On The Class of Almost β-γ-Continuous Functions

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Abstract The main purpose of the present paper is to introduce a new class of functions called almost β-γ-continuous functions which is contained in the class of almost β-continuous functions and contains the class of β-γ-continuous functions.

Keywords: β-γ-open, almost β-γ-continuous.

1 Introduction

Kasahara [10] defined an operation α on a topological space to introduce α-closed graphs. Following the same technique, Ogata [16] defined an operation γ on a topological space and introduced γ-open sets. Hariwan [7] introduced a type of continuity called β-γ-continuous function. Nasef and Noiri [13] introduced the notion of almost β-continuity.

In this paper, we introduce a new class of functions called almost β-γ-continuous functions which is contained in the class of almost β-continuous functions and contains the class of β-γ-continuous functions. We obtain basic properties of almost β-γ-continuous functions.

2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let (X, τ) be a space and A a subset of X. An operation γ [10] on a topology τ is a mapping from τ into power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [16] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open set in X. Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ-closed. The τ_γ-interior [18] of A is denoted by τ_γ-$Int(A)$ and defined to be the union of all γ-open sets of X contained in A. A subset A of a space X is said to be β-γ-open [8] if $A \subseteq Cl(\tau_\gamma\text{-}Int(Cl(A)))$. A subset A of X is called β-γ-closed [7] if and only if its complement is β-γ-open.

Definition 2.1. A subset A of a space X is said to be

1. α-open [14] if $A \subseteq Int(Cl(Int(A)))$.
2. semi-open [11] if $A \subseteq Cl(Int(A))$.
3. preopen [12] if $A \subseteq Int(Cl(A))$.
4. β-open [1] if $A \subseteq Cl(Int(Cl(A)))$.

Definition 2.2. The intersection of all preclosed (resp., semi-closed, α-closed) sets of X containing A is called the preclosure [6] (resp., semi-closure [4], α-closure [17]) of A.

Definition 2.3. [19] The δ-interior of a subset A of X is the union of all regular open sets of X contained in A. The subset A is called δ-open if $A = Int_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subseteq X$ is called δ-closed if $A = Cl_\delta(A)$, where $Cl_\delta(A) = \{x \in X : Int(Cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

Proposition 2.4. [2] *A subset A of a space X is β-open if and only if $Cl(A)$ is regular closed.*

Theorem 2.5. [1] *Let A be any subset of a space X. Then $A \in \beta O(X)$ if and only if $Cl(A) = Cl(Int(Cl(A)))$.*

Theorem 2.6. *Let A be a subset of a topological space (X, τ). Then:*

1. *If $A \in SO(X)$, then $pCl(A) = Cl(A)$* [5].
2. *If $A \in \beta O(X)$, then $\alpha Cl(A) = Cl(A)$* [3].
3. *If $A \in \beta O(X)$, then $Cl_\delta(A) = Cl(A)$* [20].

Lemma 2.7. [9] *Let A be a subset of a space (X, τ). Then $A \in PO(X, \tau)$ if and only if $sCl(A) = Int(Cl(A))$.*

Definition 2.8. Let A be any subset of a topological space (X, τ) and γ be an operation on τ. Then:

1. The union of all β-γ-open sets contained in A is called the β-γ-interior of A and is denoted by β-$\gamma Int(A)$.
2. The intersection of all β-γ-closed sets containing A is called the β-γ-closure of A and is denoted by β-$\gamma Cl(A)$.

Definition 2.9. [7] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be β-γ-continuous if for every open set V of Y, $f^{-1}(V)$ is β-γ-open in X.

Definition 2.10. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be β-γ-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a β-γ-open set U containing x such that $f(U) \subseteq V$.

Definition 2.11. [13] A function $f : (X, \tau) \to (Y, \sigma)$ is called almost β-continuous at $x \in X$ if for every open set V in Y containing $f(x)$, there exists a β-open set U in X containing x such that $f(U) \subseteq Int(Cl(V))$. If f is almost β-continuous at every point of X, then it is called almost β-continuous.

Definition 2.12. [15] A space X is said to be semi-regular if for any open set U of X and each point $x \in U$, there exists a regular open set V of X such that $x \in V \subseteq U$.

3 Almost β-γ-Continuous

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost β-γ-continuous at a point $x \in X$ if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a β-γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. If f is almost β-γ-continuous at every point of X, then it is called almost β-γ-continuous.

Example 3.2. Consider $X = \{1, 2, 3\}$ with the discrete topology τ on X. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{1, 3\} \\ X & \text{otherwise.} \end{cases}$$

And define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = 1 \\ 2 & \text{if } x = 2 \\ 3 & \text{if } x = 3 \end{cases}$$

Then, f is not β-γ-continuous.

Remark 3.3. It easily follows that β-γ-continuity implies almost β-γ-continuity and almost β-γ-continuity implies almost β-continuity. However, the converses are not true as the following example shows.

Example 3.4. Consider $X = \{a, b, c\}$ with the topology $\tau = \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} c & \text{if } x = a \\ b & \text{if } x = b \\ a & \text{if } x = c \end{cases}$$

Then f is almost β-γ-continuous but not β-γ-continuous, because $\{a\}$ is an open set in (X, σ) containing $f(c) = a$, but there exists no β-γ-open set U in (X, τ) containing c such that $f(U) \subseteq \{a\}$.
And we define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then f is almost β-continuous but is not almost β-γ-continuous.

Theorem 3.5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is almost β-γ-continuous.*
2. *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a β-γ-open set U in X containing x such that $f(U) \subseteq sCl(V)$.*
3. *For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a β-γ-open set U in X containing x such that $f(U) \subseteq V$.*
4. *For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists a β-γ-open set U in X containing x such that $f(U) \subseteq V$.*

Proof. (1) $\Rightarrow$ (2). Let $x \in X$ and let V be any open set of Y containing $f(x)$. By (1), there exists a β-γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Since V is open and hence V is preopen set. By Lemma 2.7, $Int(Cl(V)) = sCl(V)$. Therefore, $f(U) \subseteq sCl(V)$.
(2) $\Rightarrow$ (3). Let $x \in X$ and Let V be any regular open set of Y containing $f(x)$. Then V is an open set of Y containing $f(x)$. By (2), there exists a β-γ-open set U in X containing x such that $f(U) \subseteq sCl(V)$. Since V is regular open and hence is preopen set. By Lemma 2.7, $sCl(V) = Int(Cl(V))$. Therefore, $f(U) \subseteq Int(Cl(V))$. Since V is regular open, then $f(U) \subseteq V$.
(3) $\Rightarrow$ (4). Let $x \in X$ and Let V be any δ-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists an open set G containing $f(x)$ such that $G \subseteq Int(Cl(G)) \subseteq V$. Since $Int(Cl(G))$ is regular open set of Y containing $f(x)$. By (3), there exists a β-γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(G)) \subseteq V$. This completes the proof.
(4) $\Rightarrow$ (1). Let $x \in X$ and Let V be any open set of Y containing $f(x)$. Then $Int(Cl(V))$ is δ-open set of Y containing $f(x)$. By (4), there exists a β-γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(V))$. Therefore, f is almost β-γ-continuous. □

Theorem 3.6. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is almost β-γ-continuous.*
2. *$f^{-1}(Int(Cl(V)))$ is β-γ-open set in X, for each open set V in Y.*
3. *$f^{-1}(Cl(Int(F)))$ is β-γ-closed set in X, for each closed set F in Y.*
4. *$f^{-1}(F)$ is β-γ-closed set in X, for each regular closed set F of Y.*
5. *$f^{-1}(V)$ is β-γ-open set in X, for each regular open set V of Y.*

Proof. (1) $\Rightarrow$ (2). Let V be any open set in Y. We have to show that $f^{-1}(Int(Cl(V)))$ is β-γ-open set in X. Let $x \in f^{-1}(Int(Cl(V)))$. Then $f(x) \in Int(Cl(V))$ and $Int(Cl(V))$ is a regular open set in Y. Since f is almost β-γ-continuous. Then by Theorem 3.5, there exists a β-γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Which implies that $x \in U \subseteq f^{-1}(Int(Cl(V)))$. Therefore, $f^{-1}(Int(Cl(V)))$ is β-γ-open set in X.
(2) $\Rightarrow$ (3). Let F be any closed set of Y. Then $Y \setminus F$ is an open set of Y. By (2), $f^{-1}(Int(Cl(Y \setminus F)))$ is β-γ-open set in X and $f^{-1}(Int(Cl(Y \setminus F))) = f^{-1}(Int(Y \setminus Int(F))) = f^{-1}(Y \setminus Cl(Int(F))) = X \setminus f^{-1}(Cl(Int(F)))$ is β-γ-open set in X and hence $f^{-1}(Cl(Int(F)))$ is β-γ-closed set in X.
(3) $\Rightarrow$ (4). Let F be any regular closed set of Y. Then F is a closed set of Y. By (3), $f^{-1}(Cl(Int(F)))$ is β-γ-closed set in X. Since F is regular closed set. Then $f^{-1}(Cl(Int(F))) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is β-γ-closed set in X.
(4) $\Rightarrow$ (5). Let V be any regular open set of Y. Then $Y \setminus V$ is regular closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is β-γ-closed set in X and hence $f^{-1}(V)$ is β-γ-open set in X.
(5) $\Rightarrow$ (1). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$. By (5), we have $f^{-1}(V)$ is β-γ-open set in X. Therefore, we obtain $f(f^{-1}(V)) \subseteq V$. Hence by Theorem 3.5, f is almost β-γ-continuous. □

Theorem 3.7. *For a bijection function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is almost β-γ-continuous.*
2. *$f(\beta\text{-}\gamma Cl(A)) \subseteq Cl_{\delta}(f(A))$, for each subset A of X.*
3. *$\beta\text{-}\gamma Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_{\delta}(B))$, for each subset B of Y.*
4. *$f^{-1}(F)$ is β-γ-closed set in X, for each δ-closed set F of Y.*
5. *$f^{-1}(V)$ is β-γ-open set in X, for each δ-open set V of Y.*
6. *$f^{-1}(Int_{\delta}(B)) \subseteq \beta\text{-}\gamma Int(f^{-1}(B))$, for each subset B of Y.*
7. *$Int_{\delta}(f(A)) \subseteq f(\beta\text{-}\gamma Int(A))$, for each subset A of X.*

Proof. (1) $\Rightarrow$ (2). Let A be a subset of X. Since $Cl_{\delta}(f(A))$ is δ-closed set in Y, it is denoted by $\cap\{F_{\alpha} : F_{\alpha} \in RC(Y), \alpha \in \Delta\}$, where Δ is an index set. Then, we have $A \subseteq f^{-1}(Cl_{\delta}(f(A))) = f^{-1}(\cap\{F_{\alpha} : \alpha \in \Delta\}) = \cap\{f^{-1}(F_{\alpha}) : \alpha \in \Delta\}$. By (1) and Theorem 3.6, $f^{-1}(Cl_{\delta}(f(A)))$ is β-γ-closed set of X. Hence $\beta\text{-}\gamma Cl(A) \subseteq f^{-1}(Cl_{\delta}(f(A)))$. Therefore, we obtain $f(\beta\text{-}\gamma Cl(A)) \subseteq Cl_{\delta}(f(A))$.
(2) $\Rightarrow$ (3). Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (2), we have $f(\beta\text{-}\gamma Cl(f^{-1}(B))) \subseteq Cl_{\delta}(f(f^{-1}(B))) = Cl_{\delta}(B)$. Hence $\beta\text{-}\gamma Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_{\delta}(B))$.
(3) $\Rightarrow$ (4). Let F be any δ-closed set of Y. By (3), we have $\beta\text{-}\gamma Cl(f^{-1}(F))) \subseteq f^{-1}(Cl_{\delta}(F)) = f^{-1}(F)$ and hence $f^{-1}(F)$ is β-γ-closed set in X.
(4) $\Rightarrow$ (5). Let V be any δ-open set of Y. Then $Y \setminus V$ is δ-closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is β-γ-closed set in X. Hence $f^{-1}(V)$ is β-γ-open set in X.
(5) $\Rightarrow$ (6). For each subset B of Y. We have $Int_{\delta}(B) \subseteq B$. Then $f^{-1}(Int_{\delta}(B)) \subseteq f^{-1}(B)$. By (5), $f^{-1}(Int_{\delta}(B))$ is β-γ-open set in X. Then $f^{-1}(Int_{\delta}(B)) \subseteq \beta\text{-}\gamma Int(f^{-1}(B))$.
(6) $\Rightarrow$ (7). Let A be any subset of X. Then $f(A)$ is a subset of Y. By (6), we obtain that $f^{-1}(Int_{\delta}(f(A))) \subseteq \beta\text{-}\gamma Int(f^{-1}(f(A)))$. Hence $f^{-1}(Int_{\delta}(f(A))) \subseteq \beta\text{-}\gamma Int(A)$, which implies that $Int_{\delta}(f(A)) \subseteq f(\beta\text{-}\gamma Int(A))$.
(7) $\Rightarrow$ (1). Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By (7), we get $Int_{\delta}(f(f^{-1}(V))) \subseteq f(\beta\text{-}\gamma Int(f^{-1}(V)))$ which implies that $Int_{\delta}(V) \subseteq f(\beta\text{-}\gamma Int(f^{-1}(V)))$. Since V is regular open set and hence is δ-open set, then $V \subseteq f(\beta\text{-}\gamma Int(f^{-1}(V)))$. This implies that $f^{-1}(V) \subseteq \beta\text{-}\gamma Int(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is β-γ-open set in X which contains x and clearly $f(f^{-1}(V)) \subseteq V$. Hence, by Theorem 3.5, f is almost β-γ-continuous. □

Theorem 3.8. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is almost β-γ-continuous.*
2. *$\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, for each β-open set V of Y.*
3. *$f^{-1}(Int(F)) \subseteq \beta\text{-}\gamma Int(f^{-1}(F))$, for each β-closed set F of Y.*
4. *$f^{-1}(Int(F)) \subseteq \beta\text{-}\gamma Int(f^{-1}(F))$, for each semi-closed set F of Y.*
5. *$\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, for each semi-open set V of Y.*

Proof. (1) $\Rightarrow$ (2). Let V be any β-open set of Y. It follows from Proposition 2.4, that $Cl(V)$ is regular closed set in Y. Since f is almost β-γ-continuous. Then by Theorem 3.6, $f^{-1}(Cl(V))$ is β-γ-closed set in X. Therefore, we obtain $\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.
(2) $\Leftrightarrow$ (3). Let F be any β-closed set of Y. Then $Y \setminus F$ is β-open set of Y and by (2), we have $\beta\text{-}\gamma Cl(f^{-1}(Y \setminus F)) \subseteq f^{-1}(Cl(Y \setminus F)) \Leftrightarrow \beta\text{-}\gamma Cl(X \setminus f^{-1}(F)) \subseteq f^{-1}(Y \setminus Int(F)) \Leftrightarrow X \setminus \beta\text{-}\gamma Int(f^{-1}(F)) \subseteq X \setminus f^{-1}(Int(F))$. Therefore, $f^{-1}(Int(F)) \subseteq \beta\text{-}\gamma Int(f^{-1}(F))$.
(3) $\Rightarrow$ (4). This is obvious since every semi-closed set is β-closed set.
(4) $\Rightarrow$ (5). Let V be any semi-open set of Y. Then $Y \setminus V$ is semi-closed set and by (4), we have $f^{-1}(Int(Y \setminus V)) \subseteq \beta\text{-}\gamma Int(f^{-1}(Y \setminus V)) \Leftrightarrow f^{-1}(Y \setminus Cl(V)) \subseteq \beta\text{-}\gamma Int(X \setminus f^{-1}(V)) \Leftrightarrow X \setminus f^{-1}(Cl(V)) \subseteq X \setminus \beta\text{-}\gamma Cl(f^{-1}(V))$. Therefore, $\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.
(5) $\Rightarrow$ (1). Let F be any regular closed set of Y. Then F is semi-open set of Y. By (5), we have $\beta\text{-}\gamma Cl(f^{-1}(F)) \subseteq f^{-1}(Cl(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is β-γ-closed set in X. Therefore, by Theorem 3.6, f is almost β-γ-continuous.

□

Theorem 3.9. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is almost β-γ-continuous.*
2. *$\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(\alpha Cl(V))$, for each β-open set V of Y.*
3. *$\beta\text{-}\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl_{\delta}(V))$, for each β-open set V of Y.*

4. β-$\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, *for each semi-open set* V *of* Y.
5. β-$\gamma Cl(f^{-1}(V)) \subseteq f^{-1}(pCl(V))$, *for each semi-open set* V *of* Y.

Proof. (1) $\Rightarrow$ (2). Follows from Theorem 3.8 and Theorem 2.6 (2).
(2) $\Rightarrow$ (3). This is obvious since $\alpha Cl(V) \subseteq Cl_\delta(V)$ in general.
(3) $\Rightarrow$ (4) and (4) $\Rightarrow$ (5). Follows from Theorem 2.6.
(5) $\Rightarrow$ (1). Follows from Theorem 3.8 and Theorem 2.6 (1). □

Corollary 3.10. *For a function* $f : X \to Y$, *the following statements are equivalent:*

1. f *is almost* β-γ-*continuous.*
2. $f^{-1}(\alpha Int(F)) \subseteq \beta$-$\gamma Int(f^{-1}(F))$, *for each* β-*closed set* F *of* Y.
3. $f^{-1}(Int_\delta(F)) \subseteq \beta$-$\gamma Int(f^{-1}(F))$, *for each* β-*closed set* F *of* Y.
4. $f^{-1}(Int(F)) \subseteq \beta$-$\gamma Int(f^{-1}(F))$, *for each semi-closed set* F *of* Y.
5. $f^{-1}(pInt(F)) \subseteq \beta$-$\gamma Int(f^{-1}(F))$, *for each semi-closed set* F *of* Y.

Theorem 3.11. *A function* $f : X \to Y$ *is almost* β-γ-*continuous if and only if* $f^{-1}(V) \subseteq \beta$-$\gamma Int(f^{-1}(Int(Cl(V))))$ *for each preopen set* V *of* Y.

Proof. **Necessity.** Let V be any preopen set of Y. Then $V \subseteq Int(Cl(V))$ and $Int(Cl(V))$ is regular open set in Y. Since f is almost β-γ-continuous, by Theorem 3.6, $f^{-1}(Int(Cl(V)))$ is β-γ-open set in X and hence we obtain that $f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = \beta$-$\gamma Int(f^{-1}(Int(Cl(V))))$.
Sufficiency. Let V be any regular open set of Y. Then V is preopen set of Y. By hypothesis, we have $f^{-1}(V) \subseteq \beta$-$\gamma Int(f^{-1}(Int(Cl(V)))) = \beta$-$\gamma Int(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is β-γ-open set in X and hence by Theorem 3.6, f is almost β-γ-continuous. □

Corollary 3.12. *A function* $f : X \to Y$ *is almost* β-γ-*continuous if and only if* $f^{-1}(V) \subseteq \beta$-$\gamma Int(f^{-1}(sCl(V)))$ *for each preopen set* V *of* Y.

Corollary 3.13. *A function* $f : X \to Y$ *is almost* β-γ-*continuous if and only if* β-$\gamma Cl(f^{-1}(Cl(Int(F)))) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.

Corollary 3.14. *A function* $f : X \to Y$ *is almost* β-γ-*continuous if and only if* β-$\gamma Cl(f^{-1}(sInt(F)))) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y.

Theorem 3.15. *For a function* $f : X \to Y$, *the following statements are equivalent:*

1. f *is almost* β-γ-*continuous.*
2. *For each neighborhood* V *of* $f(x)$, $x \in \beta$-$\gamma Int(f^{-1}(sCl(V)))$.
3. *For each neighborhood* V *of* $f(x)$, $x \in \beta$-$\gamma Int(f^{-1}(Int(Cl(V))))$.

Proof. Follows from Theorem 3.11 and Corollary 3.12. □

Theorem 3.16. *Let* $f : X \to Y$ *is an almost* β-γ-*continuous function and let* V *be any open subset of* Y. *If* $x \in \beta$-$\gamma Cl(f^{-1}(V)) \setminus f^{-1}(V)$, *then* $f(x) \in \beta$-$\gamma Cl(V)$.

Proof. Let $x \in X$ such that $x \in \beta$-$\gamma Cl(f^{-1}(V)) \setminus f^{-1}(V)$ and suppose $f(x) \notin \beta$-$\gamma Cl(V)$. Then there exists a β-γ-open set H containing $f(x)$ such that $H \cap V = \phi$. Then $Cl(H) \cap V = \phi$ which implies $Int(Cl(H)) \cap V = \phi$ and $Int(Cl(H))$ is regular open set. Since f is almost β-γ-continuous, by Theorem 3.5, there exists a β-γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(H))$. Therefore, $f(U) \cap V = \phi$. However, since $x \in \beta$-$\gamma Cl(f^{-1}(V))$, $U \cap f^{-1}(V) \neq \phi$ for every β-γ-open set U in X containing x, so that $f(U) \cap V \neq \phi$. We have a contradiction. It follows that $f(x) \in \beta$-$\gamma Cl(V)$. □

Theorem 3.17. *If* $f : X \to Y$ *is almost* β-γ-*continuous and* $g : Y \to Z$ *is continuous and open. Then the composition function* $gof : X \to Z$ *is almost* β-γ-*continuous.*

Proof. Let $x \in X$ and W be an open set of Z containing $g(f(x))$. Since g is continuous, $g^{-1}(W)$ is an open set of Y containing $f(x)$. Since f is almost β-γ-continuous, there exists a β-γ-open set U of X containing x such that$f(U) \subseteq Int(Cl(g^{-1}(W)))$. Also, since g is continuous, then we obtain $(gof)(U) \subseteq g(Int(g^{-1}(Cl(W))))$. Since g is open, we obtain $(gof)(U) \subseteq Int(Cl(W))$. Therefore, gof is almost β-γ-continuous. □

Theorem 3.18. *If $f : X \to Y$ is an almost β-γ-continuous function and Y is semi-regular, then f is β-γ-continuous.*

Proof. Let $x \in X$ and Let V be any open set of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost β-γ-continuous. By Theorem 3.5, there exists a β-γ-open set U of X containing x such that $f(U) \subseteq G \subseteq V$. Therefore, f is β-γ-continuous. □

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