A FEM Study for Non-Newtonian Behaviour of Blood in Plaque Deposited Capillaries: Analysis of Blood Flow Structure

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ABSTRACT

Inelastic behaviour of blood is predicted by employing Power law and Carreau model along partially blocked capillaries. Numerical results for stream function have been computed for predicting the reattachment length and intensity in the capillaries at various levels of obstacle and inertia. The predicted results obtained by employing FEM (Finite Element Method) under semi-implicit Taylor-Galerkin/ pressure-correction scheme. The numerical results have been quantified in terms of reattachment length and intensity, which illustrates that their formation takes place in the downstream of a capillary segment and augment in length as increases inertia or obstacle level. The obtained results are match able with analytical results. This study is accommodating for developing devices related to heart diseases in future.

Key Words: **Inelastic, Obstacle, Capillaries and Power Law.**

1. INTRODUCTION

The accumulation in capillaries is treated as a major reason for heart failure deaths, all over the world. A number of researchers in the field of Bio-mechanics have been attracted, to study the such cases related to the heart failure problems. This study is a contribution towards flow conditions related to capillaries [1-2]. Early research relates with the Newtonian nature of blood and steady state assumption has been considered in their studies. Some researchers reported from their experimental work, that there is a considerably difference between the flow of water and blood in the capillaries, hence in this case blood is considered as a non-Newtonian fluid [3]. The non-Newtonian nature of blood is due to the variation in viscosity, therefore, mixture of glycerol and water has been investigated for simulating the inelastic non-Newtonian flow of blood [4]. Whereas, experimental results have been performed and matched with the code for predicting numerical simulations, of axi-symmetric flow of blood in capillaries and found with good agreement [5-6]. Whilst numerical simulation for three dimensional flow of blood in the human right coronary partially blocked artery has been performed by many researchers [7-9].

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Present work covers two dimensional numerical simulations for flow of blood in partially blocked capillaries at various levels of inertia. Steady state case is studied only, due to the complexity in the constitutive equation. Rheological parameters have been considered carefully so that the flow conditions are to be satisfied properly. The non-Newtonian behaviour of blood is simulated by employing finite element algorithm.

2. GOVERNING SYSTEM OF EQUATIONS

Continuity Equation (1) and momentum Equations (2-3) are defined as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rv_r\right)+\frac{\partial v_z}{\partial z}=0 \tag{1}$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+v_z\frac{dv_r}{\partial z}\right)=$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)-\frac{\tau_{\theta\theta}}{r}+\frac{\partial}{\partial z}\tau_{rz}\right)-\frac{\partial p}{\partial r} \tag{2}$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+v_z\frac{dv_z}{\partial z}\right)=$$

$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{zr}\right)+\frac{\partial}{\partial z}\tau_{zz}\right)-\frac{\partial p}{\partial z} \tag{3}$$

where

$$\tau_{rr}=2\mu(\gamma)\frac{\partial v_r}{\partial r} \qquad \tau_{rz}=\mu(\gamma)\left\{\frac{\partial v_r}{\partial_z}+\frac{\partial v_a}{\partial_r}\right\}$$

$$\tau_{zz}=2\mu(\gamma)\frac{\partial v_z}{\partial z} \qquad \tau_{\theta\theta}=2\mu(\gamma)\frac{v_r}{r}$$

for more details readers can refer [10-12].

3. PROBLEM SPECIFICATION AND BOUNDARY CONDITIONS

Initial and boundary conditions are to be addressed as:

$v(z,0) = v_0(z)$ subject to $\nabla.v_0 = 0$

The accumulated vessels by assumption termed as solid boundaries, therefore no slip boundary conditions are forced on solid boundaries. Whilst parabolic shape of obstacle is enhanced in horizontal and vertical directions. Flow of bold is assumed axi-symmetric, whereas, Neumann boundary conditions are fixed on the axis of symmetry treating zero pressure at outlet.

4. NUMERICAL SCHEME

The discrete semi-implicit system of equations for the scheme is outlined as:

Stage-1(a):

$$\left[\frac{2}{\Delta t}M+\frac{1}{2\,\mathrm{Re}}S_{rr}^{j}\right]\left(V_r^{r\,n+\frac{1}{2}}-V_r^{j\,n}\right)=$$

$$\left[-\frac{1}{\mathrm{Re}}\left\{S_{rr}V_r^j+S_{rz}V_z^j\right\}-L_1^tP_k\right]^n-N(V)V_r^j \tag{4}$$

$$\left[\frac{2}{\Delta t}M+\frac{1}{2\,\mathrm{Re}}S_{zz}^{j}\right]\left(V_z^{j\,n+\frac{1}{2}}-V_z^{j\,n}\right)=$$

$$\left[-\frac{1}{\mathrm{Re}}\left\{S_{rz}V_r^j+S_{zz}V_z^j\right\}-L_2^tP_k\right]^n-N(V)V_z^j \tag{5}$$

Stage-1(b):

$$\left[\frac{1}{\Delta t}M+\frac{1}{2\,\mathrm{Re}}S_{rr}\right]\left(V_z^{j*}-V_r^{j\,n}\right)=$$

$$\left[-\frac{1}{\mathrm{Re}}\left\{S_{rr}V_r^j+S_{rz}V_z^j\right\}-L_1^tP_k\right]^n-N(V)V_r^{j\,n+\frac{1}{2}} \tag{6}$$

$$\left[\frac{1}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{zz}\right]\left(V_z^{j*} - V_z^{j^{n}}\right) =$$

$$\left[-\frac{1}{\mathrm{Re}}\left\{S_{rz}V_r^{j} + S_{zz}V_z^{j}\right\} - L_2^{t}P_k\right]^{n} - N(V)V_r^{j^{n+\frac{1}{2}}} \qquad (7)$$

Stage- 2:

$$K\left(Q^{n+1}\right) = -\frac{2}{\Delta t}\left(L_1 V_r^{j^*} + L_2 V_z^{j^*}\right) \qquad (8)$$

Stage 3:

$$M\left(V_r^{j^{n+1}} - V_r^{j^*}\right) = \frac{\Delta t}{2}L_1^{t}\left(p^{n+1} - p^{n}\right) \qquad (9)$$

$$M\left(V_z^{j^{n+1}} - V_z^{j^*}\right) = \frac{\Delta t}{2}L_2^{t}\left(p^{n+1} - p^{n}\right) \qquad (10)$$

For description of M, S, N(V), L; K matrices readers can refer [10-12].

5. RESULTS AND DISCUSSION

Stream line projections for Power law model have been presented for defining the flow structure of blood in a partially blocked capillary segment, at various levels of inertia. The obstacle level is set at 30, 50 and 70% respectively, along with three fixed levels of inertia, i.e. 100, 200 and 300 respectively, for the purpose of simulation, illustrated in Figs.1-3. It is observed that reattachment length as well as recirculation flow rate of blood increases as obstacle or inertia levels increases. Furthermore, it is observed that formation of vortex starts with 30% level of obstacle and dominates the flow field as obstacle or inertia level increase. Whereas, by analysing the flow structure, it is found that significant changes takes place in the partially blocked capillary at Re=200 with 50% obstacle level.Table 1, presents computed results for reattachment length against various Reynolds numbers in terms of Power law and Carreau model at different obstacle levels. It is illustrated that the behaviour of reattachment length is increasing in both models with inertia and obstacle. It is further displayed that Power law covers less reattachment length in comparison with Carreau model. Whilst, inelastic Carreau model predictions for reattachment length are observed

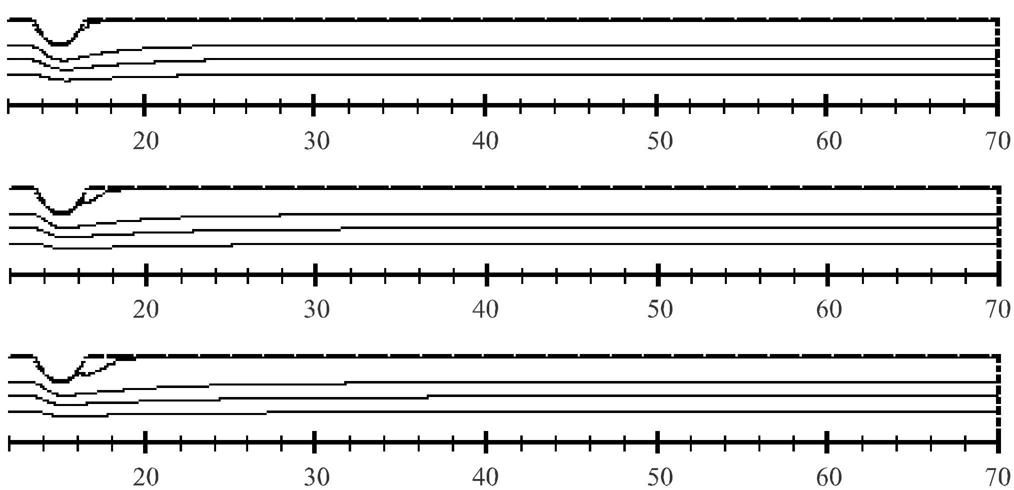

FIG. 1. STREAMLINE PROJECTIONS AT 30% LEVEL OF ACCUMULATION FOR VARIOUS REYNOLDS NUMBERS (Re=100, 200 AND 300)

identical to the predictions made for Newtonian flow of blood in partially blocked capillaries.

Recirculation flow rate of blood for both models is presented in Table 2, which illustrates that intensity of blood is increasing with increasing levels of inertia and obstacle. Furthermore, it is observed that the intensity of recirculation flow rate is very small at 30% level of deposition, moderate at medium level i.e. 30-50% level of obstacle; whilst very high at 70% and onwards levels of obstacle, with increasing levels of inertia[13-14]. The predicted numerical solutions for predicting flow structure of blood are compared against analytical solution for Power law

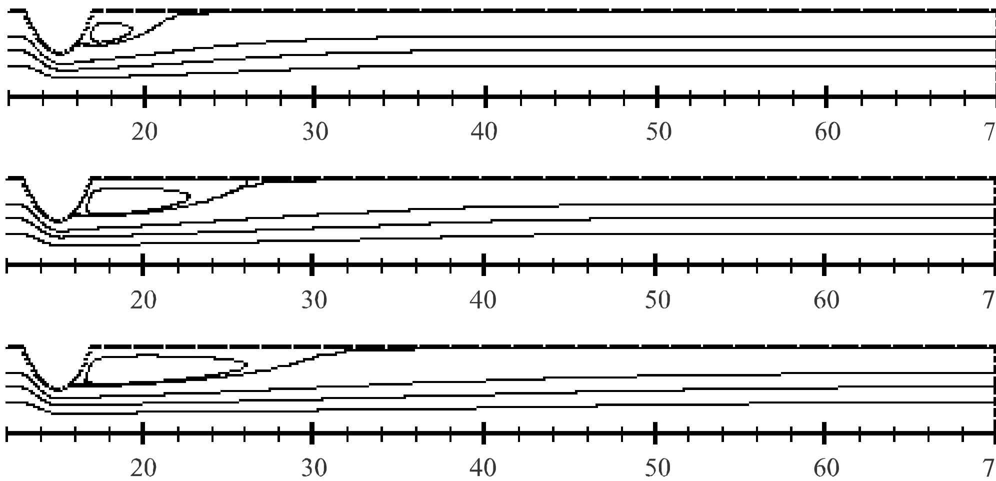

FIG. 2. STREAMLINE PROJECTIONS AT 50% LEVEL OF ACCUMULATION FOR VARIOUS REYNOLDS NUMBERS (Re=100, 200AND 300)

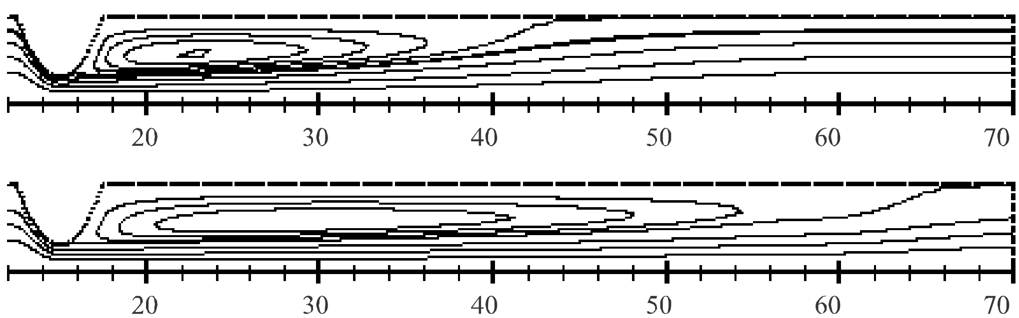

FIG. 3. STREAMLINE PROJECTIONS AT 70% LEVEL OF ACCUMULATION FOR VARIOUS REYNOLDS NUMBERS (Re=100 AND 200)

TABLE 1. REATTACHMENT LENGTH AGAINST REYNOLDS NUMBERS

Re	Reattachment Length					
	30% Blockage Level		50% Blockage Level		70% Blockage Level	
	Power Law	Carreau Model	Power Law	Carreau Model	Power Law	Carreau Model
100	0.28	0.29	2.98	3.08	14.68	14.89
200	0.83	0.85	5.69	5.86	27.57	27.92
300	1.28	1.30	8.20	8.43	-	-

TABLE 2. RECIRCULATION FLOW RATE AGAINST REYNOLDS NUMBERS

Re	Recirculation Flow Rate					
	30% Blockage Level		50% Blockage Level		70% Blockage Level	
	Power Law	Carreau Model	Power Law	Carreau Model	Power Law	Carreau Model
100	0.0000	0.00015	0.0105	0.01109	0.0664	0.06746
200	0.0006	0.00099	0.0161	0.01718	0.0739	0.07500
300	0.0015	0.00201	0.0194	0.02019	-	-

model [15], at different Reynolds numbers having various levels of obstacle. It is observed that the predicted results are in good agreement with analytical solution.

6. CONCLUSION

It is concluded that enhancement of parabolic obstacle in vertical direction from 30-70% and in horizontal direction from ±1R to ±2.5R, impacts upon recirculation length and its intensity, which develops in the downstream of the capillary segment. The trend of the interested variables is observed linearly increasing with increasing inertial level of percentage of accumulation.

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