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Quantum image classification using principal component analysis

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Abstract We present a novel quantum algorithm for the classification of images. The algorithm is constructed using principal component analysis and von Neuman quantum measurements. In order to apply the algorithm we present a new quantum representation of grayscale images.

Keywords some quantum algorithms; quantum image processing; principal component analysis;

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1 Introduction

At the end of the last century a new paradigm of computation was proposed *i.e.* quantum computation. Although it is not yet obvious whether useful quantum computers can be constructed, the field of quantum algorithm development has progressed very rapidly in recent years [1], [2]. For example, many new algorithms for quantum machine learning and quantum image processing have recently been created [3, 4].

In this work we introduce an algorithm for image classification of grayscale images, based on classical principal component analysis (PCA) and quantum measurement. The general idea behind the algorithm is as follows. Given a set of training images, using PCA we train a classifier to detect the images similar to those in the training set. Effectively we divide the image signal space

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into two orthogonal subspaces. The first one — spanned by the leading principal components — catches the most of the variability of the signal in the training set, the second one consists mostly of noise.

After the classifier is constructed the leading principal components are used to create a projector onto a subspace of quantum states. The image which is being classified is also encoded on a quantum state, and then measured using the projector defined above.

The paper is organised as follows. In Sec. 2 we recall the basic notions of quantum computation. In Sec. 3 we shortly discuss state of the art in quantum image processing. In Sec. 4 we introduce the image classification algorithm. Finally, in Sec. 5, we draw conclusions.

2 Essentials of quantum computation

Let us consider the basic model of a quantum system — a qunit — i.e. a quantum system with n basic physical states. In order to provide the mathematical description of a state of a qunit we choose an orthonormal basis in the corresponding Hilbert Space. In this case we consider n-dimensional Hilbert space. Our basis will consist of n vectors that in the braket notation take the form

$$|0\rangle = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}, \dots, \ |n-1\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{1}$$

The $|x\rangle$ vector is called 'ket' and its Hermitian conjugation $(|x\rangle)^\dagger = \langle x|$ is called 'bra'. We can represent any valid state of a qubit $|\psi\rangle$ as a normalized linear combination of the basis vectors:

$$|\psi\rangle = \alpha_1|0\rangle + \dots + \alpha_n|n-1\rangle, \tag{2}$$

where $\alpha_1, \dots, \alpha_n \in \mathbb{C}$ and $\sum_{i=1}^{n} |\alpha_i|^2 = 1$. Moreover, the most basic model of a quantum state — a qubit — is a qunit with $n = 2$.

The operation which allows joining n independent qunit systems is the tensor product. Let us take n qunit states

$$|\psi\rangle = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_n \end{bmatrix} = \psi_1|0\rangle + \dots + \psi_n|n-1\rangle, \ |\phi\rangle = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix} = \phi_1|0\rangle + \dots + \phi_n|n-1\rangle. \tag{3}$$

We can write their joint state in $\mathbb{C}^n \otimes \mathbb{C}^n$ as

$$|\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} \psi_1\phi_1 \\ \vdots \\ \psi_1\phi_n \\ \psi_2\phi_1 \\ \vdots \\ \psi_n\phi_{n-1} \\ \psi_n\phi_n \end{bmatrix}. \tag{4}$$

The other way of joining quantum systems into a bigger one is by using the direct sum. The joint state of two states $|\psi\rangle, |\phi\rangle \in \mathbb{C}^n$ is

$$|\psi\rangle \oplus |\phi\rangle = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_n \\ \phi_1 \\ \vdots \\ \phi_n \end{bmatrix}. \tag{5}$$

Let $|\psi\rangle$ (ket) be a normalised column vector from Hilbert space $\mathbb{C}^n$ with orthonormal basis $\{|i\rangle\}_{i=1}^n$. Dual vector to ket is $\langle\psi|$ (bra). In such case the state of the system is represented as $|\psi\rangle = \sum_i \psi_i |i\rangle$.

We denote the inner product of $|\psi\rangle$ and $|\phi\rangle$ by $\langle\phi|\psi\rangle = \sum_{i=1}^{n} \phi_i^* \psi_i$. It has three properties:

1. $\langle\psi|\psi\rangle \geq 0$ where equality holds iff $|\psi\rangle = 0$,
2. $\langle\phi|\psi\rangle = \langle\psi|\phi\rangle^*$,
3. $\langle\psi|(a_1|\phi_1\rangle + a_2|\phi_2\rangle) = a_1\langle\psi|\phi_1\rangle + a_2\langle\psi|\phi_2\rangle$.

Furthermore, $|\psi\rangle\langle\phi| = \sum_{i=1}^{n}\sum_{j=1}^{n} \psi_i \phi_j^* |i\rangle\langle j|$ will be their outer product.

One of the most important concepts in quantum information is the measurement. The mathematical model of the measurement is as follows. First we define a finite set of outcomes Γ. Then we assign the corresponding measurement operators $\{P_\gamma\}_{\gamma\in\Gamma}$. We request that the measurement operators satisfy the condition $P_\gamma^2 = P_\gamma$ and $\sum_\gamma P_\gamma = \mathbb{I}$.

The probability that we obtain the outcome γ when measuring a state $|\phi\rangle$ is equal to

$$P_\Gamma(\gamma, |\phi\rangle) = \langle\phi|P_\gamma|\phi\rangle. \tag{6}$$

After the measurement the state of the system changes into a state

$$\frac{P_\gamma|\phi\rangle}{\langle\phi|P_\gamma|\phi\rangle^{1/2}}.$$

3 State of the art

There are various ways in which classical data can be encoded on quantum states. The specific encoding method depends on the type of data and quantum algorithms that will be executed.

3.1 Quantum representations of digital images

Below we recall four most important representations of quantum images proposed in recent years.

In the Qubit Lattice representation of grayscale images proposed in [5] the intensity of a pixel at position y, x is encoded on a qubit $|q\rangle_{y,x}$.

The Real Ket representation introduced in [6] stores $2^n \times 2^n$ grayscale images in unnormalised quantum states of the form

$$|\Psi\rangle = \sum_{i_1,\ldots,i_n=1,\ldots,4} c_{i_1,\ldots,i_n} |i_1, \ldots, i_n\rangle,$$

where $c_{i_1,\ldots,i_n} \in \mathbb{R}$ and subsequent ququarts (a ququart is a qunit with $n = 4$) serve as the position of a pixel encoded in a quad-tree.

The flexible representation of quantum images (FRQI) captures information about pixel colours and their corresponding positions. It is inspired by the pixel representation for images in classical computers. This information is encoded into a quantum state defined as follows

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} (\cos\theta_i\,|0\rangle + \sin\theta_i\,|1\rangle) \otimes |i\rangle, \tag{7}$$

where $\theta_i \in \left[0, \frac{\pi}{2}\right]$ and constitutes of the colour encoding vector, $|0\rangle, |1\rangle$ is a fixed basis of a two-dimensional complex Hilbert space, and $|i\rangle$ is a basis of 2^{2n}-dimensional space responsible for encoding the position in the image. The colour is encoded in a $2D$-vector by $\cos\theta_i\,|0\rangle + \sin\theta_i\,|1\rangle$ which is connected by a Kronecker product with a vector $|i\rangle$ responsible for a position in the image.

A novel enhanced quantum representation (NEQR) of digital images proposed in [7] encodes a grayscale $2^n \times 2^n$ image in a quantum state of the form

$$|I\rangle = \frac{1}{2^n} \sum_{y=0}^{2^n-1} \sum_{x=0}^{2^n-1} \bigotimes_{i=0}^{q-1} |C^i_{yx}\rangle \otimes |YX\rangle,$$

where C^i_{yx} is a discrete value of image intensity, quantised with q levels of quantisations of a pixel at position (y, x).

3.2 Quantum image processing algorithms

The sub-field of quantum computation that deals with algorithm development for quantum image processing has grown very rapidly. At least a hundred papers discussing this subject have been published in the last fifteen years.

It should be noted that some of classical image transformations already have their quantum analogues. For example we can mention quantum Fourier transform [8], quantum discrete cosine transform [9, 10], and quantum wavelet transform [11].

There exist several clever techniques to process the images encoded in quantum states. For example, in [12] the authors propose a way to perform template-matching algorithm using quantum Fourier transform and amplitude amplification. In paper [13] the authors extended the use of quantum circuit models for quantum image representation and processing. They developed three strategies to extend the number of geometric transformations [14] on quantum images using the FRQI representation of quantum images. In [15] the authors proposed quantum algorithms for edge detection and image filtering based on projective measurement. In [16] the authors proposed a model for storing and operating on infra-red images.

Complex quantum image processing requires a number of basic algorithmic primitives. In [17] the authors developed a quantum image translation, which maps the position of each picture element into a new position. In [18] an algorithm for comparing colour quantum images based on FRQI model is described.

4 Algorithm for quantum image classification

The aim of the presented algorithm is the classification of quantum images. The input of the algorithm is a quantum representation of an image which will be tested. The algorithm requires a set of principal components which describe a class of pictures. The output is "yes" or "no" and answers the question whether the image exhibits the features represented by the principal components.

4.1 Principal component analysis

In order to create the described quantum classifier principal component analysis (PCA) will be applied. This technique has been successfully applied to various datasets in the domain of signal processing. In celebrated classical paper [19] it was applied to the classification of human faces.

Let us suppose that we have m sample vectors from learning set $\{|S_i\rangle\}_{i=1}^{m}$. Next we calculate mean vector of these samples

$$|\bar{S}\rangle = \frac{1}{m}\sum_{i=1}^{m}|S_i\rangle, \tag{8}$$

and then we calculate normalized sample vectors $|D_i\rangle$, where $i \in \{1,\ldots,m\}$

$$|D_i\rangle = \frac{|S_i\rangle - |\bar{S}\rangle}{|||S_i\rangle - |\bar{S}\rangle||}, \ \forall i \in \{1,\ldots,m\}. \tag{9}$$

Then we obtain the matrix of data $A \in M_{m,n}$ with rank $k \leq m$. The matrix is composed of vertically stacked horizontal vectors $|D_i\rangle$, where $i \in \{1,\ldots,m\}$.

Then, by SVD we get $A = U\Sigma V^T$. where $U \in M_m$ and $V \in M_n$ are orthogonal matrices. The matrix $\Sigma = \mathrm{diag}\{\sigma_1,\ldots,\sigma_q\}$ is such that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_k > \sigma_{k+1} = \ldots = \sigma_q = 0$, with $q = \min(m,n)$.

The numbers σ_i are called *singular values*, *i.e.* non-negative square roots of the eigenvalues of AA^T. The columns of U are eigenvectors of AA^T and the columns of V are eigenvectors of A^TA. The i-th column vector of the matrix $U_{:,i}$ is called the i-th principal component of the data.

4.2 Classification model

The developed algorithm is based on the classical model of image classification. To classify some picture X we represent it as a vector $|X\rangle$. This will be input data. We denoted the set of the principal components by $\{|V_l\rangle\}_{l=1}^{s}$. We measure the likelihood M of the input data being in the control set in the following way:

$$M = \sum_{l=1}^{s} |\langle X|V_l\rangle|^2. \tag{10}$$

The measure takes values from interval $[0,1]$, where 0 is for the images orthogonal to the considered class, and 1 for the images in the class.

4.3 Goals and assumptions

The main goal is to develop an algorithm which will keep undermentioned assumptions.

1. Underlying quantum system used for the representation of pictures dimensions grows linearly with picture size.
2. Image representation has to provide the possibility of manipulation on individual pixels without global changes or state renormalization.
3. Local image disturbances cannot cause significant change to the quantum representation state in the terms of image similarity measure.
4. The developed algorithm should allow the adaptation of classical techniques. Because of that fact we aim to achieve the linear dependency between classical inner product and inner product on considered quantum representation space.
5. The classification test should be feasible with the use of a measurement. Because of that quantum representation of principal components should provide the possibility of creating orthogonal projectors that can be used to construct a measurement.

In the following parts of this section we introduce quantum representation of pictures and image classification algorithm based on the above assumptions.

4.4 Quantum image representation

Suppose we have a normalized image vector of n values $|X\rangle = \{x_i\}_{i=1}^{n}$, where $x_i \in [0,1]$. The quantum system encoding the data from the image space will be a direct sum $\mathcal{H} = (\mathbb{C}^k)^{\oplus n}$. Quantum representation $|\Phi(X)\rangle \in \mathcal{H}$ of a picture $|X\rangle$ will be a mapping from $[0,1]^n$ to $\mathcal{H}$ defined by

$$|\Phi(X)\rangle = \frac{1}{\sqrt{n}} \bigoplus_{i=1}^{n} |\phi(x_i)\rangle, \tag{11}$$

where pixels are represented by

$$|\phi(x_i)\rangle = x_i|0\rangle + \sqrt{1-x_i^2}|1\rangle. \tag{12}$$

Using direct sum in this representation ensures that the developed algorithm keeps the first assumption.

We will construct the quantum representation of vectors from PCA in the same way. Let $\{|V_l\rangle\}_{l=1}^s$ be a set of principal components with values $v_{l,i} \in [-1,1]$ and $\{|\Phi(V_l)\rangle\}_{l=1}^s$ be a set of their quantum representations. Because we need to keep orthogonality of principal components the representation of each of them is encoded on different 2—dimensional subspace of $\mathbb{C}^k$. The spaces intersection is an axis spanned by $|0\rangle$. Each pixel of j-th component is represented by

$$|\phi(v_{j,i})\rangle = v_{l,i}|0\rangle + \sqrt{1-v_{l,i}^2}|j+1\rangle, \tag{13}$$

where $j \in \{1,2,3,\dots,s\}$ and i is a pixel index. The whole principal component representation is composed of the pixel representations in the same way as in (11). Let us take two vectors $|V_j\rangle$, $Eq.|V_l\rangle$ and their quantum representation $|\Phi(V_j)\rangle$, $|\Phi(V_l)\rangle$. The inner product of $|V_l\rangle$ and $|V_j\rangle$ is

$$\langle V_l|V_j\rangle = \sum_{i=1}^{n} v_{l,i}^* v_{j,i}, \tag{14}$$

and for corresponding quantum representations one reads

$$\begin{aligned}
\langle\Phi(V_l)|\Phi(V_j)\rangle &= \frac{1}{\sqrt{n}}\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\langle\phi(v_{l,i})|\phi(v_{j,i})\rangle \\
&= \frac{1}{n}\sum_{i=1}^{n}(v_{l,i}^* v_{j,i}\langle 0|0\rangle + \sqrt{1-v_{j,i}^2}v_{l,i}\langle 0|j+1\rangle + \\
&\quad + v_{j,i}^*\sqrt{1-v_{l,i}^2}\langle l+1|0\rangle + \sqrt{1-v_{l,i}^2}\sqrt{1-v_{j,i}^2}\langle l+1|j+1\rangle) \\
&= \frac{1}{n}\sum_{i=1}^{n} v_{l,i}^* v_{j,i},
\end{aligned} \tag{15}$$

where the first equality is from Eq. (11) and the last one is implied by orthonormality of the basis vectors. From Eq. (14) and Eq. (15) we derive that the inner product of two vectors is equal to the inner product of quantum representations of these vectors with respect to a constant factor $1/n$. This is an important feature of the introduced representation, which is significant for the algorithm.

4.5 Construction of measurement

The quantum algorithm for image classification is based on classical methods for determining the characteristic subspace of the data set in the featured space. Thus we take s principal components $\{|V_l\rangle\}_{l=1}^s$ that describe the data set crucial properties. The quantum algorithm for image classification will utilise the system $\mathcal{H}$ defined in the previous section. In order to use the classically computed components in the quantum algorithm we need to convert our principal components into the quantum representation $\{|\Phi(V_l)\rangle\}_{l=1}^s$.

The developed algorithm is based on the quantum measurement scheme. We consider two-element output set $\Gamma = \{\text{yes}, \text{no}\}$. The first of the resulting labels will correspond to the principal components subspace and the other one – to the rest of the image space. Thus we create two measurement operators Π and $\mathbb{1} - \Pi$. The principal component projection operator Π is of the form

$$\Pi = \sum_{l=1}^{s} |\Phi(V_l)\rangle\langle\Phi(V_l)|. \tag{16}$$

Projector Π corresponds to output "yes" and $\mathbb{1} - \Pi$ corresponds to output "no".

4.6 Measurement probabilities

Let $|X\rangle$ be an input image vector and $\{|V_l\rangle\}_{l=1}^{s}$ the set of the principal components.

Now let $|\Phi(X)\rangle$ be a quantum representation of the input and $\{|\Phi(V_l)\rangle\}_{l=1}^{s}$ be a quantum representation of principal components with projector Π constructed as in Eq. (16). Then the probability of the result of the measurement being "yes" for a given input is

$$P_\Gamma(\text{yes}|X) = \langle\Phi(X)|\Pi|\Phi(X)\rangle = \sum_{l=1}^{s} |\langle\Phi(X)|\Phi(V_l)\rangle|^2 = \frac{1}{n^2}M, \tag{17}$$

where the last equation results from Eq. (14) and Eq. (15). Thus the probability $P_\Gamma(\text{yes}|X)$ is linearly dependent on the classical likelihood measure M (calculated in Eq. (10)) with respect to a factor $1/n^2$, where the last equation is from Eq. (14) and Eq. (15). Thus the probability $P_\Gamma(\text{yes}|X)$ is linearly dependent on the classical likelihood measure M with factor $1/n^2$.

Because of the factor $1/n^2$ we perform n^2 tests. We assume that we have n^2 copies of the quantum representation of the vector $|X\rangle$. We perform the measurement Π on each of the copies. If any of the measurements returns "yes" then the algorithm returns positive answer. If not, the answer is negative. The probability that the algorithm will return the output "no" for a given input vector $|X\rangle$ is equal to

$$P_{\Gamma,n^2}(\text{no}|X) = (1 - P_\Gamma(\text{yes}|X))^{n^2}. \tag{18}$$

The probability of positively classifying the input image in most of the cases is close to the classical likelihood measure. In general the probability is slightly lower. Thus the algorithm trifle favors the negative answer. If the result is "yes" we can express the state after the measurement as:

$$|\Phi_{\text{yes}}(X)\rangle = \frac{\Pi|\Phi(X)\rangle}{\sqrt{\langle\Phi(X)|\Pi|\Phi(X)\rangle}},$$

if the result was "no" we can express it as:

$$|\Phi_{\text{no}}(X)\rangle = \frac{(\mathbb{1} - \Pi)|\Phi(X)\rangle}{\sqrt{\langle\Phi(X)|\Pi|\Phi(X)\rangle}}.$$

The pictures which can be obtained from decoding the state after the measurement will be defined as output pictures. By decoding we mean the operation opposite to the encoding from Eq. (11) and (12) i.e. we take subsequent values x_i from the first coordinate of $|0\rangle$ and consider them as the values of pixels of the output picture.

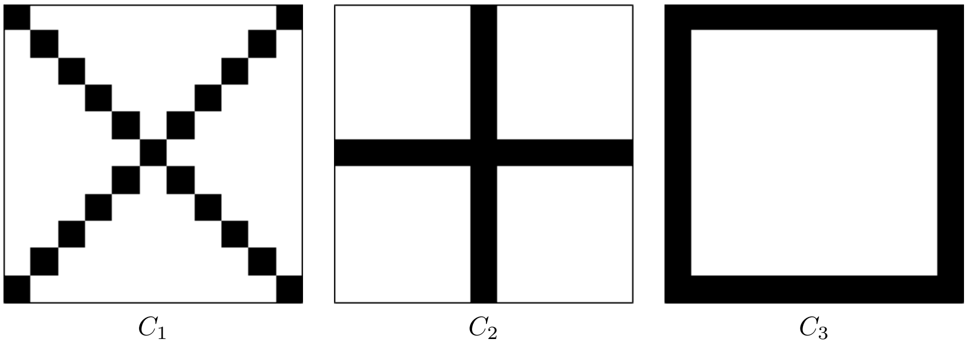


Figure 1 Pattern pictures, 11×11 pixels.

5 Example

In order to demonstrate the presented method we consider some examples. We will focus on the classification of black-white images. First, we construct a learning set and determine principal components which describe the data set. Next, we perform the results of numerical tests on three sample pictures.

5.1 Construction of the learning set

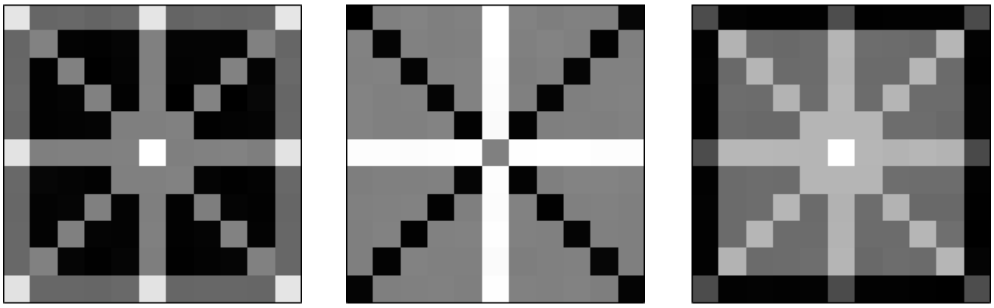
Suppose we have three pattern pictures presented in Fig. (1). Our aim is to check if some testing picture has common features with pictures $C_1,\ C_2,\ C_3$. The algorithm for PCA requires huge amount of pictures which will be used as a learning set. In this example we create the learning set based on pattern pictures $C_1,\ C_2,\ C_3$. The image from the learning set is constructed from a pattern picture as follows. Firstly, a random number with uniform distribution from interval $[0, 2/5]$ has added to every pixel. Secondly, the picture has been normalized. From each pattern picture we create 250 pictures, so the learning set consists of 750 pictures. From these pictures we create the vectors which are next vertically stacked to matrix A. This matrix is used as an input for the PCA algorithm (see Sec. 4.1).

We take three principal components which correspond to the largest singular values and transform them into the quantum representation described in Sec. 4.4. The resulting components are represented in Fig. (2). In the final step we construct the projector Π, according to Eq. (16).

5.2 Test data

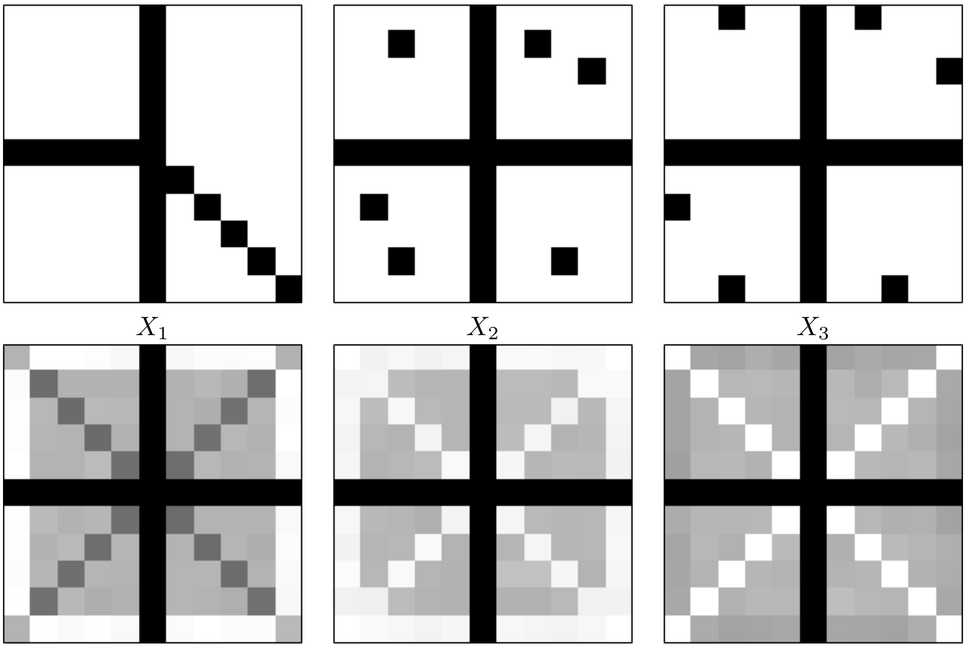
Suppose we have three images $X_1,\ X_2,\ X_3$ which we want to test separately (Fig. 3). In order to use the quantum image representation we need to normalize images $X_1,\ X_2,\ X_3$ and represent them as vectors. As in Sec. 4.4 we construct the quantum representation for each of the pictures $(|\Phi(X_1)\rangle,\ |\Phi(X_2)\rangle,\ |\Phi(X_3)\rangle)$.

In the Table 1 we present the probabilities that the algorithm returns "yes" after the measurement on one copy for each testing picture: $P_\Gamma(\text{yes}|X_i)$, where $i \in \{1, 2, 3\}$. The probabilities that the algorithm returns "yes" if we perform the measurement on 121^2 copies: $P_{\Gamma,121^2}(\text{yes}|X_i)$,



first principal component second principal component third principal component

Figure 2 First three principal components.



X_1 X_2 X_3

OX_1 OX_2 OX_3

Figure 3 Upper row: input pictures. Lower row: output pictures.

where $i \in \{1, 2, 3\}$.

Output pictures from the algorithm are presented in Fig. (3). As we can see in Fig. (2), the compliment of these three pattern pictures is distinguished in the first principal component. This explains why this compliment is visible on the output pictures.

6 Concluding remarks

In this paper we have provided a new quantum representation of digital images and the algorithm for classification of the images. The classification is performed by applying the measurement

i	$P_\Gamma(\text{yes}\|X_i)$	$P_{\Gamma,121^2}(\text{yes}\|X_i)$
1	4.0987e-05	0.45124
2	5.4741e-05	0.55133
3	5.3351e-05	0.54211

Table 1 Second column: probabilities for one copy of image X_i, where $i \in \{1,2,3\}$. Third column: probabilities for 121^2 copies of picture X_i, where $i \in \{1,2,3\}$.

apparatus on the quantum states that represent input images. The measurement is performed on multiple copies of the image sequentially. The introduced quantum representation allows the transformation of some properties from the classical model of algorithms because of linear dependency between inner products. Moreover, the quantum representation of principal components enables defining orthogonal projectors needed for the measurement construction. Therefore, the paper provides a complete system for quantum classification of digital images.

In the future work we will try to harness the possibility of manipulation on individual pixels without global changes or state renormalization in order to improve the possibilities of classification. Furthermore, we will consider other classification techniques such as linear discriminant analysis (LDA). Moreover, we will modify the algorithm in order to limit the number of used copies.

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