((u2-u1) - \int_0^h \frac{1}{\mu}.\frac{\partial p}{\partial x} z\, dz\right) / \int_0^h \partial z/\mu$$

Let $Fo = \int_0^h dz/\mu$, $F1 = \int_0^h z.dz/\mu$

Therefore, $c1 = (u2 - u1)/F0$-F1/Fo.∂p/ ∂x

Eq 2 becomes

$$U=\partial p/\partial x \int_o^z z.\frac{dz}{\mu} + (\frac{u2-u1}{fo} - \frac{f1}{fo}\partial p/\partial x)\int_0^z \partial z/\mu + u1$$

Using boundary condition in equation 2(b)

c4=v1=0 ;

$$c3=-f1/fo.\frac{\partial p}{\partial y}$$

$$v=\partial p/\partial y(\int_0^z \frac{zdz}{\mu} - f1/fo\int_0^z \partial z/\mu) \quad (18)$$

Finally we have velocity components

$$u=\partial p/\partial x\int_0^z \frac{zdz}{\mu} + (u2-u1)/fo - f1/fo.\partial p/\partial x)\int_0^z \frac{\partial z}{\mu} + u1 \quad (19)$$

$$v=\partial p/\partial y(\int_0^z \frac{zdz}{\mu} - f1/fo\int_0^z \partial z/\mu) \quad (20)$$

Where $f_o=\int_0^h \partial z/\mu 5$

$$f1=\int_0^h z\partial z/\mu) \quad (21)$$

The lubricant flow per unit width in X and Y direction may obtained using equation 19 and 20

Flow in X direction $\varepsilon x=\int_0^h u.dz$ (22)

Flow in Y direction $\varepsilon y = \int_o^h v dz$ (23)

Integrating eq. 22 part wise $\epsilon x = u.z\int_0^h - \int_0^h \frac{\partial u}{\partial z}.z.dz$

From continuity flow we have $\frac{\partial u}{\partial x} = \frac{z}{\mu}.\frac{\partial p}{\partial x} + ((u2-u1)fo - f1/fo\partial p/\partial x)1/\mu$

Substituting the expression of qx

$$q_x=u2h-\int_0^h \frac{\partial p}{\partial x}.\frac{z^2}{\mu}.dz - (\frac{(u2-u1)}{fo} - \frac{f1}{fo}.\frac{\partial p}{\partial x})\int_0^h z\partial z/\mu$$

$$q_x=u2h-\partial p/\partial x(\int_0^h \frac{z^2}{\mu}.dz - f1/fo\int_0^h zdz/\mu) - ((u2-u1)/fo)\int_0^h zdz/\mu \quad (24)$$

Integrating eq. 23 part wise (Axial component of lubricant flow velocity is zero both at runner and journal)

$$q_y=v.z\int_0^h - \int_0^h z.\frac{\partial v}{\partial z}.dz \text{ or } q_y=-\int_0^h \frac{z\partial v}{\partial z}.dz$$

using eq. 2 for substituting $\frac{\partial v}{\partial z}$

$$q_y=-\partial p/\partial y(\int_0^h \frac{z^2}{\mu}.dz + \frac{f1}{fo}.\partial p/\partial y\int_0^h zdz/\mu) \quad (25)$$

Continuity of flow in fluid column: Continuity of flow is considered to derive continuity equation in the flow field.

$$\frac{\partial \varepsilon x}{\partial x} + \frac{\partial \varepsilon y}{\partial y} + \frac{\partial h}{\partial t} = 0 \quad \text{(For constant density)}$$

or $$\partial/\partial x(\int \varepsilon x) + \frac{\partial}{\partial y(\rho\varepsilon y)} + \frac{\partial(h\rho)}{\partial t} = 0 \quad (25a)$$

Using 24, 25 and 25a

$$\partial/\partial x\{u2h - \partial p/\partial x(\int_0^h z\text{^}2/\mu dz - f1/fo\int_0^h zdz/\mu) - (u2-u1)/fo\int_0^h zdz/\mu\} + \partial/\partial y(\partial p/\partial y(\int_0^h z^2/\mu\, dz - f1/fo\int_0^h z/\mu dz)\} + \partial h/\partial t = 0$$

or $\frac{\partial}{\partial x}(u2h) - \frac{\partial}{\partial x}\{\frac{\partial p}{\partial x}\int_0^h \frac{z^2}{\mu}dz - \frac{f1}{fo}\int_0^h \frac{zdz}{\mu})\} - \frac{\partial}{\partial x}\{(u2-u1)f1/fo\} - \frac{\partial}{\partial y}\{\frac{\partial p}{\partial y}(\int_0^h \frac{z^2}{\mu}dz\} - f1/fo\int_0^h \frac{z}{\mu}dz)\} + \frac{\partial h}{\partial t} = 0$

Let $f2=(\int_0^h \frac{z^2}{\mu}dz - \frac{f1}{fo}z\frac{dz}{\mu})$

Assuming variation in u1 and u2 too finally

$$\frac{\partial}{\partial x}(\frac{f2\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{f2\partial p}{\partial y}\right) = \frac{u2\partial}{\partial x}\left(h - \frac{f1}{fo}\right) + \frac{u1\partial}{\partial x}\left(\frac{f1}{fo}\right) + \frac{\partial h}{\partial t} \quad (26)$$

Where $u2=W_jR$ surface speed of shaft u1=0 surface speed of bearing

$$\partial/\partial x\left(f2\frac{\partial p}{\partial x}\right) + \partial/\partial y\,(\frac{f2\partial p}{\partial y}) = \frac{u2\partial}{\partial x}(h - \frac{f1}{fo}) + \partial h/\partial t \quad (27)$$

Where

$$f2= (\int_0^h (z^2/\mu\, dz - \frac{f1}{fo}zdz/\mu$$

(28)

$$f1=\int_0^h \frac{z}{\mu}dz \quad (29)$$

$$f_o=\int_o^h \frac{dz}{\mu} \quad (30)$$

Non dimensionalization (Reynolds Equation)

$$\frac{\partial}{\partial x}\left(F_2 \times \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(F_2 \frac{\partial p}{\partial y}\right) = u_2 \times \frac{\partial}{\partial x}\left(h1 - \frac{F_1}{F_0}\right) + \frac{\partial h}{\partial t}$$

Velocity components: (From 19 & 20)

$$u = \frac{\partial p}{\partial x}\int_0^z \frac{zdz}{\mu} + \left(\frac{u_2 - u_1}{F_0} - \frac{F_1}{F_0}\frac{\partial p}{\partial x}\right)\int_0^z \frac{\partial z}{\mu} + u_1$$

$$v = \frac{\partial p}{\partial y}\int_0^z \frac{zdz}{\mu} - \frac{F_1}{F_0}\int_0^z \frac{\partial z}{\mu}$$

Where $F_0 = \int_0^h \frac{dz}{\mu}$ 31 $(a) F_1 = \int_0^h \frac{zdz}{\mu}$ $31(b) F_{2=} \int_0^h \frac{z^2 dz}{\mu} - \frac{F_1}{F_0}\frac{zdz}{\mu}$ $31(c)$

Non dimensional parameters:

$$x = \frac{x}{R};\ \beta = \frac{y}{R};\ \bar{z} = \frac{z}{h};\ \bar{h} = \frac{h}{c};\ \bar{p} = \frac{p}{p_s};\ \bar{u} = \frac{\mu}{\mu_r};\ \frac{\bar{w}}{\Omega} = \frac{w}{w_r}$$

$$\bar{£} = \frac{t}{\left({}^{\mu}/_{2}\, {}^{R^2}/_{C^2}\, p_s\right)} \bar{\tau} = \frac{\tau}{\left(P_s \frac{C}{R}\right)}\ ;\ F_0 = \left(\frac{ch}{\mu_r}\right)\overline{F_0} - -\overline{F_0} = \int_0^1 \frac{d\bar{z}}{\mu}$$

$$\bar{F_1} = \left(\frac{c^2 h^2}{\mu r}\right)\overline{F_1} - -\overline{F_1} = \int_0^1 \frac{\bar{z} d\bar{z}}{\bar{\mu}}\ ;\ F_2 = \left(\frac{c^3 h^3}{\mu_r}\right)\overline{F_2} - - - \overline{F_2} = \int_0^1 \frac{\bar{z}}{\bar{\mu}}\,(\bar{z} - \frac{\overline{F_1}}{\overline{F_0}})\, d\bar{z}$$

Speed of rotation of Journal is constant, Fluid film thickness is given by $h = c - hx_j \cos \propto - z_j \sin \propto$

$$\frac{\partial \bar{h}}{\partial t} = 0 - \bar{x_j} \cos \propto - \bar{z_j} \sin \propto \quad \bar{h} = \frac{h}{c} \qquad \partial \bar{h} = \frac{1}{c}\partial h$$

We have $\bar{t} = \frac{t}{\frac{\mu_{rR^2}}{c^2 p_s}}$ or $\partial \bar{t} = \frac{\partial t}{\left({}^{\mu_r R^2}/_{C^2 p_s}\right)}$

Substituting for ∂t and ∂h

$$\frac{c\partial \bar{h}}{({}^{\mu_r R^2}/_{c^2 p_s})\,\partial \bar{t}} = \left(\frac{c^3 p}{\mu R_j^2}\right)\frac{\partial \bar{h}}{\partial \bar{t}} = -\delta \bar{x} j cos \propto -\delta \bar{z} j sin \propto \frac{c^3 p_s}{\mu R_j^2} = \frac{L^{3F}/_{L^2}}{\frac{FT}{L^2}L^2} = \frac{L}{T}$$

Therefore, $$\frac{\partial \bar{h}}{\partial \bar{t}} = -\left(\frac{\mu R_j^2}{c^3 p_s}\right)\delta x_j cos \propto - \left(\frac{\mu R_j^2}{c^3 p_s}\right)\delta z j sin \propto \qquad (32)$$

(Substituting non-dimensional parameters in Reynolds Equation 5)

$$\frac{\partial}{R\partial}\ \left(\frac{C^3 \bar{h}^3}{\mu_r}\overline{F_2} P_s \frac{\partial \bar{p}}{R\partial \propto} + \frac{\partial}{R\partial \beta}\left(\frac{c^3 \bar{h}^3}{\mu_r}\bar{F}_2 \frac{p_s \partial \bar{p}}{R\partial \beta}\right)\right.$$

$$= WR \frac{\partial}{R\partial \propto}\left(c\bar{h} - \frac{c^2 \bar{h}^2}{\frac{\mu_r}{\propto \bar{h}}}\frac{\bar{F}_1}{\bar{F}_0}\right) + \frac{c\partial \bar{h}}{\left(\frac{\mu_r R^2}{c^2 p_s}\right)}\frac{1}{\partial t} \qquad = WC\left(\frac{\partial}{\theta \propto}\left(c_1 - \frac{\bar{F}_1}{\bar{F}_0}\bar{h}\right)\right) + \left(\frac{c^3 p_s}{\mu_r R^2}\right)\frac{\partial \bar{h}}{\partial t}$$

or

$$\frac{C^3 P_S}{\mu_R R^2}\left(\frac{\partial}{\partial \propto}\left(\bar{h}^3 \bar{F}_2 \frac{\partial \bar{p}}{\partial \propto}\right) + \frac{\partial}{\partial \beta}\left(\bar{h}^3 \bar{F}_2 \frac{\partial p}{\partial \beta}\right)\right) = WC\left(\frac{\partial}{\partial \propto}\left(C1 - \frac{\bar{F}_1}{\bar{F}_0}\right)\bar{h}\right) + \frac{c^3 p_s}{\mu_r R^2})\ \frac{\partial \bar{h}}{\partial \bar{t}}$$

Let $\left(\frac{C^2 p_s}{\mu_r}\right) = W_I$ (33) and $\bar{\Omega} = \frac{w}{w_i} = \frac{w}{\left(\frac{c^2 p_s}{\mu_r R^2}\right)}$ (34)

FINITE ELEMENT FORMULATION

The Reynolds equation is the non-dimensional form is (2.8)

$$\frac{\partial}{\partial \propto}\left(\bar{h}^3 \bar{F}_2 \frac{\partial p}{\partial \propto}\right) + \frac{\partial}{\partial \beta}\left(\bar{h}^3 \bar{F}_2 \frac{\partial p}{\partial \beta}\right) = \bar{\Omega}\left[\frac{\partial}{\theta \propto}\left\{\left(1 - \frac{\bar{F}_1}{\bar{F}_0}\bar{h}\right\}\right] + \frac{\partial \bar{h}}{\partial \bar{t}} \qquad (35)$$

The expression for fluid film thickness

$$h = c - x_j \cos \propto - z_j \sin \propto \qquad (36)$$

$$\bar{x}_{j=} \frac{x_j}{c}; \bar{z}_{j=} \frac{z_j}{c}; \bar{t} = \frac{t}{({}^{\mu_{rR^2}}/_{c^2 p_{s\_}})\ 2}$$

$$\bar{h} = \frac{h}{c}$$

Therefore $\bar{h}c = c - c\bar{x}_j \cos \propto - c\bar{z}_j \sin \propto$

Hence $\bar{h} = 1 - \bar{x}_j \cos \propto - \bar{z}_j \sin \propto$ (37)

Now $$\frac{dh}{dt} = \frac{cd\bar{h}}{\left(\frac{\mu_r R^2}{C^2 p_S}\right)d\bar{t}} = \left(\frac{c^3 p_S}{\mu_r R^2}\right)\frac{d\bar{h}}{d\bar{t}}$$

From Eqn 35 $$\frac{dh}{dt} = 0 - \frac{dx_j}{dt}\cos \propto - \frac{dz_j}{dt}\sin \propto$$

Using Eqn 36 $\left(\frac{c^3 p_s}{\mu_r R^2}\right)\frac{d\bar{h}}{dt} = -\frac{c}{\left(\frac{\mu_r R^2}{c^2 p_s}\right)}\frac{d\bar{x}_j}{d\bar{t}}\cos\propto - \frac{c}{\left(\frac{\mu_r R^2}{c^2 p_s}\right)}\frac{d\bar{z}_j}{dt}\sin\propto$

Hence $$\frac{d\bar{h}}{d\bar{t}} = -\frac{d\bar{x}_j}{d\bar{t}}\cos\propto - \frac{d\bar{z}_j}{d\bar{t}}\sin\propto \quad (38)$$

or $$\frac{d\bar{h}}{d\bar{t}} = -\bar{x}_j\cos\propto - \bar{z}_j\sin\propto$$

From Equation 22 and 38

$$\frac{\partial}{\partial\propto}\left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}\right) + \frac{\partial}{\partial\beta}\left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}, = \bar{\Omega}\left[\frac{\partial}{\partial\propto}\left\{\left(1-\frac{\bar{F}_1}{F_0}\right)\bar{h}\right\}\right] - \bar{x}_j\cos\propto - \bar{z}_j\sin\propto \quad (39)$$

Two dimensional iso-parametric formulations - Let n= No. of nodes per element, the fluid flow domain is discretized, Using 4-noded quadrilateral iso-parametric element. The pressure is considered to be distributed linearly over an element and expressed as

$$\bar{p} = \sum_{j=1}^{n}\bar{p}_j\,\bar{N}_j\,\frac{\partial\bar{p}}{\partial\propto} = \sum_{j=1}^{n}\frac{\partial N_j}{\partial\propto}\bar{p}j$$

$$\propto = \sum_{j=1}^{n} n_j\,d_j\frac{\partial\bar{p}}{\partial\beta} = \sum_{j=1}^{n}\frac{\partial N_j}{\partial\beta}\bar{p}\,j$$

$$\beta = \sum_{j=1}^{n} N_j B_j$$

Using Galarkin's orthogonality criterion

$$\sum_{e=1}^{ne}\int_{L\bar{A}}\int_e \{\frac{\partial}{\partial\propto}\left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}\right) + \frac{\partial}{\partial\beta}\left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}\right) - \bar{\Omega}\{\frac{\partial}{\partial\propto}\left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{h}\} + \bar{x}_j\cos\propto + \bar{z}_j\sin\propto\} N_i d\propto d\beta \quad (40)$$

Where $n_e =$ total number of element, $\mathrm{i} = 1, 2, -----\mathrm{n}$(no. of nodes per element)

Part wise integration of second order terms i.e. part 1 of equation 40

$$\int_{\bar{A}}\int_e \frac{\partial}{\partial\propto}(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}Nid\propto d\beta = \int\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}Nid\beta - \int_{\bar{A}}\int_e \bar{h}^3\,\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}\frac{\partial NC}{\partial\propto}d\propto d\beta$$

and $$\int_{\bar{A}}\int_e \frac{\partial}{\partial\beta}(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta})\,Nid\propto d\beta = \int\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}Nid\propto - \int_{\bar{A}}\int_e \bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}\frac{\partial Ni}{\partial\beta}d\propto d\beta$$

Substituting in equation 3.5

$$\int_A\int_e \{\frac{\partial}{\partial\propto}(\bar{h}^3\,\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}) + \frac{\partial}{\partial\beta}(\tilde{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta})\}\,Nid\propto d\beta = \int_{\bar{\eta}e}(\bar{h}^3\,\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}l_1 + \bar{h}^3\,\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}l_2)\,Nid\bar{\eta}e - \int_{\bar{A}}\int_e \left\{\bar{h}^3\left(\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}\frac{\partial Ni}{\partial\propto} + \bar{F}_2\frac{\partial\bar{p}}{\partial\beta}\frac{\partial Ni}{\partial\beta}\right)\right\}d\propto d\beta \quad (41)$$

Considering part 2 of equation 40

$$\int_A\int_e \bar{\Omega}\,\{\frac{\partial}{\partial\propto}\left(1-\frac{\bar{F}_0}{\bar{F}_0}\bar{h}\right\}Nid\propto d\beta = \int_\beta \bar{\Omega}\left(1-\frac{\bar{\bar{F}}_1}{\bar{F}_0}\right)\bar{h}Nid\beta - \int_{\bar{A}}\int_e \bar{\Omega}\left(1-\frac{\bar{F}_1}{F_0}\right)\bar{h}\frac{\partial Ni}{\partial\propto}d\propto d\beta$$

$$= \int_{\eta e}\bar{\Omega}\left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{n}Nil_1 d\eta^e - \int_{\bar{A}}\int_e \bar{\Omega}\left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{h}\frac{\partial Ni}{\partial\propto}.d\propto d\beta \quad (42)$$

Substituting from (3.7) and (3.8) into (3.6)

$$\sum_{e=1}^{ne}[\int_{ne}\left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}l_1 + \bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial 1^3}l_2\right)Nid\bar{\eta}^e - \int_{\bar{A}}\int_e \{\bar{h}^3(\bar{F}_2\frac{\partial\bar{p}}{\partial\propto}\frac{\partial WC}{\partial\propto} + \bar{F}_2\frac{\partial\bar{p}}{\partial\beta}\frac{\partial NC}{\partial\beta})\}\,d\propto d\beta - \int_{n^e}\bar{\Omega}\left(1-\frac{\bar{F}_1}{F_0}\right)\bar{h}Nil_1 d\bar{\eta}^e + \int_{\bar{A}}\int_e \bar{\Omega}\left(1-\frac{\bar{F}_1}{F_0}\right)\bar{h}\frac{\partial Ni}{\partial\propto}dd\beta + \int_{\bar{A}}\int_e \bar{x}\cos\propto Nid\propto d\beta + \int_{\bar{A}}\int_e \bar{z}_j\sin\propto Nid\propto d\beta = 0 \quad (43)$$

The equation 43 is written in the matrix form

$$\sum_{e=1}^{n_e}\left[[\overline{F}]^e\{\overline{p}\}^e\right] = \sum_{e=1}^{n_e}\left[\{\overline{Q}\}^e + \bar{\Omega}\,\{\bar{R}_H\}^E + \bar{x}_j\,\{\bar{R}_{x_j}\}^e + \bar{z}_1\left\{\bar{R}_{z_j}\right\}^e\right] \quad (44)$$

Where

$$\bar{f}_{ij}^e = \int_{\bar{A}}\int_e (\bar{h}^3(\bar{F}_2\frac{\partial N_i}{\partial\propto}\frac{\partial N_j}{\partial\propto} + \bar{F}_2\frac{\partial N_i}{\partial\beta}\frac{\partial N_j}{\partial\beta}))\,d\propto d\beta$$

$$\bar{Q}_i^e = \int_{\bar{n}^e}\{(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\propto} - \bar{\Omega} = \left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{h}\}\,l_1 + \left(\bar{h}^3\bar{F}_2\frac{\partial\bar{p}}{\partial\beta}\right)l_2\}\,N_i d\bar{\eta}^e$$

$$\bar{R}_{Hi}^e = \int_{\bar{A}}\int_e \left(1-\frac{\bar{F}_1}{\bar{F}_0}\right)\bar{n}\frac{\partial N_i}{\partial\propto}d\propto d\beta$$

$$\bar{R}_{x_{ji}}^e = \int_{\bar{A}}\int_e \cos\propto N_i d\propto d\beta$$

$$\bar{R}_{z_{ji}}^e = \int_{\bar{A}}\int_e \sin\propto N_i d\propto d\beta$$

Newtonian Lubricant, For Newtonian lubricant assuming $\bar{\mu} = constant$

So $\bar{F}_0 = \frac{1}{\bar{\mu}};\ \bar{F}_1 = \frac{1}{2\bar{\mu}};\ \bar{F}_2 = \frac{1}{12\bar{\mu}}$ ------3.12

For Newtonian lubricant, the matrix element is defined in Equation 3.11 are reduced to as given below

$$\bar{F}_{ij}^{e} = \int_{\bar{A}} \int_{e} \left\{ \frac{\bar{h}^3}{12\bar{\mu}} \left(\frac{\partial N_i}{\partial \propto} \frac{\partial N_j}{\partial \propto} + \frac{\partial N_i}{\partial \beta} \frac{\partial n_j}{\partial \beta} \right) \right\} d \propto d\beta$$

$$\bar{Q}_{i}^{e} = \int_{\bar{\eta}^e} \{ (\frac{\bar{h}^3}{12\bar{\mu}} \frac{\partial \bar{p}}{\partial \propto} - \frac{1}{2} \bar{\Omega}\, \bar{h})\ l_1 + \left(\frac{\bar{h}^3}{12\bar{\mu}} \frac{\partial \bar{p}}{\partial \beta} \right) l_2 \} N_i \phi \bar{p}^e$$

$$\bar{Q}_{yji}^{e} = \int_{\bar{A}} \int_{e} \frac{1}{2} \bar{h} \frac{\partial N_i}{\partial \propto} d \propto d)^3$$

$$\bar{Q}_{xj_i}^{e} = \int_{\bar{A}} \int_{e} \cos \propto N_i d \propto d\beta$$

$$\bar{Q}_{zj_i}^{e} = \int_{\bar{A}} \int_{e} \sin \propto N_i d \propto d\beta$$

Here $\bar{\mu} = 1$ for Newtonian isothermal case

After assembling the contributions of all elements, the global system equation written as

$$[\bar{F}]\,[\bar{P}] = [\bar{Q}] + \bar{\Omega}\{\bar{R}_H\} + \bar{x}_j\,\{\bar{R}_{xji} + \bar{z}_j\{\bar{R}_{zi}\}$$

The global (system) equation (3.14) in the expanded form is

$$\begin{array}{cccccc}
\bar{F}_{11}\ \bar{F}_{12} \ldots \bar{F}_{1j} \ldots \bar{F}_{1n} & \bar{p}_1 & \bar{Q}_1 & \bar{R}_{Hi} & \bar{R}_{xjj} & \bar{R}_{zji} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\bar{F}_{i1}\ \bar{F}_{i2} \ldots \bar{F}_{ij} \ldots \bar{F}_{in} & \bar{p}_i & \bar{Q}_i + \Omega & \bar{R}_{Hi} + \bar{x}_j & \bar{R}_{xji} + z_j & \bar{R}_{zji} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\bar{F}_{j1}\ \bar{F}_{j2} \ldots \bar{F}_{jj} \ldots \bar{F}_{jn} & \bar{p}_j & \bar{Q}_j & \bar{R}_{Hj} & \bar{R}_{xjj} & \bar{R}_{zjj} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\bar{F}_{n1}\ \bar{F}_{n2} \ldots \bar{F}_{nj} \ldots \bar{F}_{nn} & \bar{p}_n & \bar{Q}_n & \bar{R}_{Hn} & \bar{R}_{xjn} & \bar{R}_{zjn}
\end{array}$$

RESULTS AND DISCUSSIONS

The performance characteristics of plain hydrodynamic journal bearings for different mesh values have been computed using mathematical models presented in the previous chapter. Also the performance characteristics have been computed for comparison of plain hydrodynamic journal bearing with two axial groove and four axial groove journal bearings. These performance characteristics include load carrying capacity, maximum pressure, stiffness coefficients and fluid film damping coefficients for different eccentricity ratios.

Load Carrying Capacity ($\bar{F}_0$)

Fig. 6 and 7 shows the comparison of load carrying capacity of two axial groove and four axial groove hydrodynamic journal bearing with load carrying capacity of plain hydrodynamic journal bearing at different eccentricity ratios. From the Fig. 6 it can be observed that the load carrying capacity of two axial groove hydrodynamic journal bearing is higher than that of plain and four axial groove journal bearings for eccentricity ratios higher than 0.3. Force centricity ratios lower than 0.3 the load capacity of two axial groove bearing is lower than that of plain hydrodynamic journal bearing. The four axial groove hydrodynamic journal bearing has a lower load carrying capacity when compared to plain bearings at all eccentricity ratios as evident from the Fig. 6 and 7. From Fig. 7 it can be observed that the load carrying capacity at eccentricity ratio 0.2 in case of two axial groove journal bearing and four axial groove bearing is 82.55% and 63.19% less when compared to that of plain hydrodynamic journal bearing. As the eccentricity ratio is increased to 0.4 the load capacity of two axial groove bearing increases to 26.64% higher than that of plain hydrodynamic journal bearing and in case of four axial groove bearings the load carrying capacity is 36.22% less than that of plain hydrodynamic journal bearing.

As eccentricity ratio is increased to 0.6 the load carrying capacity of two axial groove bearing is 15.64% higher than that of plain bearing. From eccentricity ratios higher than 0.6 a continuous dip in percentage rise in load carrying capacity of two axial groove bearings with plain journal bearings can be observed. In case of four axial grooves bearings the difference in percentage with plain bearing decreases up to eccentricity ratio of 0.4 and increases further above that eccentricity ratio.

Fig. 8 and 9 shows the comparison of maximum pressure of two axial grooves and four axial groove hydrodynamic journal bearing with maximum pressure of plain hydrodynamic journal bearing at different eccentricity ratios. From the Fig. 8 it can be observed that the maximum pressure of the two axial groove bearing, four axial groove bearing and the plain bearing is approximately equal at eccentricity ratio of 0.2 At higher eccentricity ratio the maximum pressure in case of four groove bearing is found to be lesser when compared to the plain and two groove hydrodynamic journal bearing. It is found that the two groove bearing has higher maximum pressure when compared to plain bearing up to eccentricity ratio of about 0.7, above that the plain hydrodynamic journal bearing has a higher maximum pressure. From Fig. 9 it can be found that at eccentricity ratio of 0.2 the two groove and four groove bearings have about the same maximum pressure as that of plain bearing. But as the eccentricity ratio increases to about 0.4 there is a rise in percentage maximum pressure of 26.64% in case of two groove bearing, and a rise of about 8.8% in maximum pressure in case of four groove bearing. At higher eccentricity ratios it can be found that four groove bearing, have percentage decrease in pressure when compared to plain bearing and two groove bearings have higher maximum pressure when compared to plain bearing at higher eccentricity ratios.

Fig. 10 and 11 shows the comparison of load carrying capacity with eccentricity for different meshing. As seen in the Fig. 10, by increasing the mesh values i.e. for finer mesh the value of load carrying capacity is found to increase. The variation in the load carrying capacity at each eccentricity ratio with respect to Mesh A is found to increase at a greater extent. It is found that by discretizing further in the circumferential direction the load carrying capacity increases. Mesh D and Mesh E load carrying capacity values have little variation. Hence, the minimum number of elements required in this case can be equal to Mesh D. Maximum pressure (P_{max}).

Table -1 Finite Element Meshes used in Patch Test

S No	Meshes	Number of Elements in α and β directions	Total Number of Nodes
1	Mesh A	12×4	60
2	Mesh B	12×8	108
3	Mesh C	12×6	204
4	Mesh D	24×4	120
5	Mesh E	24×8	216
6	Mesh F	24×16	408

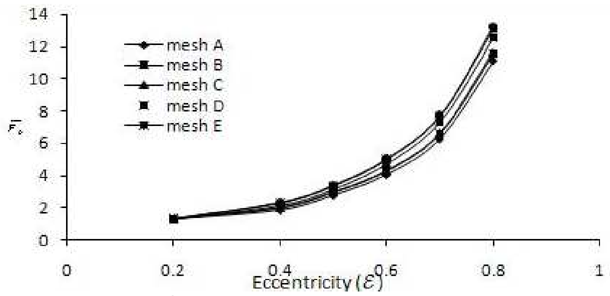

Fig.6 Variation of load carrying capacity with eccentricity ratio computed using different Meshes

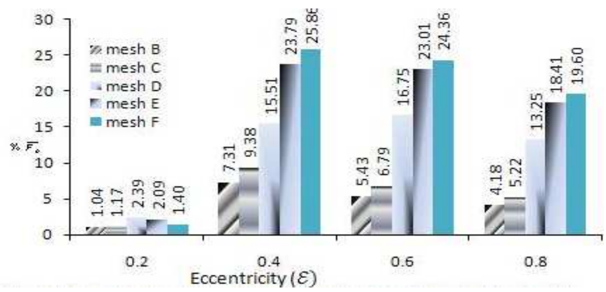

Fig.7 Percentage Change in load carrying capacity computed using different Meshes

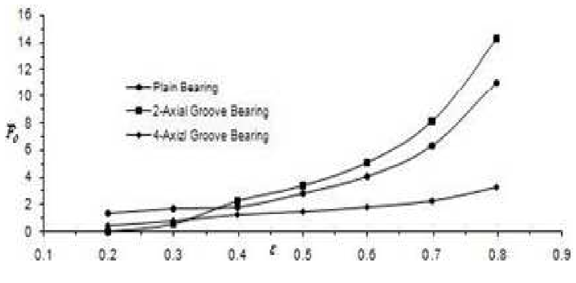

Fig.8 Load carrying capacity of plain and grooved bearings

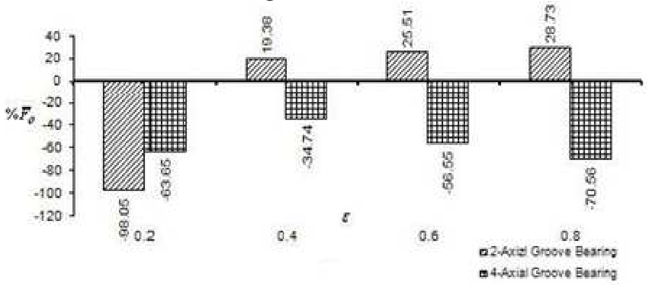

Fig. 9 Percentage change in load carrying capacity of grooved bearings with respect to plain bearings

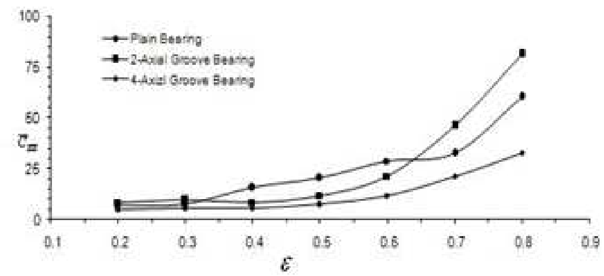

Fig. 10 Direct dumping coefficient of plain and grooved bearings

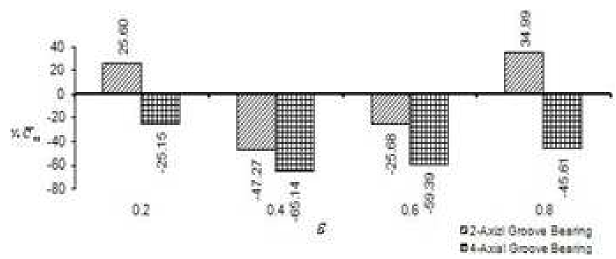

Fig. 11 Percentage change in direct dumping coefficient of grooved with respect to plain bearings

CONCLUSION

At lower eccentricity ratios, the solution of hydrodynamic journal bearing is not significantly affected from the finite element mesh size while it is significantly affected from the finite element mesh size at higher eccentricity ratios. Finer finite element mesh is to be used to analyse the bearing that operates with high eccentricity ratios. As compared to plain hydrodynamic journal bearing, two-axial groove hydrodynamic journal bearing system provides higher load carrying capacity as well as stability threshold speed at higher eccentricity ratios.

REFERENCES

[1] Saad MF, Comparative Investigation of the Performance of a Three-Lobed Journal Bearing, *Trib Int*, **1979**, 12,5, p. 214-219.

[2] Li DF, Choy KC and Allaire PE, Stability and Transient Characteristics of Four Multi lobe Journal Bearing Configurations, *Jour of Lub Tech*, **1980**, 102, p. 291-299.

[3] Radford A and Fitzgeorge D, Effects of Journal Lobingon the Performance of a Hydro dynamic Plain Journal Bearing, *Wear*, **1977**, 45,3, p. 311-322.

[4] Singh DV, Sinhasan R and Tayal SP , Analysis of Three-Lobe Bearings Having Non-Newtonian Lubricants by a Finite Element Method, *Trans Of the CSME*, **1983**,7,1,p.7-11.

[5] Soni S C, Non-Linear Analysis of Two-Lobe Bearings in Turbulent Flow Regims, *Wear*, **1985**, 103, p.11-27.

[6] Sinhasan R and Goyal KC, Transient Response of at Wolobe Journal Bearing Lubricated with Non- Newtonian Lubricant, *Trib Int*, **1995**, 28, 4, p. 233-239.

[7] Ghosh MK and Satish MR, Rotordynamic Characteristics of Multilobe Hybrid Bearings with Short Sills-Part I, *Trib Int*, **2003**,36,8,p.625-632.

[8] Ghosh MK and Satish MR, Stability of Multilobe Hybrid Bearing with Short Sills- Part II, *Trib Int*, **2003**, 36, 8, p.633-363.

[9] Ghosh MK and Nagraj A, Rotordynamic Characteristics of A Multi lobe Hybrid Journal Bearing In Turbulent Lubrication, *Proc of the Inst of Mech Engineers, Part J: Jour of Engg Trib*, **2004**, 218,1, p.61-67.

[10] Knight JD and Barrett LE, An Approximate Solution Technique for Multilobe Journal Bearings Including Thermal Effects with Comparison Experiment, *ASLE Trans*, **1983**, vol26, 4, p.501-508.

[11] Muller Karger CM, Barret LE and Flack RD, Influence of Fluid Film Nonlinearity on the Experimental Determination of Dynamic stiffness and damping Coefficients for Three-Lobe Journal bearings, *STLETribo Trans*, **1997,** vol40,1,p.49-56.