Applied Engineering Letters Vol. 1, No. 1, 1-7 (2016) ISSN: 2466-4677

COMPARATIVE ANALYSIS OF ANALYTICAL AND NUMERICAL CALCULATIONS OF CONTACT STRESSES AT ROTATIONAL ELEMENTS OF GEROTOR PUMPS

Lozica Ivanović [1], Boris Rakić [2*], Blaža Stojanović [3], Miloš Matejić [4]

[1] Lozica Ivanović, PhD, Associate Professor, company, Faculty of Engineering University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia

[2] Boris Rakić*, PhD student, Faculty of Engineering University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia

[3] Blaža Stojanović, Assistant Professor, Faculty of Engineering University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia

[4] Miloš Matejić, Teaching Assistant, Faculty of Engineering University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia

Corresponding author

Abstract:

Numerical analysis of contact stresses at rotational elements of gerotor pumps with fixed position of shafts axis and with driving rotor acting as internal element, is presented in this paper. The basic characteristics and principles of functioning of this type of pumps are explained and review of researches in the area of numerical methods and developments and design optimizations of those pumps are presented. The basic assumptions, limitation and input values are defined in order to determine maximal stresses, numerical analysis of contact stresses, which are caused by transmitting of torque from internal to external gear. The obtained results of research contact stresses by numerical and analytical method are graphical presentations. The comparisons of results for better understanding of its deviations also were performed.

ARTICLE HISTORY
Received: 03 February 2016
Accepted: 17 February 2016
Available online: 30 june 2016

KEYWORDS
trochoidal gearing,
gerotor pump,
rotational elements,
contact stresses,
numerical analysis.

1. INTRODUCTION

Determination of stress-strain state at specific elements of mechanical constructions has been identified as very important procedure in the process of reliability improvement of elements considered as well as whole construction. In this procedure, contact stresses, which are caused by contact between elements, are special area of research of stress state and those stresses directly influent to wear intensity, and by that influent to properties of gears in exploitation, its safety, reliability and duration of exploitation period.

The essential elements of large number of machines are gears and theirs quality and characteristics highly influent to exploitation and reliability of machines. Applying of gears with involute profiles, many beneficial technical and economic advantages are caused, with simultaneous causing of set of disadvantages (relatively low load capacity of teeth sides and high contact stresses) that lead to reducing of energy efficiency index. Using of trochoidal gearing profile at present constructions can provide required energy efficiency and exploitation period, with minimal elements weight and dimensions. Gearing with trochoidal profiles are used at cycloreductors, special group of planetary gearing transmitters, so as at large number of rotational multipurpose machines, such as rotational pumps, rotational motors, compressors and blowing machines. Due to large application area and possibility of realization of large number of gear combinations

CONTACT: Boris Rakić, borismfkg@gmail.com

with internal trochoidal profile gearing, research of contact stresses become essential for improving reliability and duration of exploitation period of whole machine systems. Numerical analysis of contact stresses at rotational elements with internal trochoidal profile gearing by finite elements method was done and comparisons with analytically calculated results are done. Also, comparative presentations of contact stresses at certain teeth in dependence of rotation angle of external gear are done.

Altogether with evolution of rotational motors, development of trochoidal profile gearing were done, based on kinematic principles of planetary mechanisms usually called as GEROTORS. Myron F. Hill [1] was inventor of gerotor and his first invention was realized in 1906. and after that, in 1921. he completely focused his research on development of gerotors. He registered his invention as gerotor, which was derived from phrase "GEnerated ROTOR" with detailed description of mathematical procedure for generating peritrochoidal profile of internal gear with circular arc of external profile. Due to the fact that external gear have one teeth more than internal gear, the effect of fluid pumping is caused during rotation of those gears. Gamez-Montero [2, 3] defined eight envelope for every trochoide by numerical method. He developed method for calculation of contact stresses at trochoidal pumps and he presented analysis of influence of geometrical parameters to reduction of maximal contact stresses. Ivanović et al. [4-8] developed methodology for identification of optimal trochoidal profile at gears at rotational (gerotor) pumps, so as analytical calculation of maximal contact stresses at rotational elements of those pumps. The real pump was considered by developed and described methodology and results are used as base for design optimization of considered pump. Litvin and Feng [9] developed parametric equations for equidistant of trochoid. Chmurawa and Lixing [10, 11] described the distribution of loads at cycloid disc with modified tooth profile. Blagojević at al. [12] analised the stress state of the cycloid disc by using numerical and experimental methods for the most critical case of the meshing - single meshing. Obtained experimental results showed good agreement with numerical results. Even for the most unfavourable case of the theoretical meshing (single meshing), which is almost non-existent in practice, stress values are in allowed limits and provide reliable functioning of the reducer in the foreseen work life period. Blagojević at al. [13] presented the newly designed two-stage cycloidal speed reducer which has one cycloid disc for each stage, that is, two cycloid discs in total, which means that it is rather compact. Due to its specific concept, this reducer is characterized by good load distribution and dynamic balance. Stress state analysis of cycloidal speed reducer elements was also realized, using the finite elements method (FEM), for the most critical cases of conjugate gear action (one, two, or three pairs of teeth in contact). The results showed that cycloid discs are rather uniformly loaded justifying the design solution. Calculation of forces which acting on cycloidal speed reducer elements, when machining tolerancens don`t exist, is defined in papers [14-16]. Dynamic loads are dominant at cycloidal speed reducer. Their dynamic behaviour is presented in papers [17, 18].

2. PRINCIPLE OF PUMP FUNCTIONING

Internal gear, due to rigid connection to driving shaft, is rotating in direction of shaft rotation, which is presented at Fig. 1, so as external gearing with rotational speed that depend of number of gear teeth.

Kinematic scheme of fluid distribution is presented at Fig. 1. Pump working chambers (К1, К2, К3, К4, К5 и К6) are space that is currently formed during rotation of internal gear between profiles of external and internal gear. During functioning of the pump every chamber are periodically at suction phase (consequence of internal gear), indicated with I, or at pumping phase that is indicated with II, when internal gear is functioning as pumping element.

At position that is presented at Fig. 1a), when chamber К3 passed through suctioning phase, space between gear teeth enlarged and by that maximal chamber volume (Vmax) is reached, so after that it start to function at pumping phase. Simultaneously, at other part of the pump, space between gear teeth reduced that caused chamber К6 to get its minimal volume (Vmin), both suctioning and pumping chambers. During further rotation the suction of fluid into pump is caused, that pushed other chamber trough suction phase and phase of enlarging of volume as consequence.

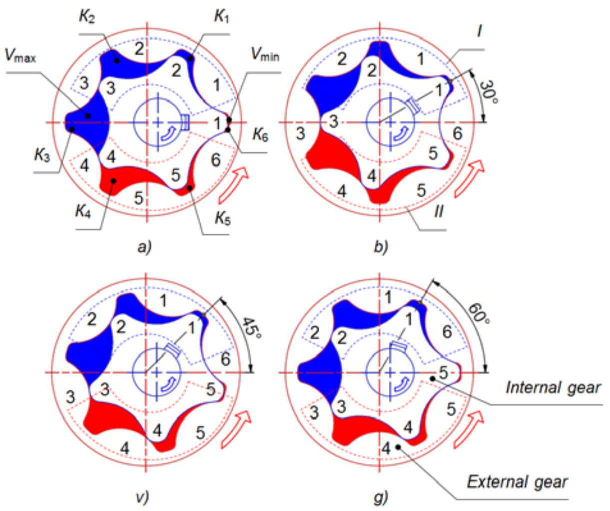

Fig. 1. Kinematic scheme of gerotor [5]

Specific positions are presented at Fig. 1b) and v), when forming of active chambers and enlargement of its volumes are done. At Fig. 1g), extreme position, during one rotation cycle of external gear, is identificated and it can be considered as starting position of other phase of pump functioning cycle, so continual forming, functioning and altering of pump chambers are done. During rotation, torque is transmitted from driving shaft to teeth of internal and external gear in mash simultaneously. Gerotors are mechanisms with design that provide simultaneous contact of all teeth both at internal and external gear.

The distribution of load at this type of construction, presented at Fig.1 is: equivalent between each other at mashed gear teeth pairs 1-1, 2-2 and 3-3, case presented at Fig 1a); equivalent between each other at mashed gear teeth pairs 1-1, 2-2 presented at Fig 1b) (inactive position) and equivalent between each other at mashed gear teeth pairs 1-1, 2-2, 5-6, presented at Fig 1v) and g), and by that one full rotation cycle is reached. Due to unequal distribution of stresses at considered gear tooth pairs, wear and damages at teeth in mash are caused that lead to reducing of exploitation period and reliability of gerotor elements. The aim of this paper is determination of maximal contact stresses and its distribution in dependence of rotation angle of external gear at gear teeth that are currently in mash.

3. ANALYTICAL CALCULATION AND ADOPTED VALUES

3.1 Calculation of gear pair contact point coordinates

As contacts are done simultaneous at all gear teeth with trochoidal profile, it is necessary to determine general relation to determine coordinates of contact point, that can be applied to all teeth in mash. The basic geometrical dimensions for calculation are presented at Fig. 2.

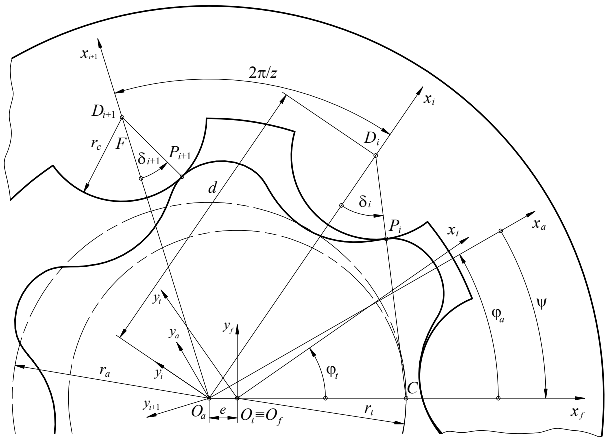

Fig. 2. Schematic presentation of gear coupling at trochoidal pump with geometric dimensions

Were is:

- $x_f O_f y_f$ – fixed coordinate system at the centre of internal gear,
- $x_t O_t y_t$ – coordinate system of the trochoide,
- $x_a O_a y_a$ – coordinate system of the envelope,
- e – center distance of internal and external gear (pump eccentricity),
- r_c – radius of curvature of external gear profile,
- r_t – radius of the internal gear pitch circle,
- r_a – radius of the external gear pitch circle,
- C – pitch point,
- i – order of current contact point,
- δ_i – involve angle,
- φ_t – rotation angle of the internal gear about its own axis,
- φ_a – rotation angle of the external gear about its own axis,
- ψ – referent rotation angle and
- d – distance between the generating point D and the centre of the external gear.

On the base of the geometrical characteristics that are presented at Fig. 2, following general relation for calculation of contact point coordinates in the coordinate system of trochoide can be defined in matrix form as [4]:

$$\overrightarrow{r_{Pi}}=\begin{bmatrix} x_{Pi}^{(t)} \\ y_{Pi}^{(t)} \\ 1 \end{bmatrix}=\begin{bmatrix} e\left\{z\lambda cos\left[\frac{\pi(2i\text{-}1)}{z}\text{-}\psi\right]\text{-}1\text{-}ccos\left[\frac{\pi(2i\text{-}1)}{z}\text{-}\psi+\delta_i\right]\right\} \\ e\left\{z\lambda sin\left[\frac{\pi(2i\text{-}1)}{z}\text{-}\psi\right]\text{-}csin\left[\frac{\pi(2i\text{-}1)}{z}\text{-}\psi+\delta_i\right]\right\} \\ 1 \end{bmatrix},$$

Where is:

- $\overrightarrow{r_{Pi}}$ – radius vector of order i contact point,
- $x_{Pi}^{(t)}$ – x coordinate of order i contact point,
- $y_{Pi}^{(t)}$ – y coordinate of order i contact point,
- z – number of teeth of external gear,
- λ – trochoide coefficient,
- i – order of contact point and
- c – equidistant coefficient ($c=\frac{r_c}{e}$).

3.2 Assumed values for analytical calculations of torques

The assumed values of torque are presented at Tab. 2, on the basis of the analytical calculations of resulting torque that is done as difference of input torque at driving shaft and torque resulted from pressure of fluid at external surface of internal gear [4].

Table 1. Assumed values of torque

	λ=1.575	λ=1.375
φ_a, [°]	T, [Nm]	T, [Nm]
0	0.632	0.621
15	0.290	0.352
25	0.041	0.059
45	0.290	0.352
60	0.632	0.621

4. NUMERICAL CALCULATION OF STRESSES BY FINITE ELEMENTS METHOD

The aim of application of finite element method is verification of analytical method concept for analysis of stress distribution at gear teeth in simultaneous mash. Numerical model that is used for simulation and determination of considered values is developed in correlation with assumptions and generalization defined on the basis of theoretical considerations.

The following assumptions are done during definition of considered problem by finite element method:

- axis of shafts are in fixed position,
- rotation is transmitted from internal to external gear,
- linear load at current contact line of gear teeth is constant
- problem is considered as static one,
- physical characteristics of material are constant,
- thermal effects are neglected and
- effects of friction are not considered.

Simulation of load (contact stresses) at gerotor gear type GP-575 (gerotor with trochoide coefficient of λ=1.575) and gerotor gear type GP-375 (gerotor with trochoide coefficient of λ=1.375) by finite element method done in software environment FEMAP v10.3.0, made primarily for structural analysis, is presented in this paper.

4.1 Visualization and graphic presentation of numerical calculation results of contact stresses

In the aim to numerical calculations to be done, it was necessary to repeat structural analysis several times, for different values of working angle φ_a. Analysis were done for different angular position of external gear, from 0° to 60° (0°, 15°, 25°, 45° and 60°), due to symmetry of distribution of results from position that correspond to one functioning cycle of the pump [5].

Visualization of considered model with resulted VonMises stresses for position that correspond to angular rotation of external gear model GP-575 and φ_a=0° [5].

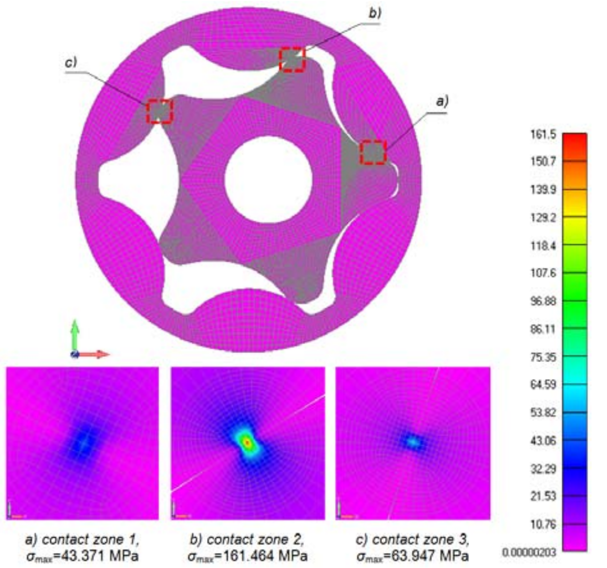

Fig. 3. Visualization of contact stresses distribution at different zones at model GP-575 for φ_a=0° with maximal values

On the basis of the numerical simulation and determined values of maximal stresses at contact points of both gerotor gears type stress-strain state at critical zones can be analysed in details.

Values of maximal stresses at contact points 1, 2 and 3 are maximal contact stresses, so those values are crucial for load capacity calculation and verification of numerical analyses. To the aim of better understanding, comparative presentations of stress distribution and value of maximal stresses at specific contact zones for both considered gerotors calculated by analytical and numerical method are presented at Fig. 4 to Fig.11.

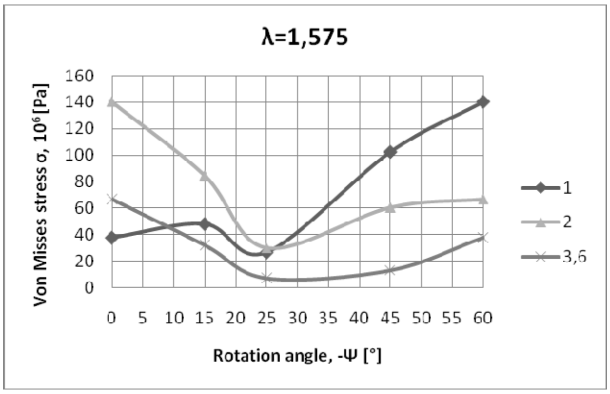

Fig. 4. Results of analytical calculations of maximal stresses at specific contact zones in dependence of external gear rotation angle

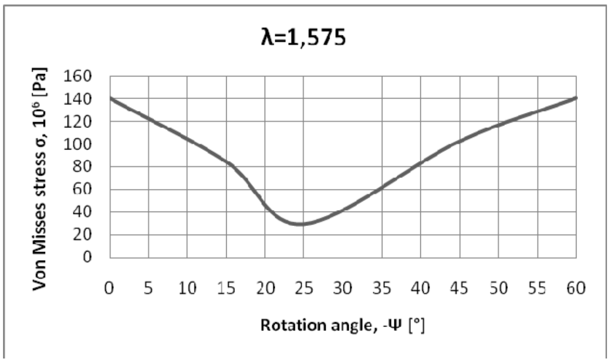

Fig. 5. Results of analytical calculations of maximal stresses in dependence of external gear rotation angle

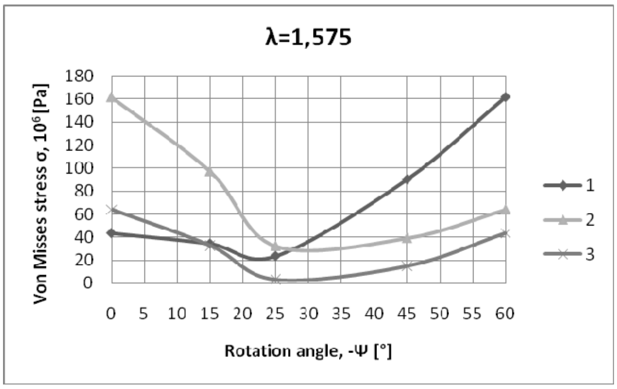

Fig. 6. Results of numerical calculations of maximal stresses at specific contact zones in dependence of external gear rotation angle

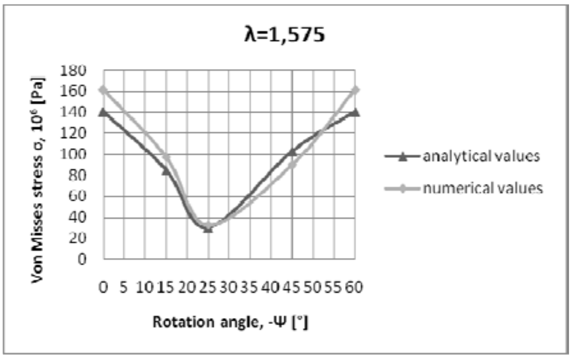

Fig. 7. Results of analytical and numerical calculations of maximal stresses in dependence of external gear rotation angle

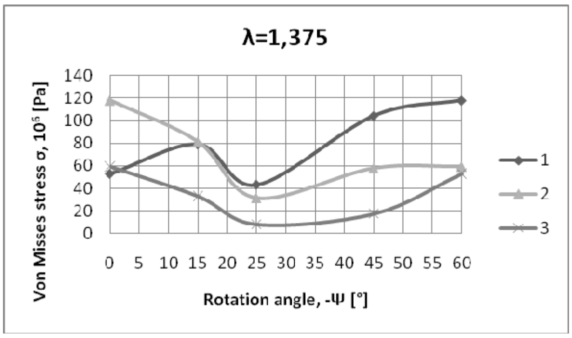

Fig. 8. Results of analytical calculations of maximal stresses at specific contact zones in dependence of external gear rotation angle

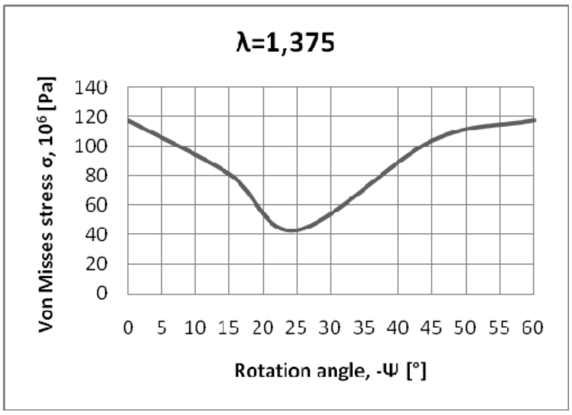

Fig. 9. Results of analytical calculations of maximal stresses in dependence of external gear rotation angle

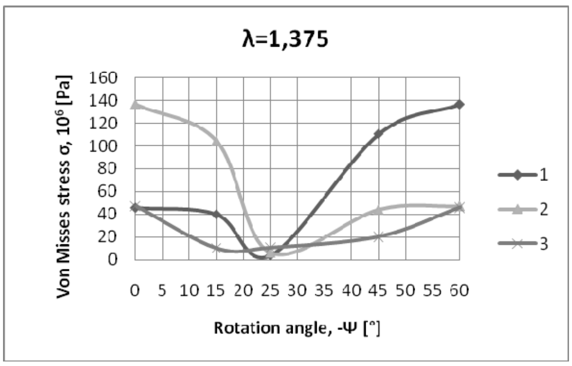

Fig. 10. Results of numerical calculations of maximal stresses at specific contact zones in dependence of external gear rotation angle

Comparative presentation of results of analytical and numerical calculations at gerotor with trochoide coefficient λ=1,375 are given at Fig. 11.

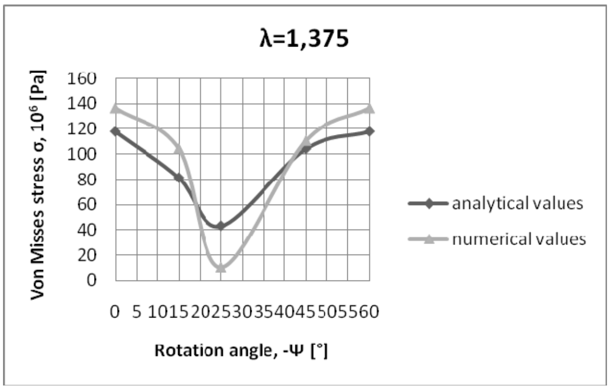

Fig. 11. Results of analytical and numerical calculations of maximal stresses in dependence of external gear rotation angle

Comparative values of maximal stresses for both gerotors, calculated by analytical and numerical method are presented at Fig. 12.

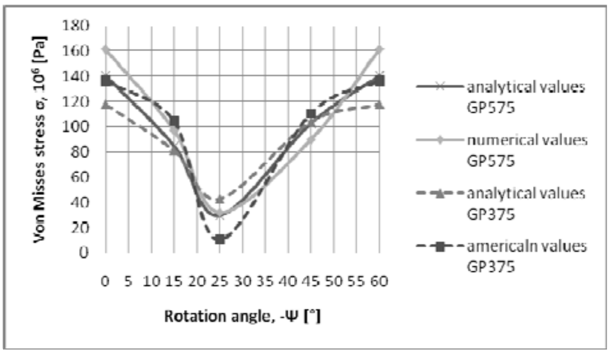

Fig. 12. Results of analytical and numerical calculations of maximal stresses in dependence of external gear rotation

5. CONCLUSIONS

On the basis of the analysis the following conclusion can be done:

- Diagrams of stresses obtained by analytical and numerical methods have same trend, so it can be concluded that numerical model is adequate for further analyses and researches to the aim of reduction of maximal stresses.
- Values of maximal stresses calculated by numerical calculations are higher than values of those stresses calculated by analytical method.
- Maximal stresses act at position that correspond to angle φ_a=0° and φ_a=60°.

On the basis of the conclusions it can be stated that analytical and numerical method provide relevant results in correlation to sensitivity to changing of input parameters. Special focus must be put on research of rotation and relative position between internal and external gear during exploitation that highly influent to changing of load and by that contact stresses that caused: wear of the rotational elements, their damages, reduction of exploitation period, reduction of reliability and other unwanted consequences.

The advanced method, presented in this paper, provide efficient and reliable calculation of load capacity at gerotor elements and form relevant base of information for research possible methods of improve load capacities of those pumps. One of the methods that consider possible changing of element oblique is finite element method. From this reason, subject of research in this paper is forming of numerical model that can be used for analysis of stress changing due to different profiles of internal gerotor gear under static load condition.

Changing of stress distribution at specific zones of the gerotor elements cause also changing of maximal stresses values. The presented analysis provide optimization of design procedures in details in order to prevent possible damages and wear at gerotor rotational elements under maximal working pressures and at critical rotational angles, φ_a=0° and φ_a=60°.

REFERENCES

[1] Company Catalogs: PPT Hydraulics, Danfoss, Nichols Portland Company, Hydraulic Products Incorporated, CATIA Version 5 Release 7 Documentation

[2] P. J. Gamez-Montero, Caracterizacion Fluidodinamica de una Bomba Oleohidraulica de Engranajes internos generados por Perfiles Trocoidales (Ph.D. Thesis), Universitat Politecnica de Catalunya, Terrassa, 2004.

[3] P. J. Gamez-Montero, E.Codina Macià, Fluid Dynamic Behaviour of an Internal Rotary Pump Generated by Trochoidal Profiles, Proc. of 1st FPNI-PhD Symposium, Hamburg, Germany, 20-22th September, 2000, pp.33-47.

[4] L. Ivanović, Identification of optimal trochoidal profile at elements of rotational pump (Ph.D. Thesis), University of Kragujevac, Faculty of Mechanical Engineering Kragujevac, Serbia, 2006.

[5] L. Ivanovic, G. Devedzic, S. Cukovic, N. Miric, Modeling of the Meshing of Trochoidal Profiles With Clearances, J Mech Des N Y, 134, 2012, No.4, 041003-1-041003-9.

[6] L. Ivanović, G. Devedžić, N. Mirić, S. Ćukovic, Analysis of forces and moments in the gerotor pumps, Proceedings of the Institution of Mechanical Engineers, Part C: JMES, 1, 2010, No.1, 1-13.

[7] L. Ivanović, D. Josifović, A. Ilić, Modelling of trochoidal gearing at the gerotor pump, 4th International Conference on Power Transmissions, Sinaia, Romania, 20-23 June, 2012, pp. 553-562.

[8] L. Ivanović, B. Rakić, D. Josifović, A. Ilić, Analysis of contact stresses at trochoidal gearing by numeric approach, The 8th International Symposium KOD 2014, Machine and Industrial Design in Mechanical Engineering, Balatonfüred, Hungary, 12-15 June, 2014, pp.205-210.

[9] F. Litvin, F. Feng Computerized design and generation of cycloidal gearings, MECH MACH THEORY, 31, 1996, No 7, pp.891-911.

[10] M. Chmurawa, A. Lokiec, Distribution of loads in cycloidal planetary gear (CYCLO) including modification of equidistant, 16th European ADAMS user conference, Berchtesgaden, Germany, 2001.

http://web.mscsoftware.com/support/library/conf/adams/euro/2001/proceedings/papers_pdf/Paper_54.pdf

[11] L. Lixing, L. Xin, H. Weidong, Q. Yuanmei, Profile modification and accurate force analysis on cycloid drive, 4th World congress on gearing and power transmission, Paris, France, 16-18th Marth, 1999, pp.1141-1146.

[12] M. Blagojevic, N. Marjanovic, Z. Djordjevic, B. Stojanovic, V. Marjanovic, R. Vujanac, A. Disic, Numerical and experimental analysis of the cycloid disc stress state, Technical Gazette, 21, 2014, No.2, pp.377-382.

[13] M. Blagojevic, N. Marjanovic, Z. Djordjevic, B. Stojanovic, A. Disic, A new design of a two-stage cycloidal speed reducer, J Mech Des N Y, 133, 2011, No.8, pp. 085001-1-085001-7.

[14] S.K. Malhotra, M.A. Parameswaran Analysis of a cycloidal speed reducer, MECH MACH THEORY, 18, 1983 No 6, pp.491-499.

[15] M. Blagojević, V. Nikolić-Stanojević, Influence of magnitude of eccentricity on tooth profile of cyclo gear and distribution of forces, IRMES, 2004, Kragujevac, Serbia, 16-17th September, 2004, pp.673-678

[16] M. Blagojević, N. Marjanović, (2007) The force distribution on two stage cyclo speed reducer with new concept, DEMI Banja Luka, Bosnia and Hercegovina, 25-26 Maj, 2007, pp.51-56.

[17] M. Blagojević, Stress and strain state of cyclo speed reducer`s elements under dynamic loads (Ph.D. Thesis), University of Kragujevac, Faculty of Mechanical Engineering Kragujevac, Serbia, 2008.

[18] M. Blagojević, V. Nikolić, N. Marjanović, LJ. Veljović, Analysis of cycloid drive dynamic behavior, Scientific Technical Review, LIX, 2009, No.1, pp 52-56.