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PROCESS CAPABILITY ESTIMATION FOR NON-NORMALLY DISTRIBUTED DATA USING ROBUST METHODS - A COMPARATIVE STUDY

Abstract: *Process capability indices are very important process quality assessment tools in automotive industries. The common process capability indices (PCIs) Cp, Cpk, Cpm are widely used in practice. The use of these PCIs based on the assumption that process is in control and its output is normally distributed. In practice, normality is not always fulfilled. Indices developed based on normality assumption are very sensitive to non- normal processes. When distribution of a product quality characteristic is non-normal, Cp and Cpk indices calculated using conventional methods often lead to erroneous interpretation of process capability. In the literature, various methods have been proposed for surrogate process capability indices under non normality but few literature sources offer their comprehensive evaluation and comparison of their ability to capture true capability in non-normal situation. In this paper, five methods have been reviewed and capability evaluation is carried out for the data pertaining to resistivity of silicon wafer. The final results revealed that the Burr based percentile method is better than Clements method. Modelling of non-normal data and Box-Cox transformation method using statistical software (Minitab 14) provides reasonably good result as they are very promising methods for non –normal and moderately skewed data (Skewness ≤ 1.5).*

Keywords: *Process capability indices, Non - normal process, Clements method, Box - Cox transformation, Burr distribution, probability plots*

1. Introduction

Process mean μ, Process standard deviation σ and product specifications are basic information used to evaluate process capability indices however, product specifications are different in different products (Pearn *et al.*, 1995). A frontline manager of a process cannot evaluate process performance using μ and σ only. For this reason Dr. Juran combined process parameters with product specifications and introduces the concept of process capability indices (PCI).Since then ,the most common indices being applied by manufacturing industry are process capability index Cp and

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process ratio for off - centre process Cpk are defined as

$$C_p = \frac{(USL-LSL)}{6\sigma'} \quad (1)$$

$$Cpk = \text{Min}\left[\frac{USL-\bar{\bar{X}}}{3\sigma'}, \frac{\bar{\bar{X}}-LSL}{3\sigma'}\right] \quad (2)$$

Capability indices are widely used to determine whether a process is capable of producing items within customer specification limits or not. The process capability indices Cp and Cpk heavily depend on an implicit assumption that the underlying quality characteristic measurements are independent and normally distributed. However, these basic assumptions are not fulfilled in actual practice as many physical processes produce non- normal data and quality practitioners need to verify that the assumptions hold before deploying any PCI techniques to determine the capability of their processes. Some authors have provided useful and insightful information regarding the mistakes in interpretation that occur with the misapplication of indices to non-normal data (Choi and Bai, 1996; Montgomery, 1996; Box and Cox, 1964). Alternatively, other authors have introduced new indices to handle the skewness in the data (Boyels, 1994) Tang and Than (1999) reported on a comparative analysis among seven indices designed for non-normal distribution.

2. Surrogate PCIs for Non-Normal Distributions

Here the following methods have been presented to compute PCIs for non-normal distribution.

- Weighted variance Method
- Clements Method
- Burr Method
- Box-Cox Transformation Method
- Modelling non-normal data using Statistical software

2.1. Weighted variance method

Hsin-Hung Wu Proposed a new process capability index applying the weighted variance control charting method for non–normal processes to improve the measurement of process performance when the process data are non-normally distributed and shows that the two weighted variance method are based on the same philosophy to split a skewed or asymmetric distribution from the mean. The main idea of the weighted variance method is to divide a skewed distribution into two normal distribution from its mean to create two new distributions which have the same mean but different standard deviations. For a population with a mean of μ and a standard deviation of σ, there are $n_1$observations out of a total observations which are less than or equal to μ. Also, there are n_2 observations out of n total observations which are greater than μ.the two new distributions can be established by using n_1 and n_2 observations respectively. That is, the two new distributions will have the same mean μ, but different standard deviations σ_1 and σ_2.For the estimation of μ, σ_1 and σ_2,μ can be estimated by $\bar{x}$,i.e.$\sum_{i=1}^{n} Xi/n$,and σ_1^2 and σ_2^2 can be estimated by S_1^2 and S_2^2 respectively. Standard deviation S_1 with n_1 observations which are less than or equal to the value of $\bar{x}$ can be computed by using similar formula to that used to calculate the sample standard deviation for n total observations (Ahmed *et al.*, 2008)

$$S^2=\sum_{i=1}^{n} \frac{(x_i-\bar{x})^2}{(n-1)} \quad (3)$$

$$S_1^2 = \frac{2\sum_{i=1}^{n_1}(x_i - \bar{x})^2}{2n_1 - 1} \quad (4)$$

Also, the sample standard deviation S_2 with n_2 observations which are greater than the value of $\bar{X}$ can be calculated as

$$S_2^2 = \frac{2\sum_{i=1}^{n_2}(x_i - \bar{x})^2}{2n_2 - 1} \quad (5)$$

The two commonly used normally-based process capability indices are $\hat{C}p, \hat{C}pk$ are modified using the weighted variance method as follows

$$Cp\ (WV) = \frac{USL-LSL}{3(S_1+S_2)} \quad (6)$$

Cpk (WV) index can be expressed as:

$$Cpk\ (WV) = \min\left[\frac{USL-\bar{x}}{3S_2}, \frac{\bar{x}-LSL}{3S_1}\right] \quad (7)$$

2.2. Clements method

For non-normal pearsonian distribution (which includes a wide class of "populations" with non-normal characteristics) Clements (1989) has proposed a novel method of non-normal percentiles to calculate process capability C_p and process capability for off centre process C_{pk} indices based on the mean, standard deviation, skewness and kurtosis. Under the assumption that these four parameters determine the type of the Pearson distribution curve, Clements utilized the table of the family of Pearson curves as a function of skewness and kurtosis. Clements replaced 6σ by $(U_P - L_p)$ in the below equation (6).

$$C_p = \frac{USL-LSL}{(U_P - L_p)} \quad (8)$$

Where, U_P is the 99.865 percentile and L_p is the 0.135 percentile. For C_{pk}, the process mean μ is estimated by median M, and the two $3\sigma_s$ are estimated by $(U_P - M)$ and $(M - L_p)$ respectively, Figure 1 depicts how a PCI are obtained for a non-normally distributed quality attribute.

$$C_{pk} = \min\left\{\frac{USL - M}{U_P - M}, \frac{M - LSL}{M - L_p}\right\} \quad (9)$$

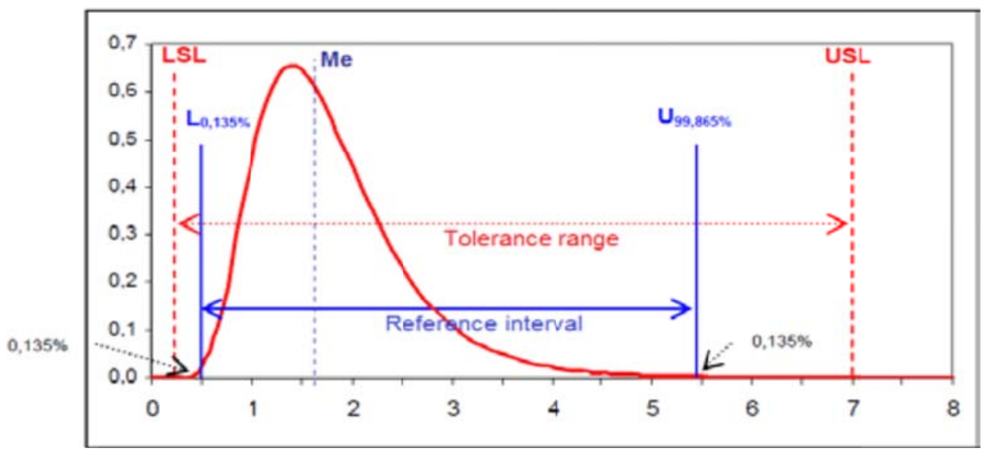

Figure 1. probability distribution curve for a non- normal data with spec. Limits

Procedure for calculating PCIs using Clements method (Boyles, 1994):

- Obtain specification limits USL and LSL for a given quality characteristic
- Estimate sample statistics for the given sample data: Sample size, Mean, Standard deviation, Skewness, Kurtosis
- Look up standardized 0.135 percentile,
- Look up standardized 99.865 percentile
- Look up standardized Median
- Calculate estimated 0.135 percentile using Eqn. $Lp = \bar{x} - s\ Lp'$
- Calculate estimated 99.865 percentile using Eqn. $Up = \bar{x} + s\ Up'$

- Calculate estimated median using Eqn. $M = \bar{x} + s M'$, for positive skewness reverse sign,
- for negative skewness leave positive
- Calculate non-normal process capability indices using Equations.
- $C_p = \frac{USL-LSL}{Up-Lp}$, $C_{pl} = \frac{M-LSL}{M-Lp}$,

  and $C_{pu} = \frac{USL-M}{Up-M}$, C_{pk}=Min $[C_{pu}, C_{pl}]$

2.3. Burr distribution

Although Clements's method is widely used in industry today, Wu *et al.* (1999) indicated that the Clements's method can not accurately measure the nominal values, especially when the underlying data distribution is skewed. To conduct the process capability analysis when the quality characteristic data is non- normally distributed, Clements's method can be modified by replacing the Pearson family of probability curves with a Burr XII distribution to improve the accuracy of the estimates of the indices for non-normal process data. Two reasons justify the use of the Burr XII distribution. First reason is that the two parameter Burr-XII distribution can be used to describe data that arise in the real world and especially those concerning non-normal processes. The second reason is that the direct use of a fitted cumulative function instead of a probability density function may avoid the need for a numerical or formal integration. It is found that a wide range of the skewness and kurtosis coefficients of various probability density functions can be covered by different combinations of c and k .Such probability density functions include most known functions, including normal, Gamma, Beta, Weibull, Logistic, Log-normal and other functions.

Burr XII distribution can be used to obtain the required percentiles of variate X .The probability density function of a Burr XII variate Y is

$$f(\frac{y}{c},k) = \left\{\frac{cky^{c-1}}{(1+y^c)^{k+1}} \text{ if } y \geq 0;\ c \geq 0 \right. \quad (10)$$

$$k \geq 0,\ f(\frac{y}{c},k) = \{0 \text{ if } y < 0 \quad (11)$$

Where c and k represent the skewness and kurtosis coefficients of the Burr XII distribution respectively .Therefore, the cumulative distribution function of the Burr distribution is derived as:

$$F(\frac{y}{c},k) = \left\{1 - \frac{1}{(1+y^c)^k} \text{ if } y \geq 0 \right. \quad (12)$$

$$F(\frac{y}{c},k) = \{0, \text{if } y < 0 \quad (13)$$

Burr-XII distribution can be applied to estimate capability indices to provide better estimate of the process capability than the commonly used Clements method. Liu and Chen introduced a modification based on the Clements method, whereby instead of using Pearson curve percentiles, they replaced them with percentiles from an appropriate Burr distribution (Castagliola, 1996).

2.3.1 Procedure for calculating PCIs using Burr XII Distribution method

Burr method involves following steps:

- Estimates the sample mean, sample standard deviation, skewness and kurtosis of the original sample data.
- Calculate standardized moments of skewness (α_3) and kurtosis (α_4) for the given sample size n, as follows:
- $\alpha_3 = \frac{n^2}{(n(n-1))^{\frac{3}{2}}} + \sum\left\{\frac{Xj-\bar{x}}{s}\right\}^3$, where, $\bar{x}$ is mean of the observations and s is the standard deviation.

  $\alpha_4 = \frac{n}{(n-1)^2} + \sum\left\{\frac{Xj-\bar{x}}{s}\right\}^4 - \frac{12(n-1)}{(n+1)(n-3)}$, where n is the number of observations in the data.
- Use the values of α_3 and α_4 to select the appropriate Burr parameters c and k.Then use the standardized $Z = (x - \bar{x}) / s = (Q - \mu)/\sigma$, where x is the

random variate of the original data. Q is the selected Burr variate, μ and σ its corresponding mean and standard deviation respectively. The mean and standard deviations as well as skewness and kurtosis coefficients, for a large collection of Burr distributions are found in the tables of Burr (Chou, 1996) and (Castaglioa, 1996). From these tables, the standardized lower, median and upper percentiles are obtained.

- Calculate estimated percentiles using Burr table for lower, median, and upper percentiles as follows: Lp = $\bar{x}$ + s $Z_{0.00135}$, M= $\bar{x}$ + s $Z_{0.5}$, Up = $\bar{x}$ + s $Z_{0.99865}$
- Calculate process capability indices using equations presented below.

$C_p = \frac{USL-LSL}{Up-Lp}$, $C_{pu} = \frac{USL-M}{Up-M}$ $C_{pl} = \frac{M-LSL}{M-Lp}$ $C_{pk} = \text{Min}\,[C_{pu}, C_{pl}]$

2.4. Box-Cox power transformation

Box and Cox (1964) provides a family of power transformations that will optimally normalize a particular variable, eliminating the need to randomly try different transformations to determine the best option. It transform non-normal data into normal data on the necessarily positive response variable X as shown in the below equation

$$X^{\lambda} = \left\{\frac{X^{\lambda}-1}{\lambda} \text{ For } \lambda \neq 0 \right. \quad (14)$$

$$X^{\lambda} = \{\ln X \text{ For } \lambda = 0 \quad (15)$$

This continuous family depends on a single parameter λ, it can on an infinite numbers of values. This family of transformations incorporates many traditional transformations like:

Square root transformation, λ= 0.50, Cube root transformation, λ=0.33.

Fourth root transformation, λ=0.25, Natural log transformation, λ =0.00

Reciprocal square root transformation, λ=-0.50, Reciprocal transformation, λ=-1.00,

No transformation needed, when λ= 1.00, it produces results identical to original data.

Most common transformations reduce positive skew but may exacerbate negative skew unless the variable is reflected prior to transformation. Box-Cox eliminates the need for it (Box and Cox, 1964).

2.5. Modelling Non-Normal data using Statistical software

Quality control engineers are frequently asked to evaluate process stability and capability for key quality characteristics that follow non-normal distributions. In the past, demonstrating process stability and capability require the assumption of normally distributed data. However, if data do not follow the normal distribution, the results generated under this assumption will be incorrect. Whether it is decided to transform data to follow the normal distribution or identify an appropriate non-normal distribution model statistical software's can be used. Identification of an appropriate non-normal distribution model is a good approach to find a non-normal distribution that fits the data. Many non-normal distribution can be used to model a response, but if an alternative to the normal distribution is going to be viable, the exponential, lognormal, and weibull distributions usually works well. Minitab statistical software can be used to verify the process stability and estimate process capability for non-normal quality characteristics.

3. Methodology

Methodology involves following steps:

- Understanding the basic concepts of process capability analysis for non-normal data
- Data Collection
- Calculate required statistics of the case study data
- Validate the critical assumptions.
- Estimation of Cp, Cpu, Cpl, Cpk using non normal methods and classical method

- Comparison of PCIs of non-normal methods with PCIs of classical method

3.1. Data collection

In order to discuss and compare the five methods to deal with non-normality issues, the data similar to an example presented by Douglas Montgomery in introduction to statistical Quality Control, fifth edition is considered in this paper. Table 1 presents consecutive measurements on the resistivity of Silicon wafers. Descriptive statistics:

Mean: 205.32; Standard deviation: 0.0405; Skewness; 0.39; Kurtosis; 0.21; Range: 0.09785

4. Construction of Control chart, Normal probability plot and histogram for validating the stability and normality assumption.

4.1. Construction of Control chart to assess the stability of the process

In this study, in order to demonstrate the applicability of the method and to make a clear decision about the capability of the production process, $\bar{X}$-R chart are constructed using Minitab 14 software to verify stability of the process. Figure 2 displays that the process is in control as all the mean and range values are within the control limits on the both charts

Table 1. Data of bore diameter using boring operation

	X1	X2	X3	X4	X5	X bar	R
1	205.324	205.275	205.356	205.349	205.343	205.329	0.081
2	205.302	205.310	205.312	205.260	205.300	205.297	0.052
3	205.346	205.280	205.336	205.315	205.346	205.325	0.066
4	205.326	205.438	205.288	205.429	205.299	205.356	0.150
5	205.330	205.397	205.305	205.368	205.354	205.351	0.092
6	205.333	205.316	205.271	205.314	205.318	205.310	0.062
7	205.282	205.396	205.306	205.348	205.297	205.326	0.114
8	205.297	205.354	205.329	205.330	205.324	205.327	0.057
9	205.409	205.313	205.269	205.323	205.319	205.327	0.140
10	205.342	205.397	205.265	205.305	205.303	205.322	0.132
11	205.368	205.397	205.295	205.262	205.315	205.327	0.135
12	205.389	205.301	205.316	205.319	205.353	205.336	0.088
13	205.356	205.298	205.356	205.270	205.294	205.315	0.086
14	205.252	205.273	205.350	205.241	205.361	205.295	0.120
15	205.326	205.297	205.377	205.371	205.316	205.337	0.080
16	205.334	205.234	205.318	205.303	205.342	205.306	0.108
17	205.287	205.262	205.316	205.383	205.312	205.312	0.121
18	205.333	205.328	205.259	205.336	205.396	205.330	0.137
19	205.325	205.297	205.320	205.335	205.285	205.312	0.050
20	205.369	205.283	205.336	205.306	205.336	205.326	0.086
						205.323	0.09785

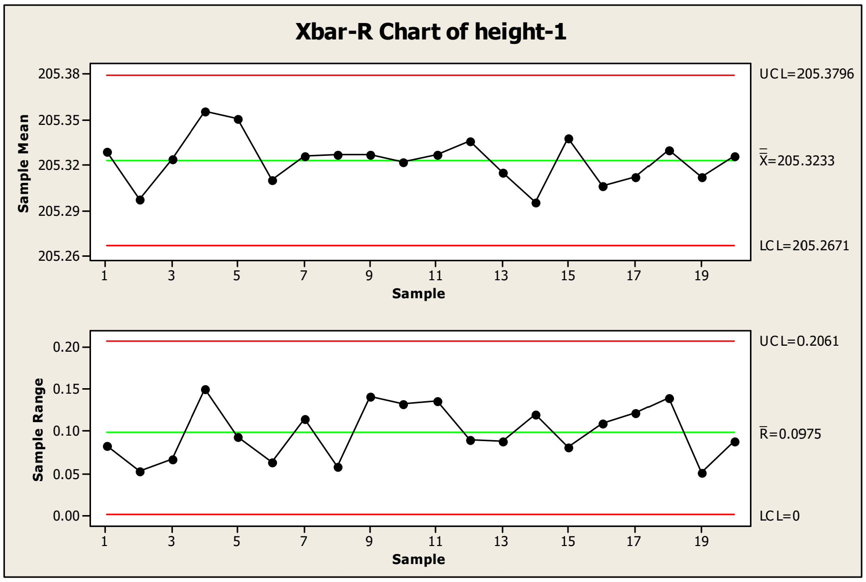

Figure 2. $\overline{\text{X}}$ and R chart

4.2. Construction of histogram and normal probability plot to check normality of the data

Graphical methods including the histogram and normal probability plot are used to check the normality of the data. Figure 3 display the histogram and Figure 4 display the normal probability plot for the data set. The histogram for sample data appears to be non-normal.

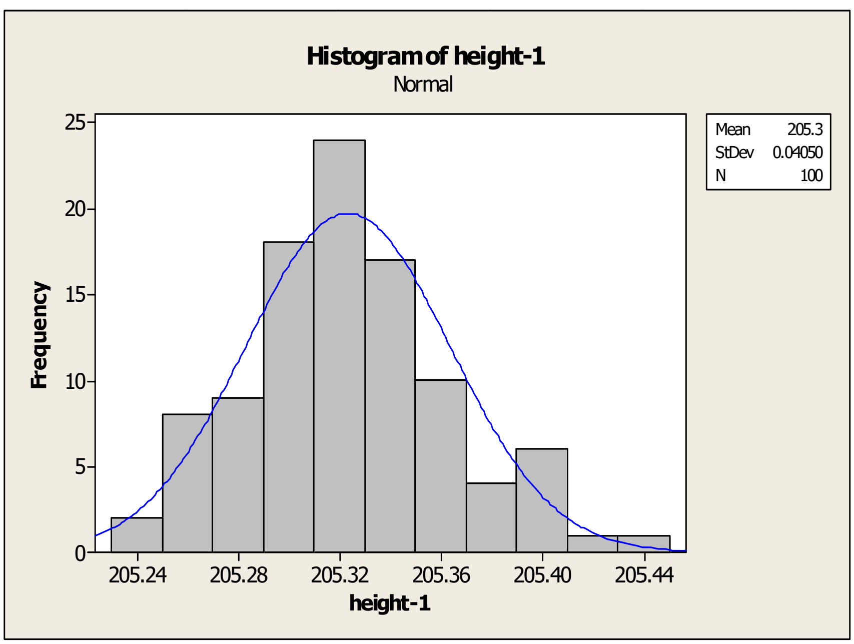

Figure 3. Histogram for case study data

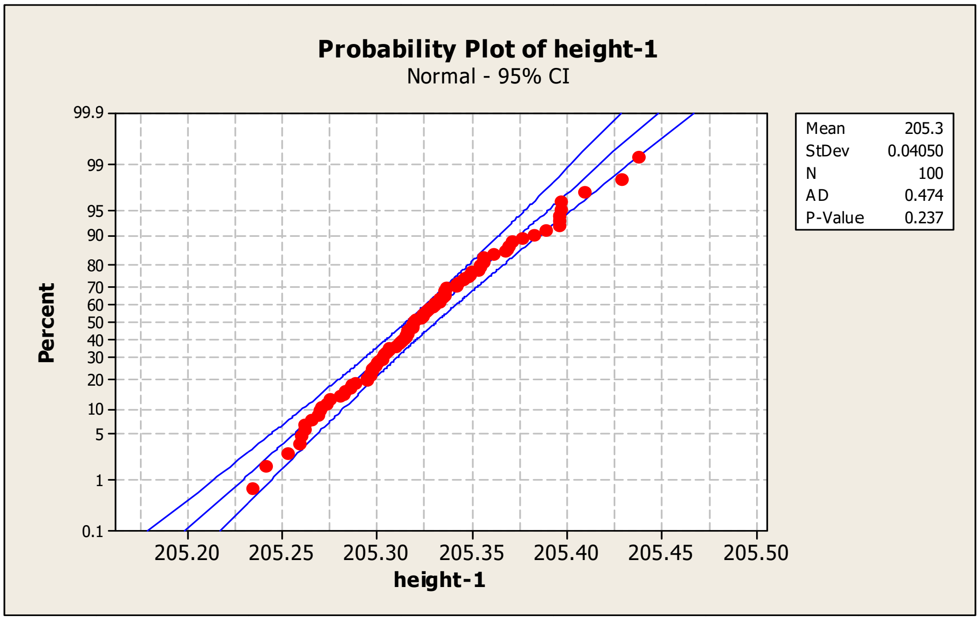

Figure 4. Normal Probability Plot

The validity of non-normality is tested by using Anderson – Darling test (AD).The hole diameter data is considered as normal as it pass normality test because, the P-value is(> 0.005),greater than critical value (0.05).This is done by using Minitab 14 software ,the result of test is shown in Figure 3and 4.

5. Computation of PCI's

For case study data using the following methods:

- Weighted variance Method
- Clements Method
- Burr Method
- Box-Cox Transformation Method

5.1. Weighted variance Method

The statistics for the obtained sample data: Std. deviation =0.0405, Mean = 205.32 and Median = 205.32, USL=205.60, LSL= 205.00

Total number of observation in the data set, n = 100

Number of observations less than or equal to the mean value in the data set, n_1 = 52

Number of observations greater than the mean value in the data set, n_2 = 48

The sample standard deviation S_1 with n_1 observations which are lower than the value of $\bar{X}$ can be calculated as:

$$S_1^2 = \frac{2\sum_{i=1}^{n_1}(x_i - \bar{x})^2}{2n_1 - 1} \quad (16)$$

S_1= 0.0700

Also, the sample standard deviation S_2 with n_2 observations which are greater than the value of $\bar{X}$ can be calculated as:

$$S_2^2 = \frac{2\sum_{i=1}^{n_2}(x_i - \bar{x})^2}{2n_2 - 1} \quad (17)$$

S_2= 0.0288

The two commonly used normally-based process capability indices Cp,Cpk are modified using the weighted variance method as follows:

$Cp\ (WV) = \frac{USL-LSL}{3(S_1+S_2)} = \frac{0.6}{3(0.02214+0.0288)}$

$= \frac{0.6}{0.15282} = 3.92$

Cpk (WV) index can be expressed as, Cpk (WV) = min $\left[\frac{USL-\bar{x}}{3S_2}, \frac{\bar{x}-LSL}{3S_1}\right]$

$= \min\left[\frac{0.28}{0.0864}, \frac{0.32}{0.066421}\right]$

= min [3.24, 4.81] =3.24

Computation of PCIs using Clements method (Table 2).

Table 2. Process capability calculation procedure using the Clements's percentile method

Step No.	Procedure	Notations	Calculations
1	Specifications : Upper specification Limit Target resistivity Lower specification Limit	USL Spec. Mean LSL	205.60 205.30 205.00
2	Estimate sample statistics: Sample size Mean Standard deviation Skewness Kurtosis	N $\bar{x}$ S Sk Ku	100 205.32 0.0405 0.39 0.21
3	Look up standardized 0.135 percentile	Lp'	2.4676
4	Look up standardized 99.865 percentile	Up'	3.5037
5	Look up standardized Median in table 2	M'	0.0652
6	Calculate estimated 0.135 percentile using Eqn. $Lp = \bar{x}$ - s Lp'	Lp	205.220
7	Calculate estimated 99.865 percentile using Eqn. $Up = \bar{x}$ + s Up'	Up	205.461
8	Calculate estimated median using Eqn. $M = \bar{x}$ + s M'	M	205.322
9	Calculate non-normal process capability indices using Equations. $C_P = \frac{USL-LSL}{Up-Lp}$, $C_{pl} = \frac{M-LSL}{M-Lp}$, $C_{pu} = \frac{USL-M}{Up-M}$, C_{pk} = Min [C_{pu}, C_{pl}]	C_P C_{pl} C_{pu} C_{pk}	2.489 3.156 2.00 2.00

Computation of PCIs using Burr's method (Table 3).

Table 3. Process capability calculation using the Burr percentile method

Step No.	Procedure	Notations	Calculations
1	Specifications : Upper specification Limit Target resistivity Lower specification Limit	 USL Spec. Mean LSL	 205.60 205.30 205.00
2	Estimate sample statistics: Sample size Mean of sample data Standard deviation (overall) Skewness Kurtosis	 N $\bar{x}$ S Sk Ku	 100 205.32 0.0405 0.39 0.21
3	Estimate standard moments of skewness (α_3) and Kurtosis (α_4) using Sk and Ku values from step 2.	α_3 α_4	0.384 3.13
4	Based on α_3 and α_4 from step 3 ,select the parameters c and k values using the Burr-XII distribution table	c k	2.5377 12.5234
5	With reference to the parameters c and k obtained in step 4, use the table of standardized tails of the Burr XII distribution to determine standardized lower, median and upper percentiles.	$Z_{0.00135}$ $Z_{0.5}$ $Z_{0.99865}$	-2.085 -0.082 3.595
6	Calculate estimated 0.135 percentile using Eqn. $Lp = \bar{x} + s\, Z_{0.00135}$	Lp	205.2355
7	Calculate estimated 99.865 percentile using Eqn. $Up = \bar{x} + s\, Z_{0.99865}$	Up	205.4655
8	Calculate estimated median using Eqn. $M = \bar{x} + s\, Z_{0.5}$	M	205.3166
9	Calculate non-normal process capability indices using equations. $C_p = \frac{USL-LSL}{Up-Lp}$, $C_{pl} = \frac{M-LSL}{M-Lp}$, $C_{pu} = \frac{USL-M}{Up-M}$, $C_{pk} = \text{Min}\,[C_{pu}, C_{pl}]$	C_p C_{pl} C_{pu} C_{pk}	2.6080 3.9038 1.9032 1.9032

5.2. Box-Cox Transformation

The Box-Cox transformation parameter (λ) is estimated by Minitab14 statistical software and corresponding process capability indices are determined. The accuracy of the Box-Cox transformation is robust to departures from normal and it avoids the trouble of having to search for a suitable method for each distribution encountered in practice.

The Lambda table as shown in figure 5 contains an estimate of lambda (-0.21) which is the value used in the transformation. It also includes the upper Confidence Interval (0.46) and lower Confidence Interval (-0.95),which are marked on the graph by vertical lines .In this case study, an optimal lambda value that corresponds to -0.21 is utilized for transforming the data and calculation of PCIs. The figure 6 shows the output of the Minitab 14 statistical software.

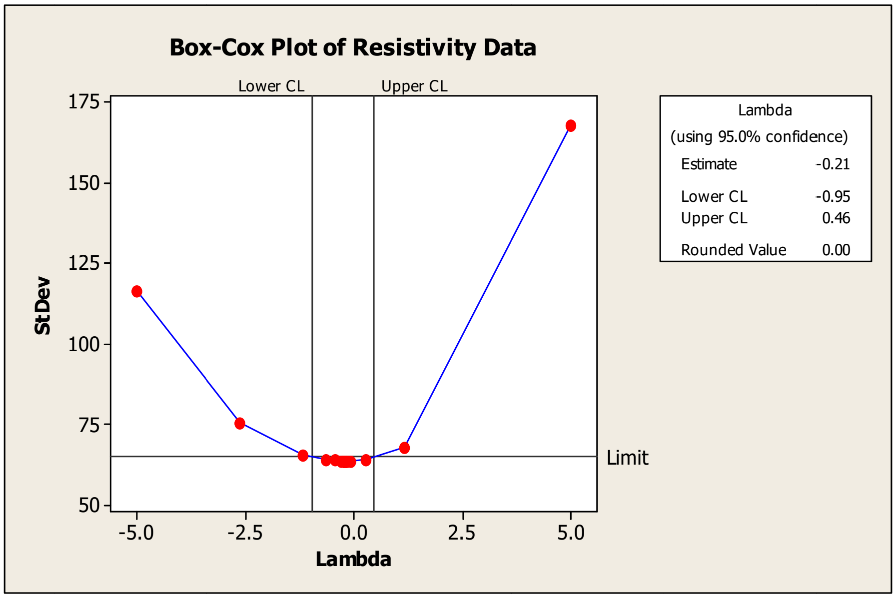

Figure 5. Box Cox plot to estimate optimal value of λ

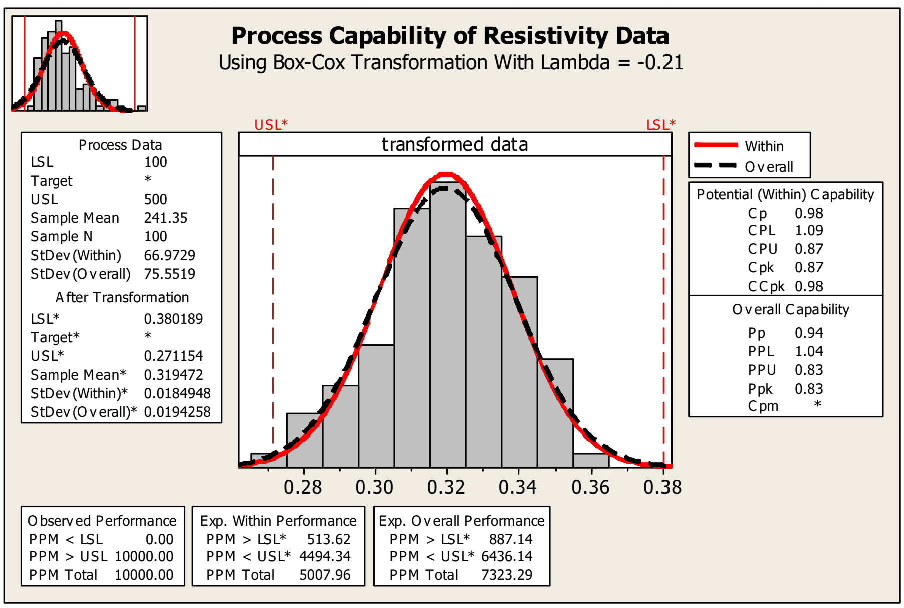

Figure 6. Process capability Analysis using Box- Cox transformation

5.3. Computation of PCIs using Burr's method

In this case study, theoretical non-normal distributions like exponential, weibull and lognormal are used to model the response (resistivity of silicon wafer) .Individual distribution identification feature in Minitab 14 is used to compare the fit of distributions as shown in the figure 7.

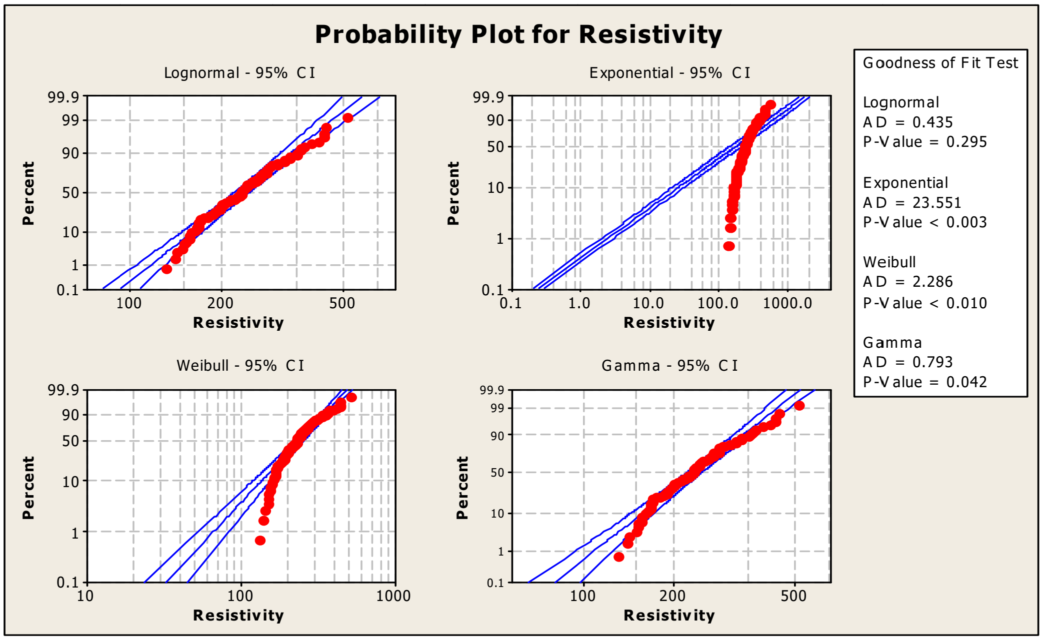

Figure 7. Probability plots for the individual distribution

5.3.1 Comparison of alternative distributions with P-values (For 95% Confidence Interval)

Individual distribution identification feature in statistical software (Minitab 14) is used to construct probability plots for said distributions in order to compare their goodness of fit with the data. In this study, seven distributions are considered to select the appropriate one that fits the data. The lognormal distribution provides the best fit in comparison with other distributions as its p-value (0.295) is greater than critical value (0.05).

Table 5. Comparison of Alternative Distributions using output from probability plot

Distribution type	AD value	P-value	
Weibull	2.286	< 0.010	
Exponential	23.55	< 0.003	
Log logistic	0.432	0.242	
Largest extreme value	0.359	> 0.250	
Lognormal	**0.435**	**0.295**	
Gamma	0.793	0.042	
Normal	2.045	< 0.005	

Process capability indices for the case study data using lognormal distribution are found through the output of Minitab 14 statistical software as shown in the figure 8.

6. Results and Discussion

The following Table 6 presents the PCIs calculation results of different methods.

Table 6. Numerical results for PCIs of Non-normal and Classical Method

PCIs	Obtained Results					
	Weighted variance method	Clements Method	Burr Distribution	Box Cox Transformation	Lognormal Model	Classical method (Normality assumption)
Cp	3.92	2.48	**2.60**	0.98	0.97	2.37
Cpl	4.81	3.15	**3.90**	1.09	1.01	2.50
Cpu	3.24	2.00	**1.90**	0.87	0.93	2.19
Cpk	3.24	2.00	**1.90**	0.87	0.93	2.19

In this paper, the Clements, Burr, Weighted variance, Box-Cox transformation methods are reviewed and used to estimate the PCIs for non-normal quality characteristic data. PCIs of Classical method are compared with the PCIs of all the non-normal methods considered in the case study. In case of classical method, Cpu is over estimated and Cpl is under estimated, when compared with PCIs of other non-normal methods. Weighted variance (WV) method gives good result but it requires manual calculations. Box- Cox transformation method gives reasonably good results compared to classical method. Burr percentile method has been used effectively and it shows better results compared to Clements method.

7. Conclusions

In practice, manufacturing processes that yields non- normally distributed data are inevitable, therefore the use of traditional process capability indices to measure capability of such processes give misleading results.

Box-Cox method is successfully used to transform the non-normal data to normally distributed data and estimated the PCIs.

The obtained values of process capability indices shows that the capability of the production process for controlling the resistivity of silicon wafer is inadequate as all values are less than 1.33 so ,the process dispersion need to be reduced and process mean have to be shifted to closer to the target value of 225 from existing mean of 241.79.

Clements method is simple extension of the traditional 6σ method, which takes into account the possible non-normality of the basic data.

Burr-based method works well under distributions that depart slightly or moderately from normality (Skewness ≤ 1.5).

Overall performance of Box-Cox method is slightly inferior than the lognormal distribution model, Lognormal distribution model exhibits that 6179 parts per million exceeding the specification limits but in case of Box –Cox transformation method 7323 parts per million exceeding the specification limits, hence it can be concluded that modeling of data to lognormal distribution approach is accurate one.

The estimates made using the Burr method, which are higher than those made using Clements's method are good indicator to help quality control engineers be more attentive to and focus on process adjustment and improvement.

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