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Compressive Sensing of Sparse Signals and MR Images

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Abstract - The sparsity of a signal means that it can be represented by a small number of non-zero coefficients in a certain basis. The reconstruction of a sparse signal can be done from sub-Nyquist samples by using nonlinear optimization, which is known as "compressive sensing (CS)". CS is a very promising technique in wide range of areas e.g.wireless healthcare systems, medical imaging such as MRI since CS enables low-power and cost-effective data processing. Most of these applications have been possible since the real world signals such as sound, image are inherently sparse. In this work, a Graphical User Interface (GUI) is developed in Matlab which can be used to do CS based reconstruction of sparse signals and MR images. This program is devoted to the scientists and researchers who desire to explore the quality of reconstruction from sub-Nyquist data and the effect of the parameters in the algorithm based on CS.

Keywords *– compressive sensing (CS), Matlab graphical user interface (GUI), magnetic resonance imaging (MRI), sparsity*

1. INTRODUCTION

In recent years, studies about reconstruction of the signal from sub-Nyquist samples has been very attractive. The technique called "compressive sensing" enables the acquisition of a signal with fewer measurements than the Nyquist theorem requires. Compressed Sensing was introduced in 2006 by two groundbreaking papers, by Donoho [1] and by Cand`es, Romberg, and Tao [2]. Since then, there are lots of applications in a wide range of areas such as electronics engineering, applied mathematics and computer science. The Nyquist Theorem is that if all the knowledge about a low-pass signal is its bandwidth, then in order to reconstruct the signal from its samples, the sampling rate should be slightly more than twice its bandwidth. However, if the signal that we want to reconstruct has some special properties, in more technical terms if we know that the signal is sparse then "compressive sensing" technique says that we can reconstruct it from fewer samples by using nonlinear optimization. The sparsity of a signal is that the signal can be represented by a small number of non-zero coefficients in a certain basis. If a signal is can be well-approximated by a sparse signal then these kind of signals are called as compressible signals.

There are lots of advantages of compressive sensing (CS). Obviously, high data rate A/D converters are computationally expensive and require more storage space. When there are limitations on the number of data capturing devices, measurements are very expensive or slow to capture such as in radiology and imaging techniques via neutron scattering, CS is very advantageous [13]. Besides, there are major applications of compressive sensing in diverse fields, ranging from image processing to gathering geophysics data. Most of this has been possible because of the inherent sparsity of many real world signals like sound, image, video etc [13].

The mathematical background of Compressed Sensing technique is given in [1] and [2]. In [1], the author asks why people spend so much effort to acquire all data when most of what they get will be thrown away, and whether one can just directly measure the part that will not end up being thrown away or not. The answers to those questions led to the technique called Compressed Sensing. Compressive Sensing Protocols allow a sensor to very efficiently get the information in a sparse signal without trying to comprehend the signal. CS is a very promising technique for advanc-

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ing devices such as "Analog-to-Information" (A/I) conversion of High-Bandwidth signals [3]. CS is used in image reconstruction for MRI in [4]. In this work, significant scan time reduction is achieved which is desirable for patients and health care economics.

In [5], the potential of CS is investigated for real time energy efficient ECG Compression on Wireless body Sensor Networks (WBSN). WBSN have very significant role on next-generation patient-centric telecardiology or mobile cardiology solutions. Since energy efficiency is very crucial, CS based ECG compression can a promising technology for WBSN-based ECG monitoring systems. The application of CS is considered for ECG and EMG biosignals in [6], in order to reduce the data rate to have ultra-low-power performance. Wireless pulse wave (PW) signal acquisition based on CS scheme is proposed and energy efficiency is evidenced by practical experiments on MICAz node in [11]. The EEG compression based on CS framework is shown to be up to eight times more energy efficient than the typical wavelet compression method in terms of compression and encoding computations and wireless transmission [14]. The CS framework mentioned in [14] is applicable to a wide range of EEG signal types, and that method is claimed to be robust to measurement noise and to packet loss. In [10], it is shown that especially for power hungry sensors, the compressed sensing based techniques can provide greater energy efficiency than transform coding and adaptive sensing in wireless sensor networks.

For real-time CS based signal reconstruction, hardware realizations are done on FPGAs in [7], [8], and [12]. The article given in [13] is a survey about Compressed Sensing, it gives information about wide range of applications with a major emphasis on communications and network domain.

In this work a Graphical User Interface is developed using MATLAB. This standardized GUI can create sparse signals, import signal data and MR images. It computes the CS based reconstruction of the related signal and shows to the user. The technical background of CS is reviewed in Section 2. Section 3 explains the software which reconstructs signals and MR images and give examples of different applications of CS using the developed GUI. In the final part, Section 4 gives the conclusions that are drawn from this work.

2. THEORY

This section is based on Gitta Kutyniok, "Theory and Applications of Compressed Sensing", GAMM-Mitteilungen, 10 July 2013. The detailed technical information can be found in [15].

Firstly, it is assumed that x itself is sparse or that there exists an orthonormal basis or a frame Φ such that $x = \Phi c$ with c being sparse. Let A be an mxn matrix, which is typically called sensing matrix or measurement matrix. For the compressed sensing problem, it will be assumed that $m < n$ and that A does not have any zero columns. Then the compressed sensing problem can formulated as:

- Recover x using the knowledge of $y = Ax$ or
- Recover c using the knowledge of $y = A\Phi c$

First approach to recovering x from $y = Ax$ is by solving:

$$\min_x \|x\|_0 \quad subject \quad to \quad y = Ax \tag{1}$$

This algorithm is NP-hard and it is computationally intractable. The main idea of CS is to relax the reconstruction to a convex optimization problem, in other words substitute the l_0 'norm' by the closest convex norm, which is the l_1 norm. This approach is also known as Basis Pursuit:

$$\min_x \|x\|_1 \quad subject \quad to \quad y = Ax \tag{2}$$

For solving this problem, convex optimization, greedy, and combinatorial algorithms are proposed, each one has its own advantages and disadvantages.

To summary, CS has basically needs three requirements:

- Sparsity or Transform Sparsity: The signal that will be reconstructed should be sparse, or it has a sparse representation in a known transform domain (e.g Fourier domain, Wavelet domain, etc.)
- Incoherence of Undersampling Artifacts: The artifacts in the reconstruction caused by undersampling should be incoherent (noise-like) in the sparsifying domain.
- Nonlinear reconstruction: The signal should be reconstructed using nonlinear methods which enforces both

sparsity of the signal representation and consistency of the reconstruction with the acquired samples [4].

As it is explained above, the theory of compressed sensing suggests random undersampling. To understand the reason behind this situation, in this part equispaced undersampling will be investigated and it will be compared with the random undersampling. A 1x128 vector is generated with 3 nonzero coefficients and permute them randomly. As it is obvious, the signal is sparse. The original sparse signal is shown Figure 1 Sparse Signal with 3 nonzero coefficients.

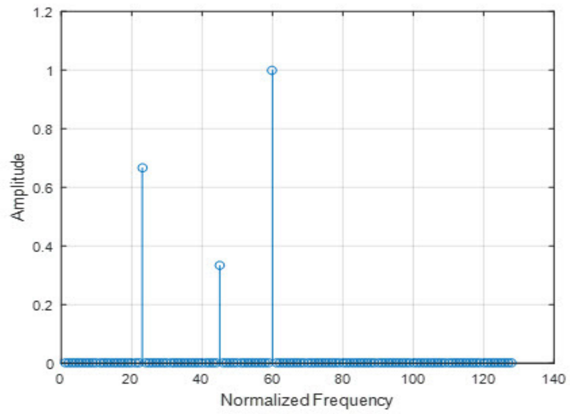

Figure 1 *Sparse Signal with 3 nonzero coefficients*

If the sparse signal is undersampled by taking 32 equispaced samples in time domain (note that Figure 1 Sparse Signal with 3 nonzero coefficients shows frequency domain data), and after that the Fourier transform is computed by filling the missing data with zeroes. The plot of the absolute value of the result is shown in Figure 2 Fourier Transform is computed from 32 equispaced samples (sub-Nyquist). As it is seen, there is ambiguity and the original signal cannot be able to reconstructed from the result.

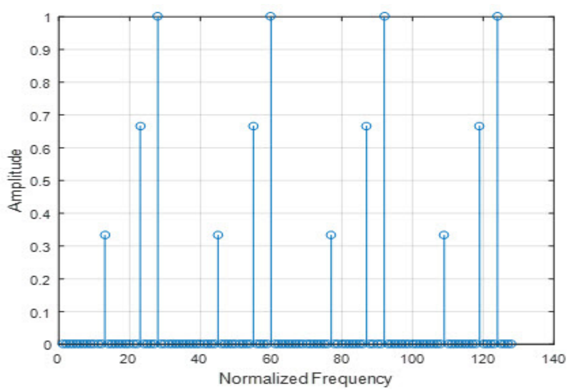

Figure 2 *Fourier Transform is computed from 32 equispaced samples (sub-Nyquist)*

The time domain data can be undersampled by taking 32 samples at random, and then the zero-filled Fourier transform is computed. The absolute value of the result is plotted in Figure 3 Fourier Transform is computed from 32 random samples (sub-Nyquist). As it is seen, by doing undersampling **randomly**, the Fourier transform is like noisy signal. Therefore, the ill-conditioned problem is turned into a sparse signal denoising problem. But "that noise" is not a really noise, it is caused by incoherent aliasing. As a result, the sparse signal can be recovered exactly.

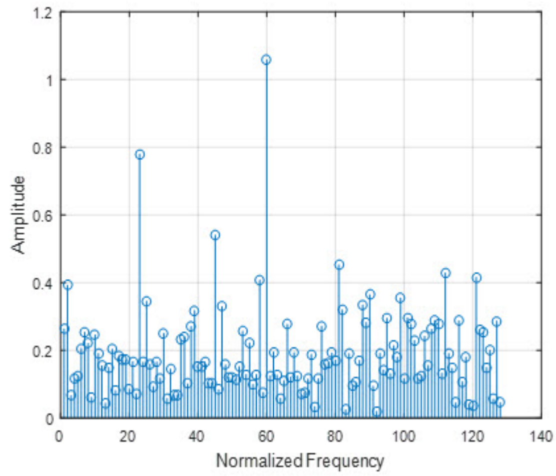

Figure 3 *Fourier Transform is computed from 32 random samples (sub-Nyquist)*

In this part, the randomly sampled data is used to reconstruct the sparse signal. To do this, the equation given below is solved:

$$\arg\min \frac{1}{2}\left\|F_u\hat{x} - y\right\|_2^2 + \lambda\left|\hat{x}\right|_1 \qquad (3)$$

In this equation $\hat{x}$ is the estimated signal, $F\hat{x}$ is the undersampled Fourier transform of the estimate, y are the samples of the Fourier transform that is acquired. There is no closed form solution for this problem. Because of this situation,it will be solved iteratively by applying soft-thresholding, considering the data consistency. The implemented Matlab code for this part is based on Projection Over Convex Sets (POCS) type algorithm.

As it is shown in Figure 5 Reconstructed sparse signal from 32 randomly selected undersampled data, the recovery of the sparse signal from randomly selected undersampled data is quite satisfactory.

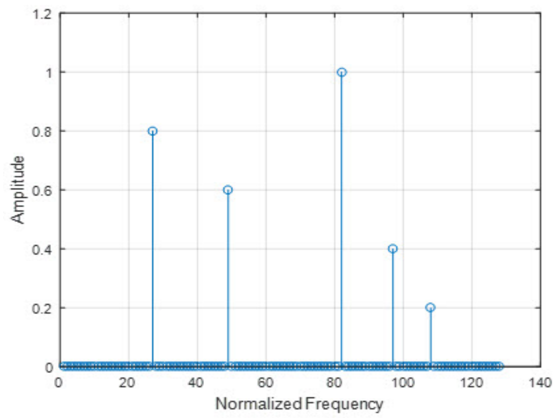

Figure 4 *Sparse signal with 5 nonzero coefficients*

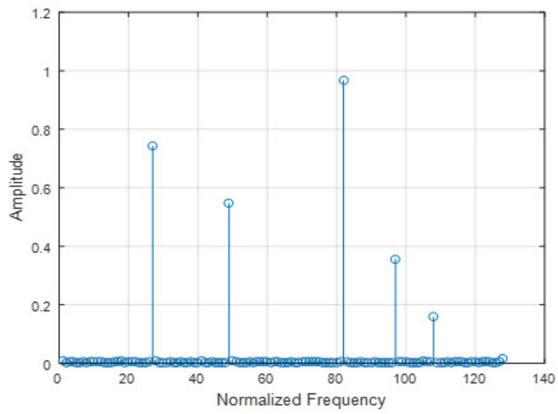

Figure 5 *Reconstructed sparse signal from 32 randomly selected undersampled data*

3. MATLAB GUI AND RELATED WORK

The Matlab GUI which analyzes the reconstruction of signals and MR images with using Compressive Sensing is developed as a result of this work. The related work can also do the equispaced sampling in order to compare it with Compressive Sensing which requires random sampling for signals. The general showing of the GUI is shown below:

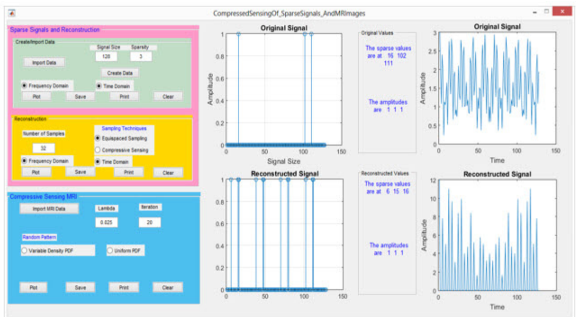

Figure 6 *The image of the related MATLAB GUI analyzing CS*

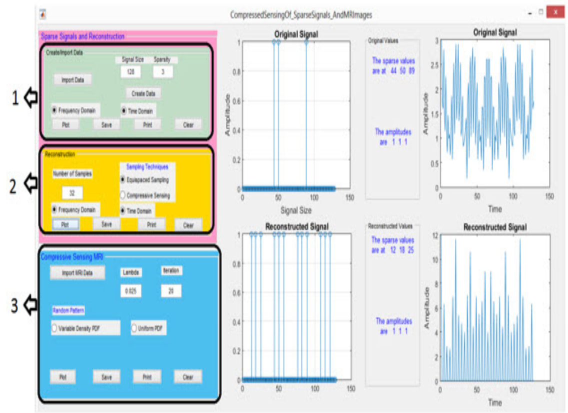

Figure 7 *The MATLAB GUI and its parts*

This GUI is very easy to use and can handle the following instructions. The part that is shown as 1 is related to sparse signal creation. This part can do the instructions given as:

- Import 1D signals
- Create 1D signals by specifying signal size and sparsity
- Save the created 1D data
- Plot the time and frequency response of the 1D signal
- Measure and give the amplitude and frequency information about 1D signal
- Print out the figures about 1D signal
- Clear the figures and the related data about 1D signal

The part that is shown as 2 is reconstruction of sparse signals. This part can do the instructions given as:

- Choose one of the sampling techniques and specify the number of samples that will be used in reconstruction
- Save the reconstructed 1D data
- Plot the time and frequency response of the reconstructed signal
- Measure and give the amplitude and frequency information about reconstructed signal
- Print out the figures about the reconstructed signal
- Clear the figures and the related data about reconstructed signal

The last part that is shown as 3 is related to MR image processing part. This part is independent from first and second parts. The instructions given below can be done using this part:

- Import MR image data
- Plot the image of the original MR

data
- Give pixel information about the original data
- Choose one of the random 3-fold undersampling patterns such as Variable Density PDF and Uniform PDF
- Specify the iteration number and lambda which is related to Compressive Sensing scheme
- Plot the image of the CS-reconstructed MR image
- Give the information about number of pixels used for reconstruction
- Plot the difference image of CS-reconstructed from the original image
- Save the reconstructed MR image
- Print out the reconstructed images
- Clear the related data and figures about images

An example of a simulation with created signal is shown below. User can import or create signal data. To create data, user should enter the signal size as a vector and sparsity. This denotes a sparse signal in frequency domain. By pushing create data, the user gets the signal. If he/she desires to plot the frequency domain and time domain respresentation, he/she should select the related radiobuttons and click the “Plot” pushbutton.

In the Reconstruction part, The user specifies the number of samples that will taken while doing reconstruction and he/she should select one of the Sampling Techniques: Equispaced Sampling or Compressive Sensing. He/She can see the related frequency and time domain figures by selecting the related radiobuttons and pushing “Plot” as it is explained in Creating data section. In this simulation given in Figure 8 The Reconstruction with Equispaced Samples, Equispaced Sampling is selected. As is seen from frequency and time domain responses the reconstruction is not successful, there is aliasing and ambiguity.

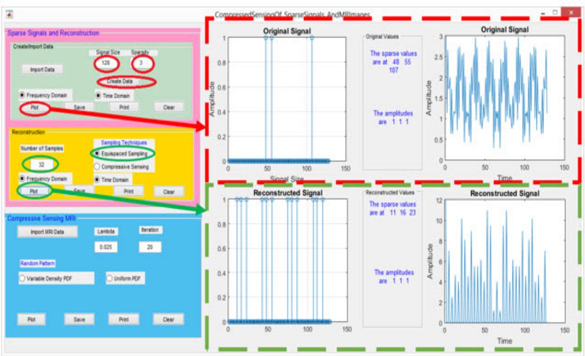

Figure 8 *The Reconstruction with Equispaced Samples*

If the simulation is repeated with Compressive Sensing selection, the reconstruction is quite successful. The screenshot of one such simulation is shown below in Figure 9 The Reconstruction with Compressive Sensing Algorithm.

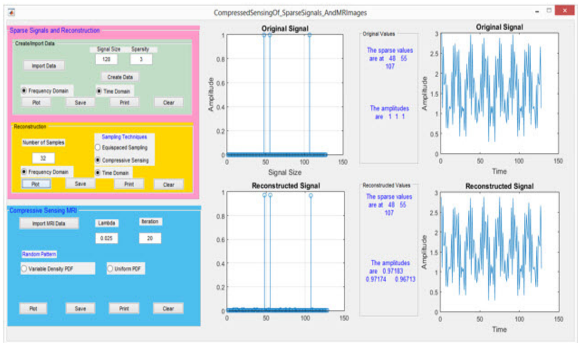

Figure 9 *The Reconstruction with Compressive Sensing Algorithm*

By using Compressive Sensing MRI part of the GUI, the user can import MRI data, see its image and do compressive sensing. The user specifies the parameter lambda mentioned in Eq (3) and the number of iterations. There is a tradeoff between iterations and time. When user does more iterations then he/she needs more time to get the reconstructed image, but the quality of the reconstructed image will be better. In the first simulation, Variable Density PDF is selected. 20 iterations are done with POCS algorithm for 2D images and $\lambda = 0.025$ .As it is seen in the difference image the reconstruction is quite satisfactory. In the reconstructed MRI image, the algorithm uses 33.3% of the original data, but there is no big difference with original image as difference image plot shows.

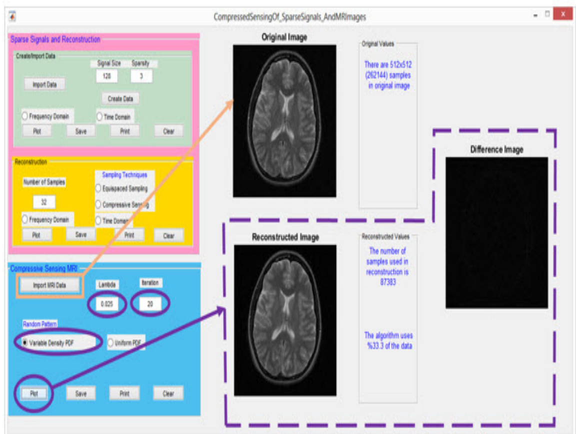

Figure 10 *The Reconstruction of MR image with with Variable Density PDF*

However, if the simulation is repeated with the same iteration number and same but with Uniform PDF, the reconstruction is not satisfactory. The result is shown below:

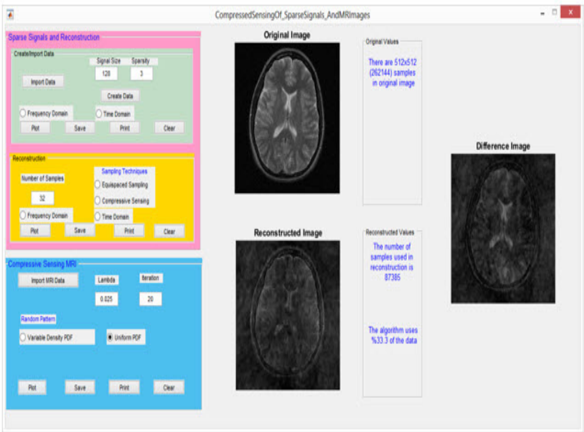

Figure 11 *The Reconstruction of MR image with Uniform PDF*

4. CONCLUSIONS

Compressed sensing is a novel sampling issue which has gained a lot of attention in recent years. Contrary to Nyquist sampling paradigm, CS tells that sparse signals can be reconstructed from far fewer samples than Nyquist rate requires. Basically, CS needs sparsity of signals in some transform domain and the incoherency of these measurements with the original domain. Although CS paradigm is recent, there are lots of applications in diverse fields [13]. The reason behind this situation is that many real world signals like sound, image, video are approximately sparse naturally. Some headlines of the applications can be pointed as: Compressive Imaging, Medical Imaging, Seismic Imaging, Compressive RADAR, Analog-to-Information Converters and Applications in Communications and Networks. In this work, we have developed a MATLAB GUI which analyzes CS scheme for 1D signals and MR images. The related GUI is very user friendly, and the researchers who wants to explore CS scheme and see whether the reconstruction is successful or not can utilize it. The clinicians that desires to use CS for MRI in future can make researches to see if the reconstruction is satisfactory to make diagnosis. Besides, the effects of parameters such as iteration, and random sampling pattern to CS can be investigated using this MATLAB GUI.

5. REFERENCES

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