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THE BUCKLING ANALYSIS OF A RECTANGULAR PLATE ELASTICALLY CLAMPED ALONG THE LONGITUDINAL EDGES

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Abstract:
The paper analyses the stability of a rectangular plate which is elastically buckled along longitudinal edges and pressed by equally distributed forces. A general case is analyzed – different stiffness elastic clamping and then special simpler cases are considered.
Energy method is used in order to determine critical stress. Deflection function is introduced in a convenient way so that it reflects the actual state of the plate deformation in the best manner. In this way, critical stress is determined in analytic form suitable for analysis.
With help of the equation it is easy to conclude how certain parameters influence the value of critical stress. The paper indicates how the obtained solution could be utilized for determining local buckling critical stress in considerably more complex systems – pressed thin-walled beams of an arbitrary length.

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1. INTRODUCTION

In studying so-called local buckling of thin-walled prismatic beams pressed by axial force evenly distributed on the surface of cross section, the problem of determining critical stress arises, both in the whole beam as well as in individual rectangular plates that constitute the beam. The beam plates are connected along the connection lines, so while studying the stability of individual plates, the influence of other adjacent plates cannot be disregarded. Adjacent plates of beams loaded in this way have a role of elastic clamps by loading the examined plate with bending moments equally distributed along the connection lines. These moments are proportional to the plate curvature [1, 2, 3] on those edges, and coefficients of proportionality C – stiffness of elastic clamping – depend on adjacent plates with which the examined plate is connected. Case like this occurs in analyzing the stability of any plate of the thin-walled beam of rectangular cross section contour, as well as in examining the rib stability of thin-walled beams with cross section in the form of U and Z profile with unequal legs.

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The examination of the buckling problem of these beams comes down to the buckling analysis of the elastically supported rectangular plate which length is equal to the length of the beam, and b width is equal to the length of the profile contour line (Fig. 1). The plate is elastically clamped along the longitudinal edges.

Stiffness of elastic clamping of longitudinal edges is different (there is no system symmetry, in the general case) and constant along the plate edges. Along the transverse (loaded) edges, the plate is simply supported.

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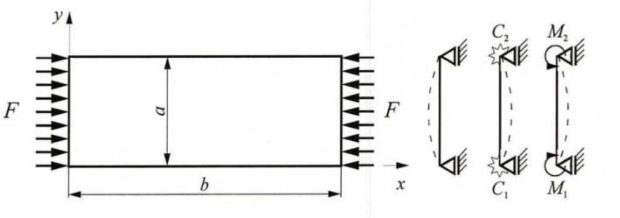

Fig. 1. Plate supporting and elastic clamping position

The law on plate deflection in longitudinal direction, because of the way of support, could be represented by sinusoid function $f(x)=\sin\frac{m\pi x}{a}$.

The law on plate deflection in transverse direction is determined by application of the superposition System of simple support, whereby the deflection could be represented by sinusoid function, and deflection of two "elastic" moments:

$$f(y)=\sin\frac{\pi y}{b}-N_1\left(2b^2y-by^2+y^3\right)-N_2\left(by^2-y^3\right). \quad (1)$$

The deflection of the buckled plate could be represented by the following form:

$$w=Af(x)f(y)=A\sin\frac{m\pi x}{a}\cdot \left[\sin\frac{\pi y}{b}-N_1\left(2b^2y-by^2+y^3\right)-N_2\left(by^2-y^3\right)\right] \quad (2)$$

where:

- A – constant (deflection amplitude);
- x, y – longitudinal and transverse coordinate;
- m – the number of longitudinal semi-waves of the deformed plate ($m\in N$);
- N_1 i N_2 – constants which are determined from boundary conditions.

Plate bending moments on plate edges have to be equal to the moments of elastic clamping:

$$-D\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right)_{y=0}=C_1\left(\frac{\partial^3 w}{\partial x^2\partial y}\right)_{y=0}, \quad -D\left(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}\right)_{y=b}=-C_2\left(\frac{\partial^3 w}{\partial x^2\partial y}\right)_{y=b}. \quad (3)$$

where:

- $D=\frac{E\delta^3}{12\left(1-\mu^2\right)}$ – plate bending stiffness;
- E – elasticity module;
- μ – Poisson's coefficient.

It follows from these conditions (3) that constants N_1 and N_2 have the following values:

$$N_1=\frac{m^2\pi^3C_1\left(6a^2D+bm^2\pi^2C_2\right)}{3b^2\left(12a^4D^2+4a^2bDm^2\pi^2C_1+4a^2bDm^2\pi^2C_2+b^2m^4\pi^4C_1C_2\right)}, \quad N_2=\frac{m^2\pi^3C_2\left(6a^2D+bm^2\pi^2C_1\right)}{3b^2\left(12a^4D^2+4a^2bDm^2\pi^2C_1+4a^2bDm^2\pi^2C_2+b^2m^4\pi^4C_1C_2\right)}, \quad (4)$$

that is:

$$N_1=\frac{m^2\pi^3\overline{C_1}\left(6+m^2\pi^2\overline{C_2}\right)}{3b^3\left(12+4m^2\pi^2\overline{C_1}+4m^2\pi^2\overline{C_2}+m^4\pi^4\overline{C_1}\overline{C_2}\right)}, \quad N_2=\frac{m^2\pi^3\overline{C_2}\left(6+m^2\pi^2\overline{C_1}\right)}{3b^3\left(12+4m^2\pi^2\overline{C_1}+4m^2\pi^2\overline{C_2}+m^4\pi^4\overline{C_1}\overline{C_2}\right)}. \quad (5)$$

where $\overline{C}_1=\frac{bC_1}{a^2D}$ and $\overline{C}_2=\frac{bC_2}{a^2D}$ – are reduced (non-dimensional) stiffness of elastic clamping. With joint supports $C_1=C_2=0$, $\overline{C}_1=\overline{C}_2=0$ and $N_1=N_2=0,$ while clamping $C_1=C_2=\infty$, $\overline{C}_1=\overline{C}_2=\infty$ and $N_1=N_2=\frac{\pi}{3b^3}.$ In general case $0\le N_1\le\frac{\pi}{3b^3}$ i $0\le N_2\le\frac{\pi}{3b^3}.$

The system's potential energy is:

$$Ep=\frac{D}{2}\int_0^a\int_0^b\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dxdy+ +\int_0^a\left(\frac{M_1^2}{2C_1}\right)dx+\int_0^a\left(\frac{M_2^2}{2C_2}\right)dx. \quad (6)$$

The operation of external forces:

$$A_d=\frac{1}{2}\sigma_k\delta\int_0^a\int_0^b\left(\frac{\partial w}{x}\right)^2dxdy\ . \quad (7)$$

Equaling the system's potential energy (6) and the operation of external forces (7) the equation for critical stress is obtained:

$$\sigma_k=\frac{D\pi^2}{b^2\delta}k\ . \quad (8)$$

Where k is the buckling coefficient:

$$k=m^2\left(\frac{b}{a}\right)^2+\frac{1}{m^2}\left(\frac{a}{b}\right)^2P+Q\ , \quad (9)$$

and if we don't use reduced (non-dimensional) stiffness of elastic clamping:

$$k=m^2\left(\frac{b}{a}\right)^2+\frac{1}{m^2}\left(\frac{a}{b}\right)^2R+S\ , \quad (10)$$

and P, Q, R and S are non-dimensional coefficients:

$$\begin{aligned} P = & \frac{105}{\overline{C}_1^2\left[16\left(4\pi^4+105\pi^2-1260\right)+42\overline{C}_2\left(\pi^4+20\pi^2-280\right)+7\overline{C}_2^2\left(\pi^4+15\pi^2-240\right)\right]} \\ & \frac{\left[4\left(3+\overline{C}_2\right)+\overline{C}_1\left(4+\overline{C}_2\right)\right]}{2\overline{C}_1\left[5040\left(\pi^2-6\right)+\overline{C}_2\left(64\pi^4+2940\pi^2-30240\right)+21\overline{C}_2^2\left(\pi^4+20\pi^2-280\right)\right]} \\ & \frac{\left\{\overline{C}_1\left[4\left(\pi^2-6\right)+\overline{C}_2\left(\pi^2-8\right)\right]+4\left[3\pi^2+\overline{C}_2\left(\pi^2-6\right)\right]\right\}}{16\left[945\pi^2+630\overline{C}_2\left(\pi^2-6\right)+\overline{C}_2^2\left(4\pi^4+105\pi^2-1260\right)\right]}, \end{aligned} \tag{11}$$

$$\begin{aligned} Q = & \frac{14\left\{12\overline{C}_1\left[120\left(\pi^2-6\right)+12\overline{C}_2\left(7\pi^2-60\right)+5\overline{C}_2^2\left(3\pi^2-28\right)\right]\right\}+}{\overline{C}_1^2\left[16\left(4\pi^4+105\pi^2-1260\right)+42\overline{C}_2\left(\pi^4+20\pi^2-280\right)+7\overline{C}_2^2\left(\pi^4+15\pi^2-240\right)\right]} \\ & \frac{+\overline{C}_1^2\left[48\left(7\pi^2-60\right)+60\overline{C}_2\left(3\pi^2-28\right)+5\overline{C}_2^2\left(5\pi^2-48\right)\right]+}{2\overline{C}_1\left[5040\left(\pi^2-6\right)+\overline{C}_2\left(64\pi^4+2940\pi^2-30240\right)+21\overline{C}_2^2\left(\pi^4+20\pi^2-280\right)\right]} \\ & \frac{+48\left[45\pi^2+30\overline{C}_2\left(\pi^2-6\right)\left(7\pi^2-60\right)+\overline{C}_2^2\left(7\pi^2-60\right)\right]}{16\left[945\pi^2+630\overline{C}_2\left(\pi^2-6\right)+\overline{C}_2^2\left(4\pi^4+105\pi^2-1260\right)\right]}, \end{aligned} \tag{12}$$

$$\begin{aligned} R = & \frac{105}{bC_1^2\left[16D^2\left(4\pi^4+105\pi^2-1260\right)+42bDC_2\left(\pi^4+20\pi^2-280\right)+7b^2C_2^2\left(\pi^4+15\pi^2-240\right)\right]} \\ & \frac{\left[4D\left(3D+bC_2\right)+bC_1\left(4D+bC_2\right)\right]}{2bDC_1\left[5040b^2\left(\pi^2-6\right)+bDC_2\left(64\pi^4+2940\pi^2-30240\right)+21b^2C_2^2\left(\pi^4+20\pi^2-280\right)\right]} \\ & \frac{\left\{bC_1\left[4D\left(\pi^2-6\right)+bC_2\left(\pi^2-8\right)\right]+4D\left[3D\pi^2+bC_2\left(\pi^2-6\right)\right]\right\}}{16D^2\left[945D^2\pi^2+630bDC_2\left(\pi^2-6\right)+b^2\overline{C}_2^2\left(4\pi^4+105\pi^2-1260\right)\right]}, \end{aligned} \tag{13}$$

$$\begin{aligned} S = & \frac{14\left\{12bDC_1\left[120D^2\left(\pi^2-6\right)+12bDC_2\left(7\pi^2-60\right)+5b^2C_2^2\left(3\pi^2-28\right)\right]\right\}+}{b^2C_1^2\left[16D^2\left(4\pi^4+105\pi^2-1260\right)+42bDC_2\left(\pi^4+20\pi^2-280\right)+7b^2C_2^2\left(\pi^4+15\pi^2-240\right)\right]} \\ & \frac{+b^2\overline{C}_1^2\left[48D^2\left(7\pi^2-60\right)+60bDC_2\left(3\pi^2-28\right)+5b^2C_2^2\left(5\pi^2-48\right)\right]+}{2bDC_1\left[5040D^2\left(\pi^2-6\right)+bDC_2\left(64\pi^4+2940\pi^2-30240\right)+21b^2C_2^2\left(\pi^4+20\pi^2-280\right)\right]} \\ & \frac{+48D^2\left[45D^2\pi^2+30bDC_2\left(\pi^2-6\right)\left(7\pi^2-60\right)+b^2C_2^2\left(7\pi^2-60\right)\right]}{16D^2\left[945D^2\pi^2+630bDC_2\left(\pi^2-6\right)+b^2C_2^2\left(4\pi^4+105\pi^2-1260\right)\right]}. \end{aligned} \tag{14}$$

Because of the complexity of the mathematical problem for determining the symbolic solution of critical stress and coefficients k, P, Q, R and S, software package Matematica® was used [4].

2. RESULTS

The buckling coefficients for certain extreme cases of supporting are known in the literature and match with the obtained values [5, 6, 7]. The values between those boundary values are represented on diagrams in Figures 2-6.

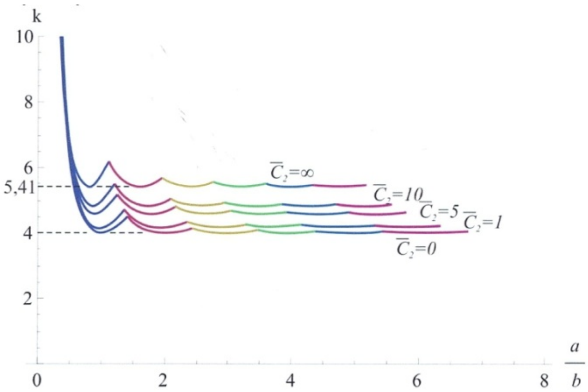

Fig. 2. Buckling coefficient for clamping stiffness $\overline{C_1}=0$

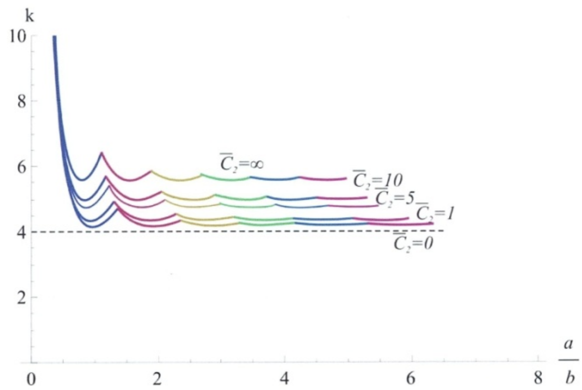

Fig. 3. Buckling coefficient for clamping stiffness $\overline{C_1}=1$

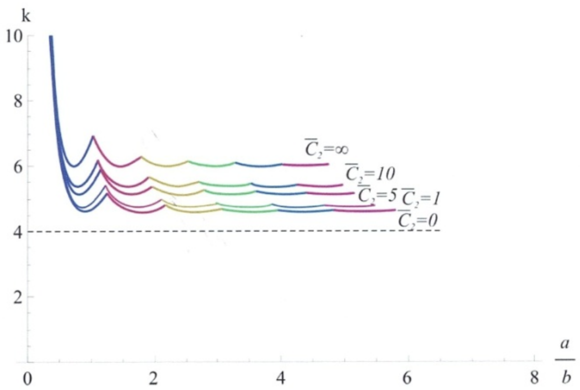

Fig. 4. Buckling coefficient for clamping stiffness $\overline{C_1}=5$

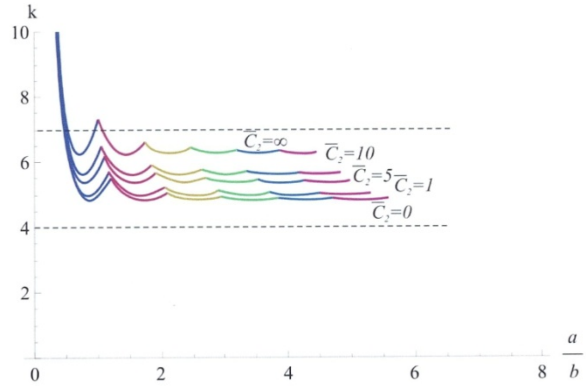

Fig. 5. Buckling coefficient for clamping stiffness $\overline{C_1}=10$

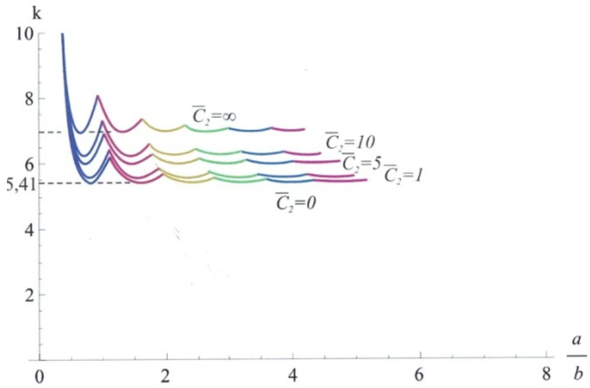

Fig. 6. Buckling coefficient for clamping stiffness $\overline{C_1}=\infty$

3. CONCLUSION

The type of support along the transverse (shorter) edges has significance only with relatively short plates. If a plate is long, and such are almost all plates constituting thin-walled beam, the type of support of these edges only influences the deformation of the plate parts around those edges.

In the middle part of the plate, the influence of support is lost, i.e. all previous results for critical stress obtained for joint support along the loaded plate edges are also valid for other types of supports (elastic and rigid support).

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