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Comparative Study of Universal Function Approximators (Neural Network, Fuzzy Logic, ANFIS) for Non-Linear Systems

Hamit Erdem, Institute of Science and Engineering, Baskent University, Baglıca Kampusu, Ankara, TURKEY
E-mail: herdem@baskent.edu.tr
Ali Berkol, Institute of Science and Engineering, Baskent University, Baglıca Kampusu, Ankara, TURKEY
E-mail: ali.berkol@yahoo.com
Mustafa Sert, Institute of Science and Engineering, Baskent University, Baglıca Kampusu, Ankara, TURKEY
E-mail: msert@baskent.edu.tr

Abstract - The Fuzzy Logic (FL), Artificial Neural Networks (ANN) and Adaptive Neuro Fuzzy Inference System (ANFIS) are known as universal function approximator which has been used in various applications. In general, a function approximator needs to select a function or a mapping algorithm among the well-defined methods that closely capture the input – output variables relation. This study compares the application of aforementioned artificial intelligence approximators by using two non-linear functions. The curve-fitting capability of approximators has been compared considering three main metrics. These metrics are; fitting accuracy (Root Mean Square Error (RMS)), memory occupation, program code size and running time. Additionally, the parameters which affect the performance of each system have been investigated in details. The entire analysis has been developed and accomplished by MATLAB.

Keywords *– ANFIS, Function Approximation, Fuzzy Logic, Neural Network, Non- Linear Systems.*

1. INTRODUCTION

The significance of Function Approximation (FA) becomes obvious in the wide spectrum of computational activities for modelling and analysis ranging from applied mathematics to soft computing. Function Approximation is a method of generalization in such a way that it specifies a function in an aim to develop a representative function being approximate to the target function.

Soft computing or artificial intelligence methods have been widely used for nonlinear modelling and curve fitting or function approximation. From these intelligence methods or algorithms, Fuzzy Logic, ANN and ANFIS are the most well-known methods which have been used in engineering and applied mathematics. FL tries to simulate human brain from the point of view the software and ANN as an artificial brain imitates the human brain's hardware. Each of these two methods has advantages and disadvantage regarding to training, rule and structure definition. Also these methods have been used for control, pattern recognition signal processing, time series analysis and etc., one of the common application areas for these methods is curve fitting or function approximation. Fuzzy Logic's capability for modelling of a single input-single output (SISO) system has been investigated in [1]. Another study shows that, Fuzzy logic as member of approximate reasoning and soft computing branch can fit a nonlinear curve with imprecise data to determine acceptable outputs [2]. Considering ANN, an earlier study [3] has investigated the application ANN for function approximation. Using different model of ANN, in [4] a radial based function ANN has been applied for function approximation. In later studies hybrid neuro-fuzzy system has been investigated in many researches.

Adaptive Neuro-Fuzzy Inference System which combines the advantages of the Fuzzy Logic has been used in many engineering applications. For example, the application of ANFIS has been applied to a nonlinear robotic system [5]. In order to compare the curve fitting capability of these systems, in [6] two ANN methods, fuzzy and ANFIS have been used for function approximation. The study supports experimental implementation, but the generated code size code size

Corresponding Author
Hamit Erdem, Institute of Science and Engineering, Baskent University, Baglıca Kampusu, Ankara, TURKEY
E-mail: herdem@baskent.edu.tr

and run time has not been compared. The applied methods have been compared regarding to curve fitting accuracy due to RMS of the sampled error. Considering above studies, the proposed study tries to compare ANN, Fuzzy Logic and ANFIS for function approximation considering function approximation error, code size, and run time. Two nonlinear functions have been selected and performance of the applied methods has been evaluated using MATLAB software.

This paper is organized as follows: soft computing methods for function approximation (2), a simulation and comparison with experimental data is shown in section (3), conclusions are presented in section (4).

2. SOFT COMPUTING METHODS FOR FUNCTION APPROXIMATION

Previous studies show that, the most used soft computing function approximations are Fuzzy Logic, ANN and ANFIS. Each method has its own structure, for example Fuzzy Logic is a rule based method and present human knowledge for system modeling and control. On the other hand, ANN tries to simulate human brain containing artificial neurons. The ANFIS structure combines the advantages of the two methods. Structures of these methods for SISO modeling or function approximation have been given in figures 1, 2.

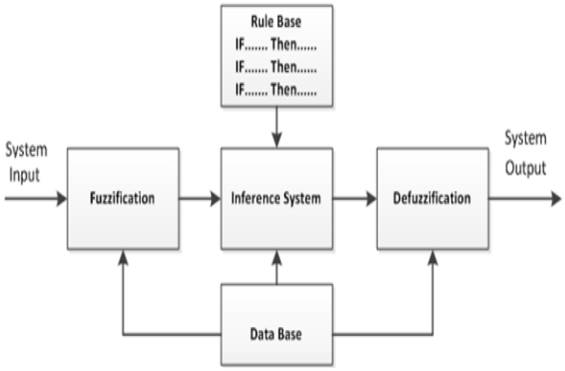


Figure 1. *Fuzzy Logic Architecture for SISO Function Approximation*

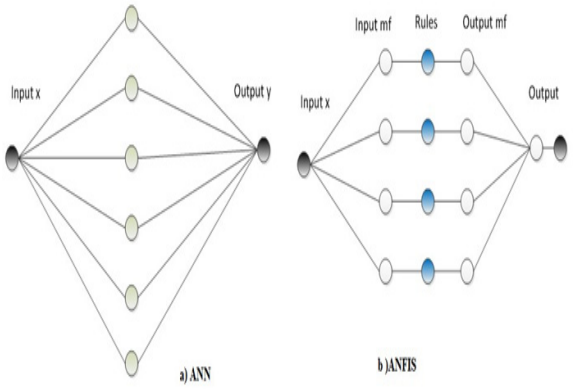


Figure 2. *Structures of ANN (a) and ANFIS (b) for SISO function approximation*

2.1. Fuzzy Logic Systems

Fuzzy logic (in Fig.1) idea is similar to the human being's feeling and inference process. Unlike classical modelling and mapping algorithms, which is a point-to-point and mathematical based, FL is a range-to-point or range-to-range using human knowledge. The output of a fuzzy controller is derived from fuzzifications of both inputs and outputs using the associated membership functions and expert defined rules. A crisp input will be converted to the different members of the associated membership functions based on its value. From this point of view, the output of a fuzzy logic controller is based on its memberships of the different membership functions, which can be considered as a range of inputs. FL uses verbal (linguistic) variables and an expert defined rule to model or relate the input-output variable. As given in figure 1, Fuzzy logic system contains 4 main blocks which are fuzzification, inference system, defuzzification and data base. Fuzzification block converts real values to fuzzy values by defining membership's values and defuzzification section converts the result of fuzzy processing to a real value [7]. Considering function approximation problem, the performance of FL is depended on defended membership function and rules.

2.2. Artificial Neural Network

ANN as one of the most used soft computing or intelligent systems has been used in many application area. ANN can be used for classification, clustering, function approximation. ANN acts like a black box and tries to imitate the behavior of interconnected electro-chemical neuron. Although various ANN

structure have been used in many application, multilayer feedforward ANN is the most used ANN structure [8]. The main concept in using of an ANN is learning and generalization capabilities. Different from the FL which suffers from learning or generalization and needs expert knowledge, ANN need data about the proposed system. AS presented above, function approximation, cure fitting, or mapping can be provided using multi-layer perceptron structure. The performance of the multilayer feedforward ANN depends on numbers of hidden layer and how many samples have been used for learning. For applying ANN tot a problem, the following step must be considered

- Selecting sufficient and sampled learning examples from the hole of input-output data
- Selecting of hidden layers and related neurons
- Selecting of activation function, leaning algorithm and learning rate
- Dividing examples to train and test sets

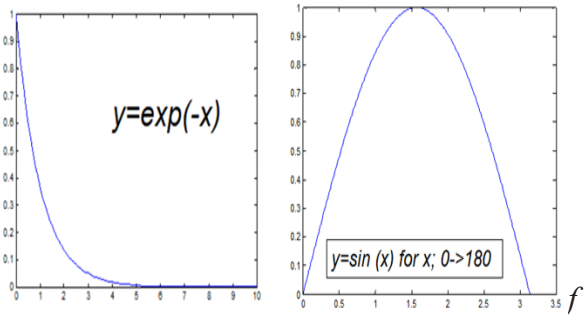
Following the selecting ANN structure and related parameters, the train stage can be started. Batch (offline) or online learning can be applied considering the advantages of each method, and difference between two methods is regarding to weigh updating approach which can be changed after each sample or each epoch. The learning process of ANN is an optimization problem. Cost function or objective function of this optimization problem is RMS of model error. Reaching to desired minimum error, the learning step can be terminated. Another method for end of learning is generalization test during learning step.

2.3. ANFIS

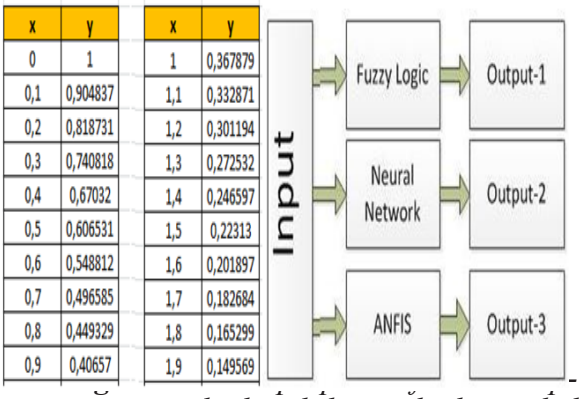
Referred to ANFIS, adaptive neuro fuzzy inference system or adaptive network-based fuzzy inference system represents a class of adaptive networks based on Takagi-Sugano type fuzzy model (Fig. 3). In 1993, Jang embodied both fuzzy techniques and neural networks in a single framework of a hybrid intelligent system by taking advantage of their best features. Its inference system constructs an input-output mapping in the form of simple fuzzy if-then rules based on linguistic expressions as data pairs. ANFIS contains the learning ability ANN and each step of ANFIS structure represents one step of FL. General structure of ANFIS is composed of five distinctive layers being built into Sugeno type fuzzy system with two inputs, x1 and x2, and one input namely ∑. It implements a feed forward inference model of systematic approach in generating fuzzy rules from a given input-output dataset. During the training, the position of membership function, rule numbers and output function coefficients has been changed [9-10]. A more simple structure of ANFIS for a SISO system which given in figure 2 has been used in the proposed study.

3. PROPOSED SIMULATION STUDY

In order to compare the suggested soft computing methods two nonlinear simple mathematical function has been selected, these function are ***y=exp(-x)*** and ***y=sin (x) for x; 0->180*** which graphical representation has been given in figure-3. Additionally some sampled data and the proposed system methodology has been given in figure-4. To apply the simulations, firstly datasets should be generated for functions. Secondly, the structure of each system must be defined and input/output parameters should be given to the systems. As shown in figure 4, the related table shows small example I/O data for the function. As given in table 1, 101 and 181 sample selected for functions used in ANN and ANFIS application. Function approximation rules for the FL based function approximation has been given in Table 2, Moreover, this table presents the parameters selected for ANN and ANFIS. In FL based cure fitting, the fuzzy rules for each function are presented in Table 3.



f the proposed functions in defined ranges



proximation method and the applied sampled data

Table 1. *Simulation Environment and Database*

Function	y=exp(-x)	y=sin (x) for x; 0->180
Sample	101	181
Applied PC	Intel ® Core ™ i5 CPU M460 @2.53 GHz	

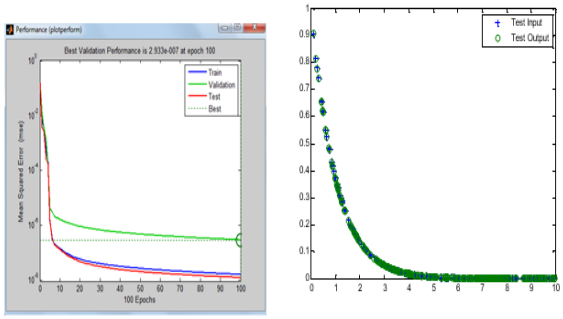
Table 2. *Information for the Methods*

Fuzzy logic	*y=exp(-x)*	Triangular MF for input, Singelton MF for output variable, 4 Rules
	y=sin (x)	Triangular MF for input, Singelton MF for output variable, 3 Rules
ANN	*y=exp(-x)*	2 Neurons for hidden layer, GD learning algorithm, training & test samples= 101
	y=sin (x)	2 Neurons for hidden layer, GD learning algorithm, training samples= 181
ANFIS	*y=exp(-x)*	Number of nodes: 24 Linear and Nonlinear parameters :25, 5 Rules, training samples =101
	y=sin (x)	Number of nodes: 12 Linear and Nonlinear parameters: 10, 2 Rules. , training samples =90

Table 3. Fuzzy rules for function approximation

Fuzzy rules for y=exp(-x),	*Fuzzy rules for* y=sin(x) and x; 0->180
If input1 is Very Small then (output1 is Very Big If input1 is Medium then output1 is Medium If input1 is Big) then (output1 is Small If input1 is Small then (output1 is Big	If input is Very Small then output is Very Big If (input is Medium then (output is Medium If input is Big then output is Small If input1 is Small then output is Big

There is no way to determine a good network topology just from the number of inputs and outputs. It depends critically on the number of training examples and the complexity of the classification you are trying to learn [8]. Although some research has been published about determining the number of neuron in hidden layer, the most used method is based on trial and error. Some studies suggest, start training with small number of neurons in hidden layer and increase number of hidden neurons during simulation regarding to learning performance. On the other hand, in the other studies the authors prefer to begin training approach with more neurons. If the learning is possible decrease the number of neurons. This process continued until desired performance has been obtained with less neuron. In this study the number of neurons in hidden layer has been obtained by trial and error. All the proposed simulations have been applied in MATLAB [11-12]. The following figures and tables show the results for function approximation using the mentioned functions. Figure 5a give error function change regarding to epochs. Figure 5b presents the function and approximated one.



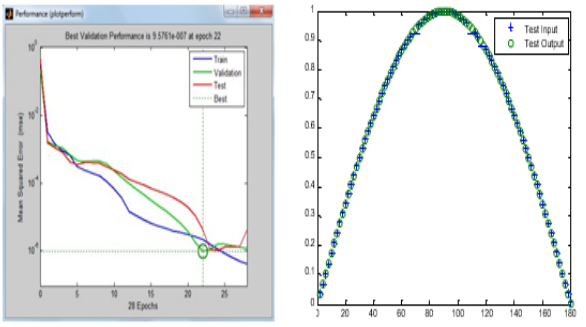
(a) (b)

Figure 5: *Error function* **(a)** *Plotted func. for y=exp(-x)* **(b)** *Test input&output values y=exp(-x)*

Figures 5 through 7 gives the graphical representation error change during training step for ANN and ANFIS, fitted function in compare with the desired function. Additionally, figure 8 presents the curve fitting result based on fuzzy logic. Comparing of all result regarding to performance criteria has been given in table 4.All simulation parameters have been given in table 2 and the applied methods has been constructed using this parameters. The performance of the applied method has been evaluated regarding to, fitting error, program size and program run time in table

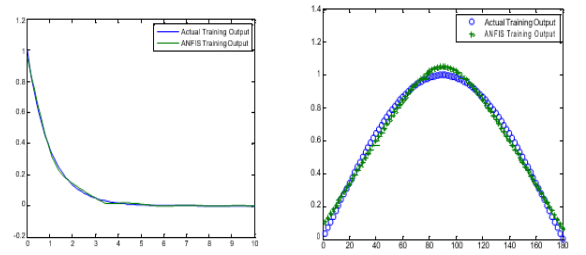
Regarding to table, ANN has the lowest RMS error for two functions and ANFIS follow ANN. FL presents the most fitting error. The performance of FL can be improved using more rules which tends to increase program size. As given in table FL has the biggest program size in memory. Similarly, the performance of ANFIS can be increased by changing parameters. From point view of the program run time, ANFIS performance is very better than other methods. Considering all result, it may be said; ANFIS shows bet-

ter performance in compare to other methods. Although, the fitting error of ANN is little better than ANFIS in this study, this criteria can be improved by selecting suitable parameters. We can say, this result may be an expected result, because, ANFIS as a hybrid system combines the advantage of FL andANN.



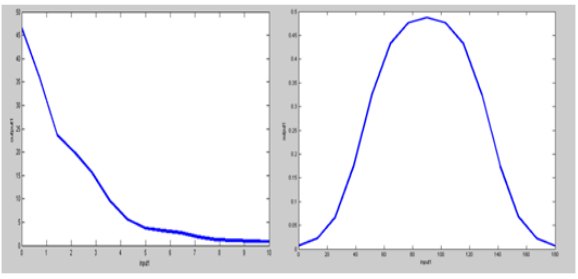
(a) (b)

function for y=exp(-x) **(b)** *Test input&output values y=sin (x) for x; 0->180*



(a) (b)

x) **(b)** for *y=sin (x) for x; 0->180*



(a) (b)

curves (a) function one, (b) function two

Table 4. *Comparison table*

Function	Title	ANN	FL	ANFIS
Function-1	*RMS Error*	0.023	0.527	0.298
	Program Size	384 Bytes	1.13Kbyte	642 Byte
	Run Time	4 sec	3 sec	0.221 Sec
Function-2	*RMS Error*	0.048	0.428	0.314
	Program Size	630 Bytes	506 Bytes	682 Bytes
	Run Time	3 sec	3 sec	0,293 Sec

4. CONCLUSION

In this work we compared three intelligent function approximator methods. To compare these methods, two simple nonlinear functions have been selected. In MATLAB simulation environment, function has been sampled and cure fitting has been applied in equal condition and selected parameters. As discussed above the result of simulation has been given in Table 4. Overview of the result showed that ANFIS as a function approximator acts better than the other methods. The run time and program size of ANFIS is better than the others. Accuracy or fitting error can be reduced by proper parameter selection. Fuzzy Logic has very acceptable application in control area. Although this method can be used as function approximator, the performance is very weak in comparison with other methods. Additionally, considering all results, ANFIS will be better than ANN in function approximation, especially for multi input, multi output functions.

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