Probability-Based Analysis to Determine the Performance of Multilevel Feedback Queue Scheduling

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ABSTRACT

Operating System may work on different types of CPU scheduling algorithms with different mechanism and concepts. The Multilevel Feedback Queue (MLFQ) Scheduling manages a variety of processes among various queues in a better and efficient manner. CPU scheduler appears transition mechanism over various queues. This paper is presented with various schemes of under a probability-based model. The scheduler has random movement over queues with given time quantum. This paper designs general transition model for its functioning and justifying comparison under different scheduling schemes through a simulation study applied on different data sets in particular cases.

Keywords - **Markov chain model, Multi-level feedback queue scheduling, Process queue, Process scheduling, Transition probability matrix.**

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1. INTRODUCTION

MLFQ scheduling mechanism should provide a structure which favors short jobs, I/O-bound jobs to get good I/O device utilization and determine the nature of a job as quickly as possible and schedule the job accordingly. When a new process enters at the tail of the top priority queue. It moves through that queue in FIFO manner until it gets the CPU. If the job relinquishes the CPU to wait for I/O completion or some event completion, the job leaves the queuing network. If the quantum expires before the process voluntarily relinquishes the CPU, the process is placed at the back of the next low-level priority queue. The process is next serviced when it reaches the head of that queue if the first queue is empty. As long as the process uses the full quantum provided at each level, it continues to move to the back of the next lower queue. Usually, there is some bottom-level queue through which the process circulates round-robin until it completes. Jain et al. (2015) presented a Linear Data Model based study of Improved Round Robin CPU Scheduling algorithm with features of Shortest Job First scheduling with varying time quantum whereas Chavan and Tikekar (2013) derived an Optimum Multilevel Dynamic Round Robin scheduling algorithm, which calculates intelligent time slice and changes after every round of execution.

The operating system (OS) has a large number of processes arriving to the processor at a time that causes waiting queue. Suranauwarat (2007) used simulator to learn scheduling algorithms in an easier and a more effective way. Sindhu et al. (2010) proposed an algorithm which can handle all types of process with optimum scheduling criteria. Li et al. (2009) presented a new scheduling algorithm called Distributed Weighted Round-Robin (DWRR). Major task of OS is to manage processes in the multiple queues. The process arrival is randomized along with its different categories and types in terms of size, memory requirement, time etc. This randomization involved in scheduling procedure leads to perform a probabilistic study over the movement phenomenon. The movement of scheduler over multiple queues of processes is according to priority and preferences to analyze under probability and stochastic study of system.

Although MLFQ is the combination of basic scheduling algorithms such as FCFS and RR scheduling algorithm. Yadav and Upadhayay (2012) suggested a novel approach which will improve the performance of MLFQ. Chahar and Raheja (2013) analyzed basic multilevel queue and multilevel feedback queue scheduling techniques and thereafter discussed a review of techniques proposed by different authors. Rao and Shet (2014) articulated the task states of New Multi Level Feedback Queue [NMLFQ] Scheduler and (2010) also analysed distinguishing problems with existing MLFQ scheduling algorithm to develop a New Multi Level Feedback Queue (NMLFQ) describing object oriented code to justify the algorithm. Hieh and Lam (2003) discussed smart schedulers for multimedia users. Saleem and Javed (2000) developed a comprehensive tool which runs a simulation in real time. Raheja et al. (2013) and (2014) proposed a new scheduling algorithm called Vague Oriented Highest Response Ratio Next (VHRRN) scheduling algorithm and a 2-layered architecture of multilevel queue scheduler based on vague set theory (VMLQ) respectively. Shukla and Jain (2007 a) have discussed the use of Markov chain model for multilevel queue scheduler and (2007 b) also designed a scheduling scheme and compared through deadlock-waiting index measure.

Shukla et al. (2009) analyzed round robin scheme using Markov chain model. Helmy and Dekdouk (2007) introduced Burst Round Robin, a proportional-share scheduling algorithm as an attempt to combine the low scheduling overhead of round robin algorithms and favor shortest jobs. Maste et al. (2013) proposed a new variant of MLFQ algorithm using dynamic time quantum and neural network with static time slice for each queue. Jain and Jain (2015) discussed the various approaches of scheduling algorithm and probability-based Markov chain analysis to determine the performance of these algorithms. Jain and Jain (2016) proposed a Markov chain model to analyze this transition phenomenon in MLFQ scheduling scheme with simulation study. This paper referred different CPU scheduling and their various aspects by Silberschatz and Galvin (2010), Stalling (2004), Tanenbaum and Woodhull (2000), Dhamdhere (2009) and Deitel(1999) but stochastic processes and Markov chain model by Medhi(1991).

This paper proposes different schemes of MLFQ with the assumption of random jumps of scheduler on different queue taking states and a wait state under the assumption of Markov chain model and comparing them to determine the performance over MLFQ. along with various data sets.

2. GENERALIZED MULTI-LEVEL FEEDBACK QUEUE SCHEDULING

This paper propose a general class of multilevel feedback queue scheduling procedure with free entry of any new process to any queue at any time. Consider five queues Q_1, Q_2, Q_3, Q_4, Q_5, each having large number of processes Pj, Pj', Pj", Pj"', Pj"" (j=1, 2, 3, 4, 5….) respectively for processing and one more queue Q_6 for waiting. Characterizing and organizing these queues are on the basis of priority, size, or weight. Define Q_i (i=1, 2, 3, 4, 5) are states of scheduling system and a specific states Q_6 which is a waiting state. First five states are for arrival and inputting of processes while the last one associate with waiting of the scheduler. A quantum is a small pre-defined slot of time given for processing in various queues to the processes. So few steps for the model are assumed as follows:

- A new process can enter in any of the five queues Q_1, Q_2, Q_3, Q_4 and Q_5 and the scheduler is allowed to accept for processing to pick any of the queue with initial probabilities pr_1, pr_2, pr_3, pr_4 and pr_5 satisfying this probability condition

$$\sum_{i=1}^{5} pr_i = 1$$

- The leftover of a process with the CPU until the quantum time is ended. If a process finishes in the quantum, then it puts off the queue Q_i and if an incomplete process in the quantum, scheduler gives next quantum to the next process of the same queue.
- The previous incomplete process moves to next queue Q_{i+1} where $(i+1) \leq 6$ and waits there for next quantum to be allotted for its processing.
- The movement of scheduler is random over different states Q_i (i=1, 2, 3, 4, 5) and to waiting states through quantum variation.
- Arrival of a new process is selected with priority given of any queue Q_i and assigns a quantum time by the scheduler.
- The scheduler jumps from one state to other state at the end of a quantum. In this quantum allotment procedure continues by scheduler within Q_i until Q_i is empty. When Q_1, Q_2, Q_3, Q_4, Q_5 are empty, scheduler moves towards processing in queue Q_6 in FCFS manner.
- Q_6=W is considered as waiting state in the transition system. Any of the specific conditions over waiting or restricting transition can be associated within this scheduling scheme.
- Define Q_1 as state 1, Q_2 as state 2, Q_3 as state 3, Q_4 as state 4, Q_5 as state 5 and Q_6 as waiting state W. The symbol n indicates to the n^{th} quantum of time consumed by scheduler for executing a process (n = 1, 2, 3, 4…..).

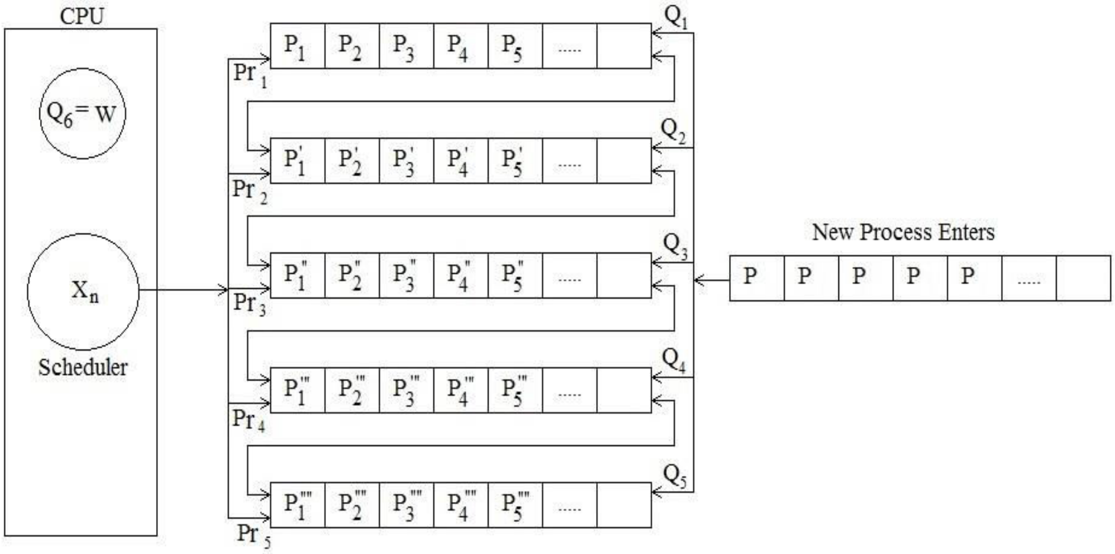


Figure 2.1: Generalized Multilevel Feedback Queue System

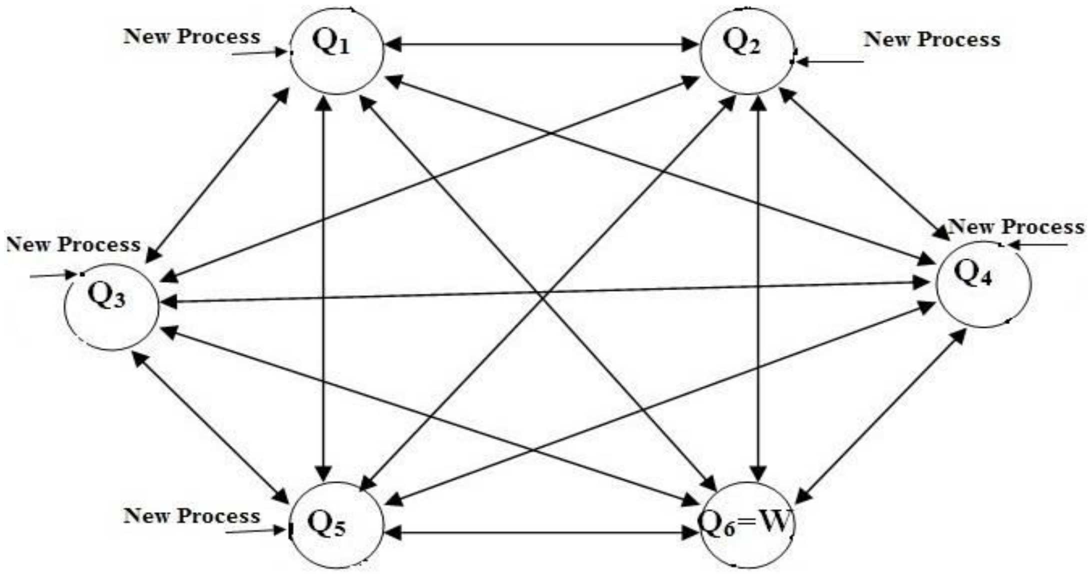


Figure 2.2: Unrestricted Transition Diagram

Fig.2.2 shows the transition diagram performing transition from one state to another state according to MLFQ

3. PROPOSED SYSTEM

Let X $^{(n)}$, n≥1} be a Markov chain where X $^{(n)}$ denotes the state of the scheduler at the quantum of time. The state space for the random variable X $^{(n)}$ is{ Q_1, Q_2, Q_3, Q_4, Q_5, Q_6} where Q_6=W is waiting state and scheduler X moves stochastically over different processing states and waiting states within different quantum of time. Predefined selections for initial probabilities of states are:

$$\left.\begin{aligned} P[X^{(0)}=Q_1] &= pr_1\\ P[X^{(0)}=Q_2] &= pr_2\\ P[X^{(0)}=Q_3] &= pr_3\\ P[X^{(0)}=Q_4] &= pr_4\\ P[X^{(0)}=Q_5] &= pr_5\\ P[X^{(0)}=Q_6] &= pr_6 \end{aligned}\right\} \text{.......... 3.1}$$

With $pr_1+pr_2+pr_3+pr_4+pr_5+pr_6=\sum_{i=1}^{6} pr_i=1$, where $pr_6=0$.

Let Sij (i, j=1,2,3,4,5,6) be the unit step transition probabilities of scheduler over six proposed states then transition probability matrix for :

$$\begin{array}{c|c|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow}\\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6\\ \hline \updownarrow & Q_1 & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ & Q_2 & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ X^{(n-1)} & Q_3 & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ & Q_4 & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ & Q_5 & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ \updownarrow & Q_6 & S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{array}$$

Figure 3.1: Transition Probability Matrix

If S_{ij} (i, j=1,2,3,4,5) be the unit-step transition probabilities of scheduler over proposed six states then transition probability matrix for $X^{(n)}$ will be

$$S_{ij} = P\ [X^{(n)} = Q_i / X^{(n-1)} = Q_j]$$

Unit-step Transition Probabilities for the wait state W are as follows:

$$\left.\begin{array}{ll} S_{16} = (1 - \sum_{i=1}^{5} S_{1i}), & S_{26} = (1 - \sum_{i=1}^{5} S_{2i})\\ S_{36} = (1 - \sum_{i=1}^{5} S_{3i}), & S_{46} = (1 - \sum_{i=1}^{5} S_{4i})\\ S_{56} = (1 - \sum_{i=1}^{5} S_{5i}), & S_{66} = (1 - \sum_{i=1}^{5} S_{6i})\\ \multicolumn{2}{c}{0 \le S_{ij} \le 1} \end{array}\right] \text{.......... 3.2}$$

After first quantum, the state probabilities can be determined by the following expressions:

$$\left.\begin{aligned} P[X^{(1)}=Q_1] = {} & P[X^{(0)}=Q_1].\ P[X^{(1)}=Q_1 / X^{(0)}=Q_1] +\\ & P[X^{(0)}=Q_2].\ P[X^{(1)}=Q_1 / X^{(0)}=Q_2] +\\ & P[X^{(0)}=Q_3].\ P[X^{(1)}=Q_1 / X^{(0)}=Q_3] +\\ & P[X^{(0)}=Q_4].\ P[X^{(1)}=Q_1 / X^{(0)}=Q_4] +\\ & P[X^{(0)}=Q_5].\ P[X^{(1)}=Q_1 / X^{(0)}=Q_5] +\\ & P[X^{(0)}=W].\ P[X^{(1)}=Q_1 / X^{(0)}=W]\\ = {} & \sum_{i=1}^{6} pr_i S_{i1}\\ P[X^{(1)}=Q_2] = {} & \sum_{i=1}^{6} pr_i S_{i2}\\ P[X^{(1)}=Q_3] = {} & \sum_{i=1}^{6} pr_i S_{i3}\\ P[X^{(1)}=Q_4] = {} & \sum_{i=1}^{6} pr_i S_{i4}\\ P[X^{(1)}=Q_5] = {} & \sum_{i=1}^{6} pr_i S_{i5}\\ P[X^{(1)}=Q_6] = {} & \sum_{i=1}^{6} pr_i S_{i6} \end{aligned}\right] \text{.......... 3.3}$$

Similarly, after second quantum, the state probabilities can be determined by the following expressions:

$$\left.\begin{aligned} P[X^{(2)}=Q_1] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j1}\\ P[X^{(2)}=Q_2] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j2}\\ P[X^{(2)}=Q_3] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j3}\\ P[X^{(2)}=Q_4] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j4}\\ P[X^{(2)}=Q_5] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j5}\\ P[X^{(2)}=Q_6] &= \sum_{j=1}^{6} \Big\{ \sum_{i=1}^{6} (\ pr_i S_{ij}\ ) \Big\} S_{j6} \end{aligned}\right] \text{.......... 3.4}$$

In a similar way, the generalized expression for the nth quantum:

$$\left.\begin{aligned}
P[X^{(n)} = Q_1] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m1} \\
P[X^{(n)} = Q_2] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m2} \\
P[X^{(n)} = Q_3] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m3} \\
P[X^{(n)} = Q_4] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m4} \\
P[X^{(n)} = Q_5] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m5} \\
P[X^{(n)} = Q_6] &= \sum_{m=1}^{6} \cdots\cdots \sum_{l=1}^{6} \sum_{k=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} pr_i S_{ij} \right) S_{jk} \right\} S_{kl} \cdots\cdots S_{m6}
\end{aligned}\right\} \cdots\cdots 3.5$$

4. PROPOSED MULTI LEVEL FEEDBACK QUEUE SCHEDULING SCHEMES

Some specifications for the proposed model:

- Up-gradation of the processes of lower order queues if five upper order queues are empty. This will provide a approach to control the accessibility of a resource that is available infrequently.
- In fact, transition takes place from W that signifies the situation when it provides as the waiting of the processes. Waiting state W is where system can achieve in any quantum while processing to a job but can put out back to the same queue in any quantum.

By applying few restrictions and conditions that can produce various scheduling schemes from above mentioned generalized Multi-level feedback queue scheme. These schemes are discussed as follows

4.1 SCHEME-I: Under process entry restriction, the scheme-I is described in fig 4.1

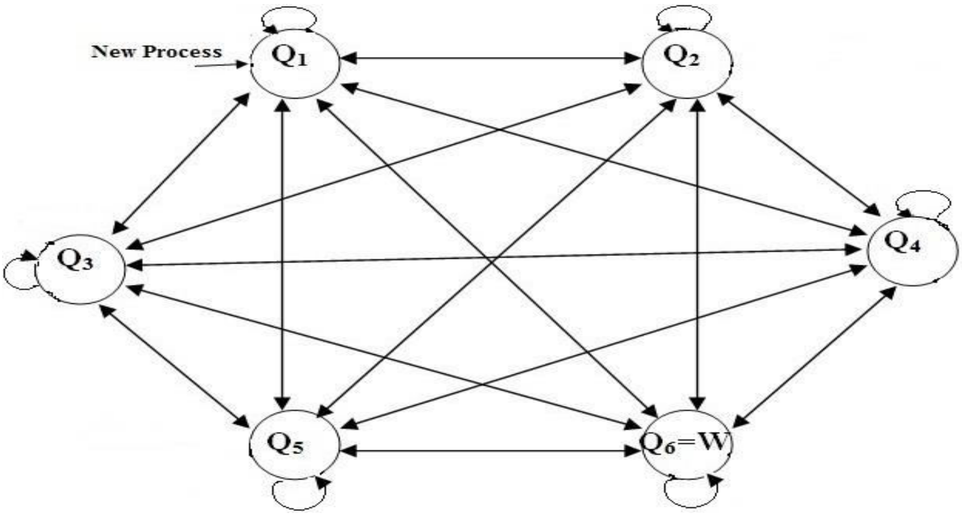


Figure 4.1: Transition Diagram of Scheme-I

- A new Process can only enter to first queue Q_1.
- Define Q_6=W is a waiting state.

$$P[X^{(0)}=Q_1]=1;$$
$$P[X^{(0)}=Q_2]=0;$$
$$P[X^{(0)}=Q_3]=0;$$
$$P[X^{(0)}=Q_4]=0;$$
$$P[X^{(0)}=Q_5]=0;$$
$$P[X^{(0)}=Q_6]=0;$$

$$\overleftrightarrow{\quad X^{(n)}\quad}$$

$$X^{(n-1)}\ \updownarrow\ \begin{array}{c|cccccc} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ Q_2 & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ Q_3 & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ Q_4 & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ Q_5 & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ Q_6 & S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{array}$$

Remark 4.1.1: Using equation (3.3), the state probabilities of scheme-I, after the first quantum is:

Unit Step Transition Probability Matrix for $\mathbf{x^{(n)}}$ under scheme-I:

$$P[X^{(1)}=Q_1]=S_{11}$$
$$P[X^{(1)}=Q_2]=S_{12}$$
$$P[X^{(1)}=Q_3]=S_{13}$$
$$P[X^{(1)}=Q_4]=S_{14}$$
$$P[X^{(1)}=Q_5]=S_{15}$$
$$P[X^{(1)}=Q_6]=S_{16}$$

Remark 4.1.2: Using equation (3.4), the state probabilities after the second quantum are:

$$P[X^{(2)}=Q_1]=\sum_{j=1}^{6} S_{1j}S_{j1}$$
$$P[X^{(2)}=Q_2]=\sum_{j=1}^{6} S_{1j}S_{j2}$$
$$P[X^{(2)}=Q_3]=\sum_{j=1}^{6} S_{1j}S_{j3}$$
$$P[X^{(2)}=Q_4]=\sum_{j=1}^{6} S_{1j}S_{j4}$$
$$P[X^{(2)}=Q_5]=\sum_{j=1}^{6} S_{1j}S_{j5}$$
$$P[X^{(2)}=Q_6]=\sum_{j=1}^{6} S_{1j}S_{j6}$$

Remark 4.1.3: Using (3.5), the generalized expressions for nth quantum of scheme-I are:

$$P[X^{(n)}=Q_1]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m1}$$
$$P[X^{(n)}=Q_2]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m2}$$
$$P[X^{(n)}=Q_3]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m3}$$
$$P[X^{(n)}=Q_4]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m4}$$
$$P[X^{(n)}=Q_5]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m5}$$
$$P[X^{(n)}=Q_6]=\sum_{m=1}^{6}\cdots\cdots\left[\sum_{1=1}^{6}\left\{\sum_{j=1}^{6}\left(\sum_{i=1}^{6}S_{1i}\right)S_{ij}\right\}S_{j1}\right]\cdots\cdots S_{m6}$$

4.2 SCHEME-II: In the general class of MLFQ, following assumption is restricted and the scheme-II is described in fig.4.2:

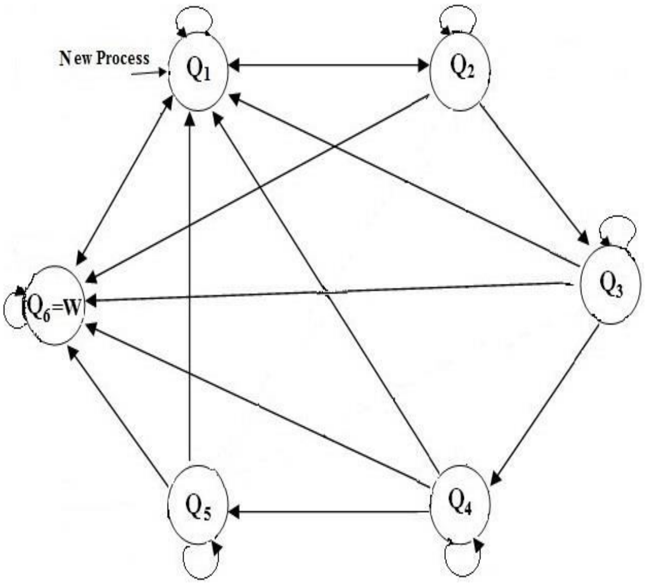


Figure 4.2: Transition Diagram Scheme-II

- A new process can only enter to Q_1.
- Scheduler cannot move to
  - Q_3 from Q_1 without passing Q_2
  - Q_4 from Q_1 without passing Q_2 and Q_3
  - Q_5 from Q_1 without passing Q_2, Q_3 and Q_4
- Scheduler comes to
  - Q_3 only if Q_1 and Q_2 are empty; it restricts the transition from Q_3 to Q_2; however, the transition from Q_3 to Q_1 is allowed only if a new process enters to Q_1; Q_4 only if Q_1, Q_2 and Q_3 are empty; it restricts the transition from Q_4 to Q_3; however, the transition from Q_4 to Q_1 is allowed only if a new process enters to Q_1;
  - Q_5 only if Q_1, Q_2, Q_3 and Q_4 are empty; it restricts the transition from Q_5 to Q_4; however, the transition from Q_5 to Q_1 is allowed only if a new process enters to Q_1;
- Resting of scheduler on state W ends up only if a new process enters in Q_1, otherwise resting continues.
- Define Q_6=W is a waiting State.

Remark 4.2.1: The scheme-II is same as the multi-level feedback scheduling discussed in literature [See Stallings (2005), Silberschatz and Galvin (1999), Tannenbaum (2000)].

Remark 4.2.2: The initial probabilities and transition probability matrix under scheme-II are:

$$P[X^{(0)}=Q_1]=1;$$
$$P[X^{(0)}=Q_2]=0;$$
$$P[X^{(0)}=Q_3]=0;$$
$$P[X^{(0)}=Q_4]=0;$$
$$P[X^{(0)}=Q_5]=0;$$
$$P[X^{(0)}=Q_6]=0;$$

$$X^{(n-1)}\ \begin{array}{c|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ Q_2 & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ Q_3 & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ Q_4 & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ Q_5 & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ Q_6 & S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{array}$$

Remark 4.2.3: Using (3.4), state probabilities after the first quantum for scheme-II are:

$$P[X^{(1)}=Q_1]=S_{11}$$
$$P[X^{(1)}=Q_2]=S_{12}$$
$$P[X^{(1)}=Q_3]=0$$
$$P[X^{(1)}=Q_4]=0$$
$$P[X^{(1)}=Q_5]=0$$
$$P[X^{(1)}=Q_6]=S_{16}$$

Define an indicator function bij (i, j = 1, 2, 3, 4, 5, 6) such that

$$b_{ij}=0 \text{ if } \begin{cases} (i=1, j=3, 4, 5), (i=2, j=1, 4, 5), \\ (i=3, j=2, 5), (i=4, j=2, 3), \\ (i=5, j=2, 3, 4) \text{ and } (i=6, j=2, 3, 4, 5) \end{cases}$$

$$b_{ij}=1 \qquad \text{otherwise.}$$

Then, using (3.4) state probabilities after second quantum of scheme-II:

$$P[X^{(2)}=Q_1]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j1})$$
$$P[X^{(2)}=Q_2]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j2})$$
$$P[X^{(2)}=Q_3]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j3})$$
$$P[X^{(2)}=Q_4]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j4})$$
$$P[X^{(2)}=Q_5]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j5})$$
$$P[X^{(2)}=Q_6]=\sum_{j=1}^{6}(b_{1j}S_{1j})(b_{j1}S_{j6})$$

Remark 4.2.4: Using (3.5) the generalized expressions for n quantum of scheme II are:

$$P[X^{(n)} = Q_1] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m1} b_{m1}$$

$$P[X^{(n)} = Q_2] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m2} b_{m2}$$

$$P[X^{(n)} = Q_3] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m3} b_{m3}$$

$$P[X^{(n)} = Q_4] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m4} b_{m4}$$

$$P[X^{(n)} = Q_5] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m5} b_{m5}$$

$$P[X^{(n)} = Q_6] = \sum_{m=1}^{6} \cdots\cdots \left[\sum_{1=1}^{6} \left\{ \sum_{j=1}^{6} \left(\sum_{i=1}^{6} s_{1i} b_{1i} \right) s_{ij} b_{ij} \right\} s_{j1} b_{j1} \right] \cdots\cdots s_{m6} b_{m6}$$

4.3 SCHEME-III:

The following transitions are restricted in scheme-III:

- A new process can only enter to Q_1.
- Transition from Q_1 to W is restricted.
- Transitions must occur in sequence from Q_1 to Q_2, Q_2 to Q_3, Q_3 to Q_4, Q_4 to Q_5 and then Q_5 to Q_6 to be shown in fig 4.3.

This gives a security for the scheduler because it cannot be on waiting state unless all the queues are empty.

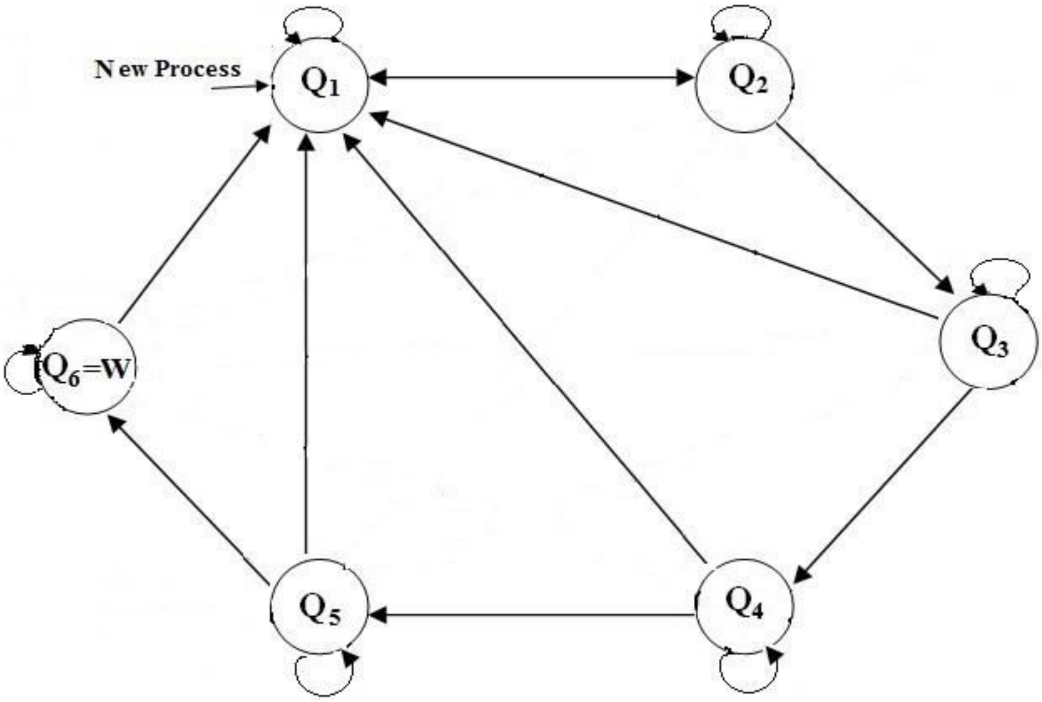


Figure 4.3: Transition Diagram in Scheme-III

For scheme-III, initial probabilities and the transition probability matrix are:

$$\begin{aligned} P[X^{(0)} = Q_1] &= 1; \\ P[X^{(0)} = Q_2] &= 0; \\ P[X^{(0)} = Q_3] &= 0; \\ P[X^{(0)} = Q_4] &= 0; \\ P[X^{(0)} = Q_5] &= 0; \\ P[X^{(0)} = Q_6] &= 0; \end{aligned}$$

$$\begin{array}{cc|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline \uparrow & Q_1 & S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & Q_2 & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ X^{(n-1)} & Q_3 & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ & Q_4 & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ & Q_5 & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ \downarrow & Q_6 & S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{array}$$

Using (3.3), (3.4) and (3.5) the state probabilities after the first, second and third quantum are:

$$\begin{aligned} P[X^{(1)} = Q_1] &= S_{11}; \\ P[X^{(1)} = Q_2] &= S_{12}; \\ P[X^{(1)} = Q_3] &= 0; \\ P[X^{(1)} = Q_4] &= 0; \\ P[X^{(1)} = Q_5] &= 0; \\ P[X^{(1)} = Q_6] &= 0; \end{aligned}$$

$$\begin{aligned} P[X^{(2)} = Q_1] &= S_{11} S_{11} + S_{12} S_{21} \\ P[X^{(2)} = Q_2] &= S_{11} S_{12} + S_{12} S_{22} \\ P[X^{(2)} = Q_3] &= S_{12} S_{23} \\ P[X^{(2)} = Q_4] &= 0 \\ P[X^{(2)} = Q_5] &= 0 \\ P[X^{(2)} = Q_6] &= 0 \end{aligned}$$

$$\begin{aligned} P[X^{(3)} = Q_1] &= (S_{11} S_{11} + S_{12} S_{21}) S_{11} + (S_{11} S_{12} + S_{12} S_{22}) S_{21} + (S_{21} S_{23}) S_{31} \\ P[X^{(3)} = Q_2] &= (S_{11} S_{12} + S_{12} S_{21}) S_{12} + (S_{11} S_{12} + S_{12} S_{22}) S_{22} \\ P[X^{(3)} = Q_3] &= (S_{11} S_{12} + S_{12} S_{22}) S_{23} + (S_{12} S_{23}) S_{34} \\ P[X^{(3)} = Q_4] &= (S_{12} S_{23}) S_{34} \\ P[X^{(3)} = Q_5] &= 0 \\ P[X^{(3)} = Q_6] &= 0 \end{aligned}$$

Using similar pattern, the generalized expression for nth quantum is:

$$\begin{aligned} P[X^{(n)} = Q_1] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i1} \\ P[X^{(n)} = Q_2] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i2} \\ P[X^{(n)} = Q_3] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i3} \\ P[X^{(n)} = Q_4] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i4} \\ P[X^{(n)} = Q_5] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i5} \\ P[X^{(n)} = Q_6] &= \sum_{i=1}^{6} P[X^{(n-1)} = Q_i]\, S_{i6} \end{aligned}$$

5. FORMULATE AND CALCULATE THE EQUAL VALUE TRANSITION PROBABILITIES

Consider equal transition probability matrix for a constant number ‘c’, $0 \leq c < 1$ and $5c < 1$.

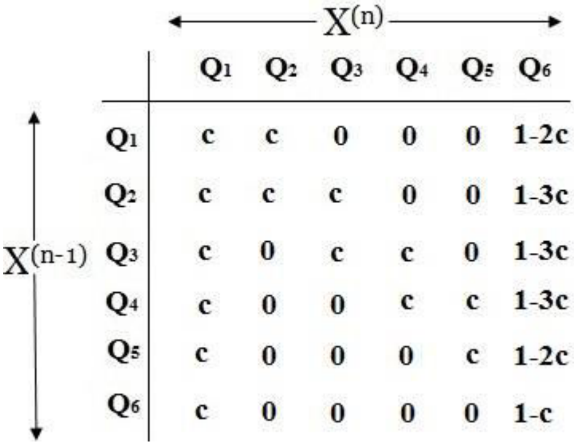
5.1: The equal transition matrix for scheme-I is expressed as:

$$\begin{array}{cc|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline \uparrow & Q_1 & c & c & c & c & c & 1-5c \\ & Q_2 & c & c & c & c & c & 1-5c \\ X^{(n-1)} & Q_3 & c & c & c & c & c & 1-5c \\ & Q_4 & c & c & c & c & c & 1-5c \\ & Q_5 & c & c & c & c & c & 1-5c \\ \downarrow & Q_6 & c & c & c & c & c & 1-5c \end{array}$$

Therefore the nth quantum under scheme-I is determined as:

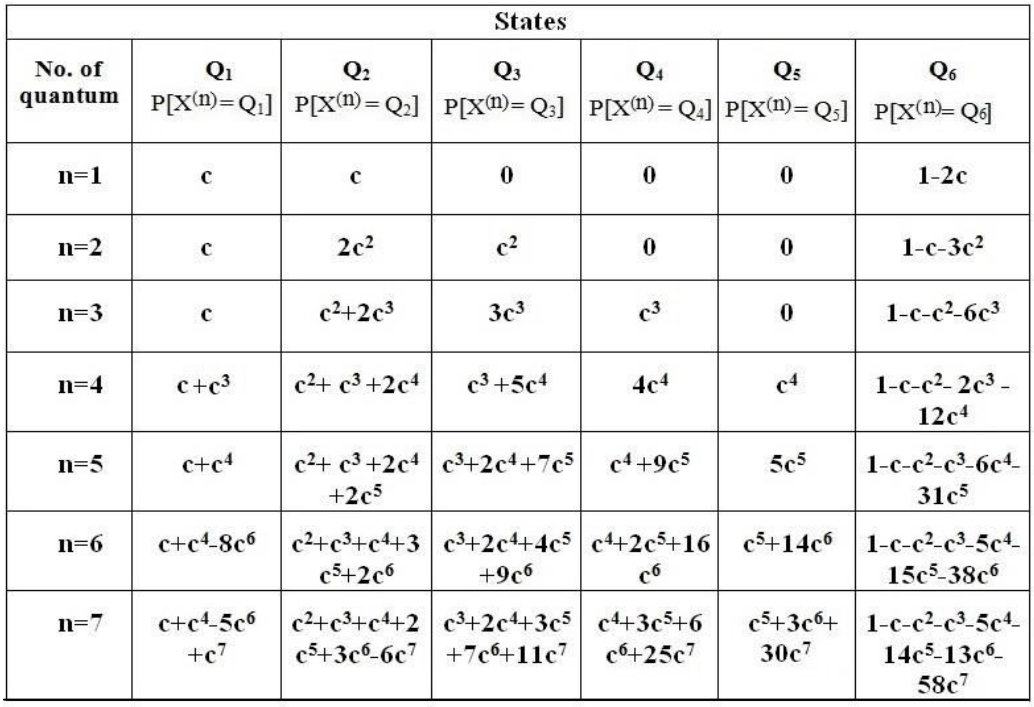
$$P[X^{(n)} = Q_1] = c$$
$$P[X^{(n)} = Q_2] = c$$
$$P[X^{(n)} = Q_3] = c$$
$$P[X^{(n)} = Q_4] = c$$
$$P[X^{(n)} = Q_5] = c$$
$$P[X^{(n)} = Q_6] = 1\text{-}5c$$

5.2: In scheme-II, the equal transition matrix is:



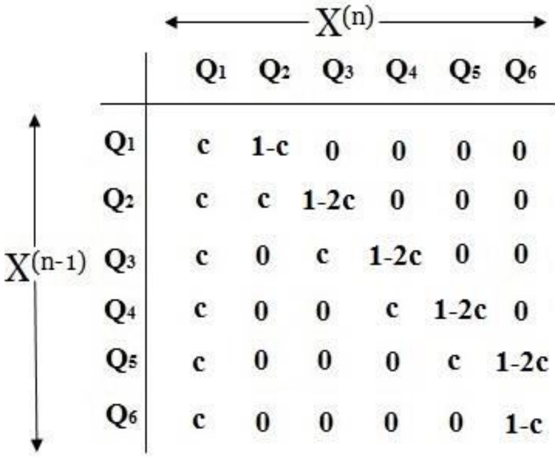
$$\xleftrightarrow{\quad X^{(n)} \quad}$$
$$X^{(n-1)} \updownarrow \begin{array}{c|cccccc} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & c & c & 0 & 0 & 0 & 1-2c \\ Q_2 & c & c & c & 0 & 0 & 1-3c \\ Q_3 & c & 0 & c & c & 0 & 1-3c \\ Q_4 & c & 0 & 0 & c & c & 1-3c \\ Q_5 & c & 0 & 0 & 0 & c & 1-2c \\ Q_6 & c & 0 & 0 & 0 & 0 & 1-c \end{array}$$

Table 5.2 (Seven Quantum Transition Probabilities under Scheme-II)



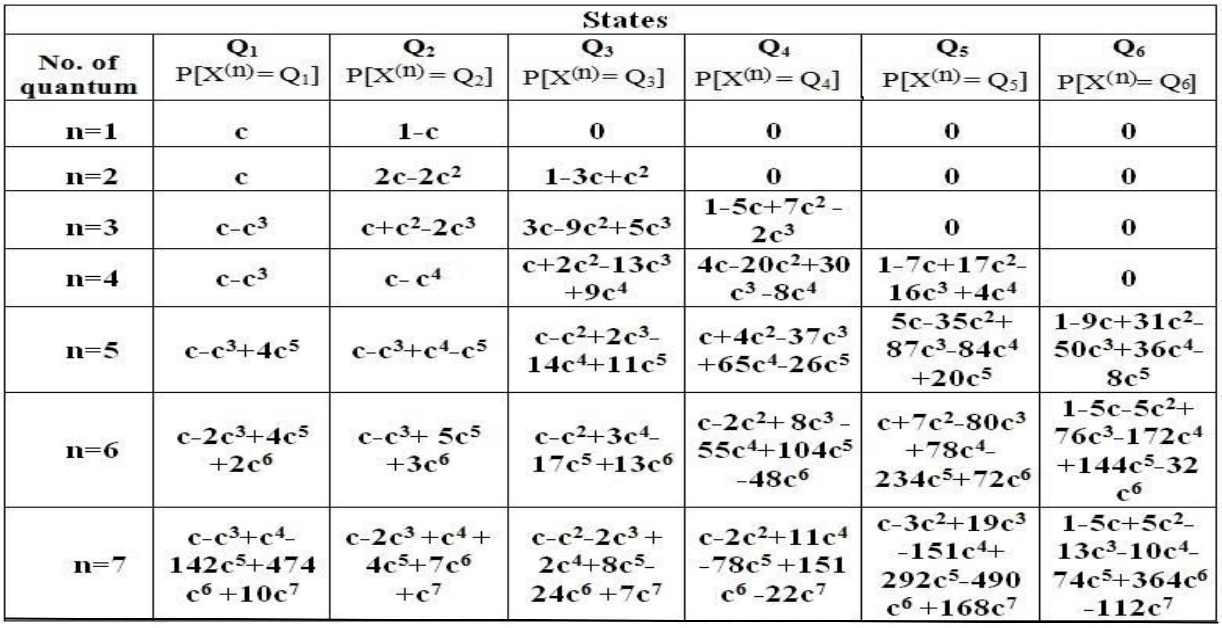
	States					
No. of quantum	**Q_1** $P[X^{(n)}=Q_1]$	**Q_2** $P[X^{(n)}=Q_2]$	**Q_3** $P[X^{(n)}=Q_3]$	**Q_4** $P[X^{(n)}=Q_4]$	**Q_5** $P[X^{(n)}=Q_5]$	**Q_6** $P[X^{(n)}=Q_6]$
n=1	c	c	0	0	0	$1-2c$
n=2	c	$2c^2$	c^2	0	0	$1-c-3c^2$
n=3	c	c^2+2c^3	$3c^3$	c^3	0	$1-c-c^2-6c^3$
n=4	$c+c^3$	$c^2+c^3+2c^4$	c^3+5c^4	$4c^4$	c^4	$1-c-c^2-2c^3-12c^4$
n=5	$c+c^4$	$c^2+c^3+2c^4+2c^5$	$c^3+2c^4+7c^5$	c^4+9c^5	$5c^5$	$1-c-c^2-c^3-6c^4-31c^5$
n=6	$c+c^4-8c^6$	$c^2+c^3+c^4+3c^5+2c^6$	$c^3+2c^4+4c^5+9c^6$	$c^4+2c^5+16c^6$	c^5+14c^6	$1-c-c^2-c^3-5c^4-15c^5-38c^6$
n=7	$c+c^4-5c^6+c^7$	$c^2+c^3+c^4+2c^5+3c^6-6c^7$	$c^3+2c^4+3c^5+7c^6+11c^7$	$c^4+3c^5+6c^6+25c^7$	$c^5+3c^6+30c^7$	$1-c-c^2-c^3-5c^4-14c^5-13c^6-58c^7$

5.3: Using Scheme-III, the equal transition matrix is as:



$$X^{(n-1)}\ \begin{array}{c|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & c & 1-c & 0 & 0 & 0 & 0 \\ Q_2 & c & c & 1-2c & 0 & 0 & 0 \\ Q_3 & c & 0 & c & 1-2c & 0 & 0 \\ Q_4 & c & 0 & 0 & c & 1-2c & 0 \\ Q_5 & c & 0 & 0 & 0 & c & 1-2c \\ Q_6 & c & 0 & 0 & 0 & 0 & 1-c \end{array}$$

Table 5.3 (Seven Quantum Transition Probabilities under Scheme-III)



	States					
No. of quantum	Q_1 $P[X^{(n)}=Q_1]$	Q_2 $P[X^{(n)}=Q_2]$	Q_3 $P[X^{(n)}=Q_3]$	Q_4 $P[X^{(n)}=Q_4]$	Q_5 $P[X^{(n)}=Q_5]$	Q_6 $P[X^{(n)}=Q_6]$
n=1	c	$1-c$	0	0	0	0
n=2	c	$2c-2c^2$	$1-3c+c^2$	0	0	0
n=3	$c-c^3$	$c+c^2-2c^3$	$3c-9c^2+5c^3$	$1-5c+7c^2-2c^3$	0	0
n=4	$c-c^3$	$c-c^4$	$c+2c^2-13c^3+9c^4$	$4c-20c^2+30c^3-8c^4$	$1-7c+17c^2-16c^3+4c^4$	0
n=5	$c-c^3+4c^5$	$c-c^3+c^4-c^5$	$c-c^2+2c^3-14c^4+11c^5$	$c+4c^2-37c^3+65c^4-26c^5$	$5c-35c^2+87c^3-84c^4+20c^5$	$1-9c+31c^2-50c^3+36c^4-8c^5$
n=6	$c-2c^3+4c^5+2c^6$	$c-c^3+5c^5+3c^6$	$c-c^2+3c^4-17c^5+13c^6$	$c-2c^2+8c^3-55c^4+104c^5-48c^6$	$c+7c^2-80c^3+78c^4-234c^5+72c^6$	$1-5c-5c^2+76c^3-172c^4+144c^5-32c^6$
n=7	$c-c^3+c^4-142c^5+474c^6+10c^7$	$c-2c^3+c^4+4c^5+7c^6+c^7$	$c-c^2-2c^3+2c^4+8c^5-24c^6+7c^7$	$c-2c^2+11c^4-78c^5+151c^6-22c^7$	$c-3c^2+19c^3-151c^4+292c^5-490c^6+168c^7$	$1-5c+5c^2-13c^3-10c^4-74c^5+364c^6-112c^7$

6. SIMULATION STUDY WITH NUMERICAL ANALYSIS USING DATA SETS

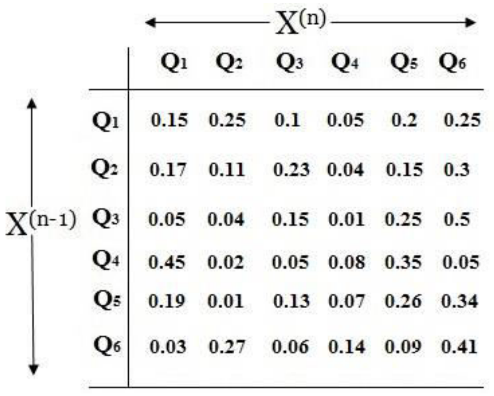
In order to analyze three schemes mentioned in section 4.1, 4.2 and 4.3 under Markov Chain Model with Equal and Unequal Transition elements (section 5.1, 5.2, 5.3 and table 5.2, 5.3) using different data sets:

6.1: Data Set- I

Scheme I: Let initial probabilities are $pr_1= 0.2$, $pr_2= 0.1$, $pr_3= 0.25$, $pr_4= 0.3$ and $pr_5= 0.15$

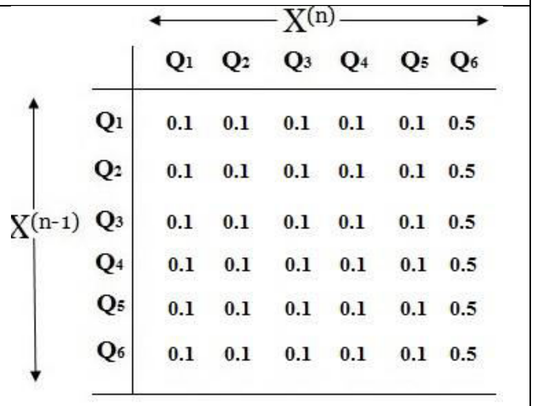
Equal and Unequal probabilities Matrix are follows:

UNEQUAL



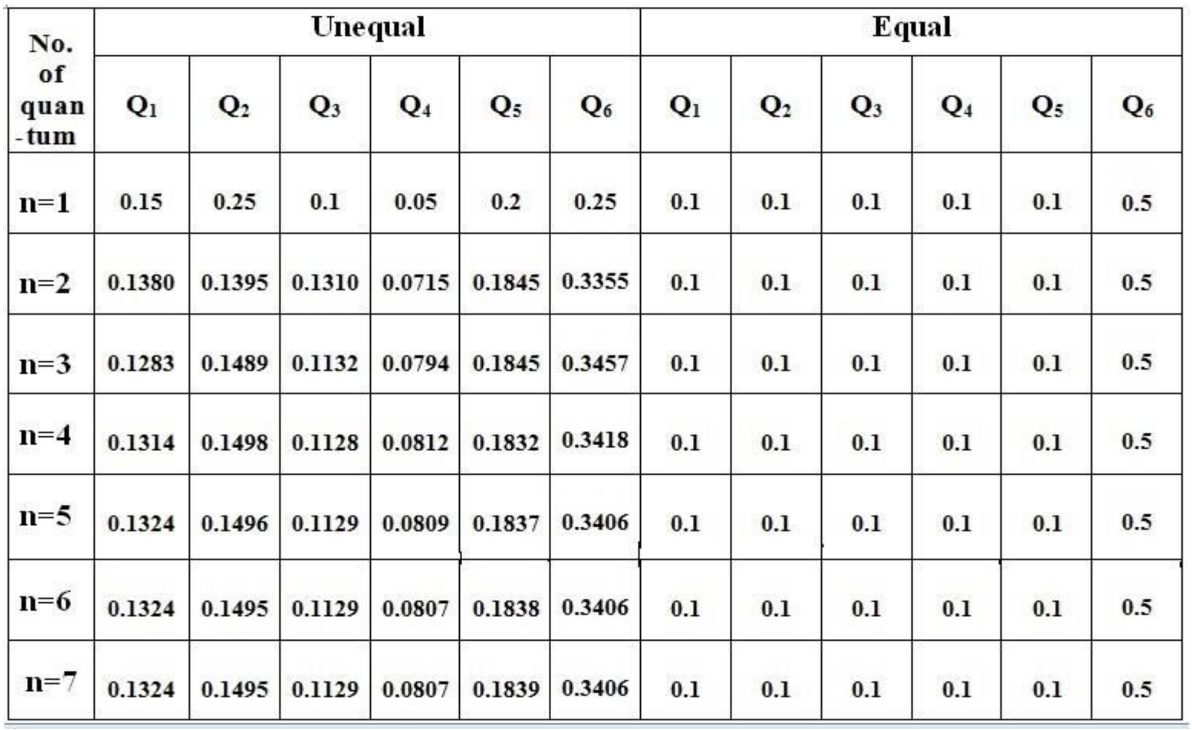
$$\begin{array}{cc|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.15 & 0.25 & 0.1 & 0.05 & 0.2 & 0.25 \\ & Q_2 & 0.17 & 0.11 & 0.23 & 0.04 & 0.15 & 0.3 \\ X^{(n-1)} & Q_3 & 0.05 & 0.04 & 0.15 & 0.01 & 0.25 & 0.5 \\ & Q_4 & 0.45 & 0.02 & 0.05 & 0.08 & 0.35 & 0.05 \\ & Q_5 & 0.19 & 0.01 & 0.13 & 0.07 & 0.26 & 0.34 \\ & Q_6 & 0.03 & 0.27 & 0.06 & 0.14 & 0.09 & 0.41 \end{array}$$

EQUAL



$$\begin{array}{cc|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \\ & Q_2 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \\ X^{(n-1)} & Q_3 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \\ & Q_4 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \\ & Q_5 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \\ & Q_6 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.5 \end{array}$$

Table 6.1.1: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases



No. of quan -tum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.15	0.25	0.1	0.05	0.2	0.25	0.1	0.1	0.1	0.1	0.1	0.5
n=2	0.1380	0.1395	0.1310	0.0715	0.1845	0.3355	0.1	0.1	0.1	0.1	0.1	0.5
n=3	0.1283	0.1489	0.1132	0.0794	0.1845	0.3457	0.1	0.1	0.1	0.1	0.1	0.5
n=4	0.1314	0.1498	0.1128	0.0812	0.1832	0.3418	0.1	0.1	0.1	0.1	0.1	0.5
n=5	0.1324	0.1496	0.1129	0.0809	0.1837	0.3406	0.1	0.1	0.1	0.1	0.1	0.5
n=6	0.1324	0.1495	0.1129	0.0807	0.1838	0.3406	0.1	0.1	0.1	0.1	0.1	0.5
n=7	0.1324	0.1495	0.1129	0.0807	0.1839	0.3406	0.1	0.1	0.1	0.1	0.1	0.5

Scheme II: Let initial probabilities are pr_1= 1.0, pr_2= 0.0, pr_3= 0.0, pr_4= 0.0 and pr_5= 0.0

Equal and Unequal probabilities Matrix are follows:

UNEQUAL	EQUAL
$\begin{array}{c\|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ X^{(n-1)} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.5 & 0.2 & 0 & 0 & 0 & 0.3 \\ Q_2 & 0.2 & 0.45 & 0.1 & 0 & 0 & 0.25 \\ Q_3 & 0.11 & 0 & 0.39 & 0.07 & 0 & 0.43 \\ Q_4 & 0.19 & 0 & 0 & 0.2 & 0.12 & 0.29 \\ Q_5 & 0.15 & 0 & 0 & 0 & .09 & 0.64 \\ Q_6 & 0.08 & 0 & 0 & 0 & 0 & 0.92 \end{array}$	$\begin{array}{c\|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ X^{(n-1)} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.1 & 0.1 & 0 & 0 & 0 & 0.8 \\ Q_2 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0.7 \\ Q_3 & 0.1 & 0 & 0.1 & 0.1 & 0 & 0.7 \\ Q_4 & 0.1 & 0 & 0 & 0.1 & 0.1 & 0.7 \\ Q_5 & 0.1 & 0 & 0 & 0 & 0.1 & 0.8 \\ Q_6 & 0.1 & 0 & 0 & 0 & 0 & 0.9 \end{array}$

Table 6.1.2: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases

No. of quan-tum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.5	0.2	0	0	0	0.30	0.1	0.1	0	0	0	0.8
n=2	0.3140	0.1900	0.0200	0	0	0.4760	0.1	0.02	0.01	0	0	0.87
n=3	0.2353	0.1483	0.0268	0.0014	0	0.5882	0.1	0.012	0.0030	0.0010	0	0.8840
n=4	0.1976	0.1138	0.0253	0.0022	0.0002	0.6608	0.1	0.0112	0.0015	0.0004	0.0001	0.8868
n=5	0.1776	0.0907	0.0212	0.0022	0.0003	0.7072	0.1	0.0111	0.0013	0.0002	0.0001	0.8874
n=6	0.1663	0.0763	0.0174	0.0019	0.0003	0.7365	0.1	0.0111	0.0012	0.0001	0.0000	0.8875
n=7	0.1597	0.0676	0.0144	0.0016	0.0003	0.7548	0.1	0.0111	0.0012	0.0001	0.0000	0.8875

Scheme III: Let initial probabilities are pr_1= 1.0, pr_2= 0.0, pr_3= 0.0, pr_4= 0.0 and pr_5=0.0

Equal and Unequal probabilities Matrix are follows:

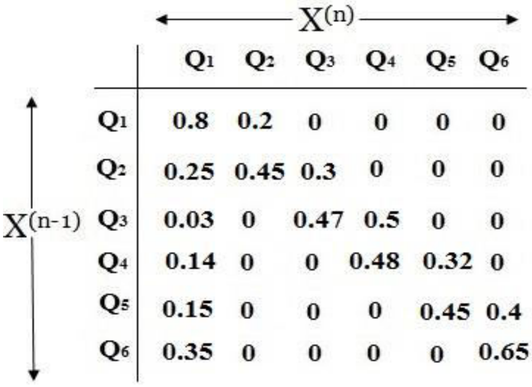
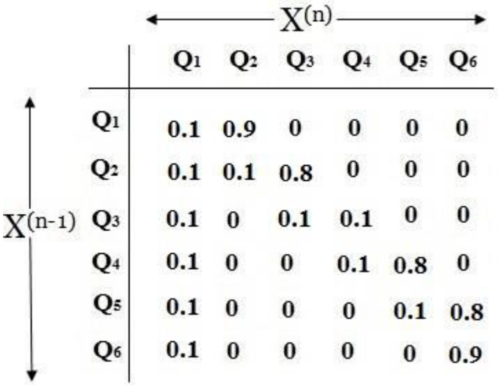
UNEQUAL	EQUAL
 $\begin{array}{cccccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ & Q_2 & 0.25 & 0.45 & 0.3 & 0 & 0 & 0 \\ X^{(n-1)} & Q_3 & 0.03 & 0 & 0.47 & 0.5 & 0 & 0 \\ & Q_4 & 0.14 & 0 & 0 & 0.48 & 0.32 & 0 \\ & Q_5 & 0.15 & 0 & 0 & 0 & 0.45 & 0.4 \\ & Q_6 & 0.35 & 0 & 0 & 0 & 0 & 0.65 \end{array}$	 $\begin{array}{cccccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.1 & 0.9 & 0 & 0 & 0 & 0 \\ & Q_2 & 0.1 & 0.1 & 0.8 & 0 & 0 & 0 \\ X^{(n-1)} & Q_3 & 0.1 & 0 & 0.1 & 0.1 & 0 & 0 \\ & Q_4 & 0.1 & 0 & 0 & 0.1 & 0.8 & 0 \\ & Q_5 & 0.1 & 0 & 0 & 0 & 0.1 & 0.8 \\ & Q_6 & 0.1 & 0 & 0 & 0 & 0 & 0.9 \end{array}$

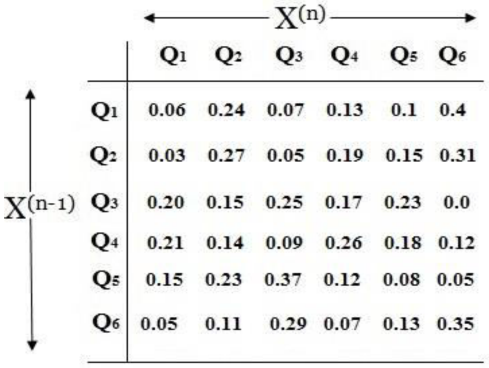
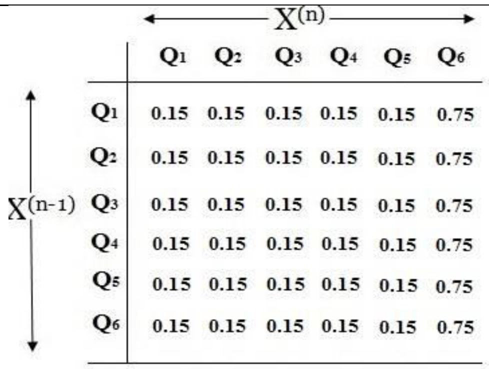
Table 6.1.3: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases

No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.8	0.2	0	0	0	0	0.1	0.1	0.9	0	0	0
n=2	0.69	0.25	0.06	0	0	0	0.1	0.18	0.72	0	0	0
n=3	0.6163	0.2505	0.1032	0.0300	0	0	0.1	0.1080	0.2160	0.5760	0	0
n=4	0.5630	0.2360	0.1237	0.0660	0.0096	0	0.1	0.1008	0.1080	0.2304	0.4608	0
n=5	0.5238	0.2188	0.1289	0.0935	0.0254	0.0038	0.1	0.1001	0.0914	0.1094	0.2304	0.3686
n=6	0.4958	0.2032	0.1262	0.1093	0.0414	0.0127	0.1	0.1	0.0892	0.0841	0.1106	0.5161
n=7	0.4772	0.1906	0.1203	0.1156	0.0536	0.0248	0.1	0.1	0.0889	0.0798	0.0783	0.5530

6.2: Data Set- II

Scheme I: Let initial probabilities are $pr_1 = 0.15$, $pr_2 = 0.3$, $pr_3 = 0.1$, $pr_4 = 0.25$ and $pr_5 = 0.2$

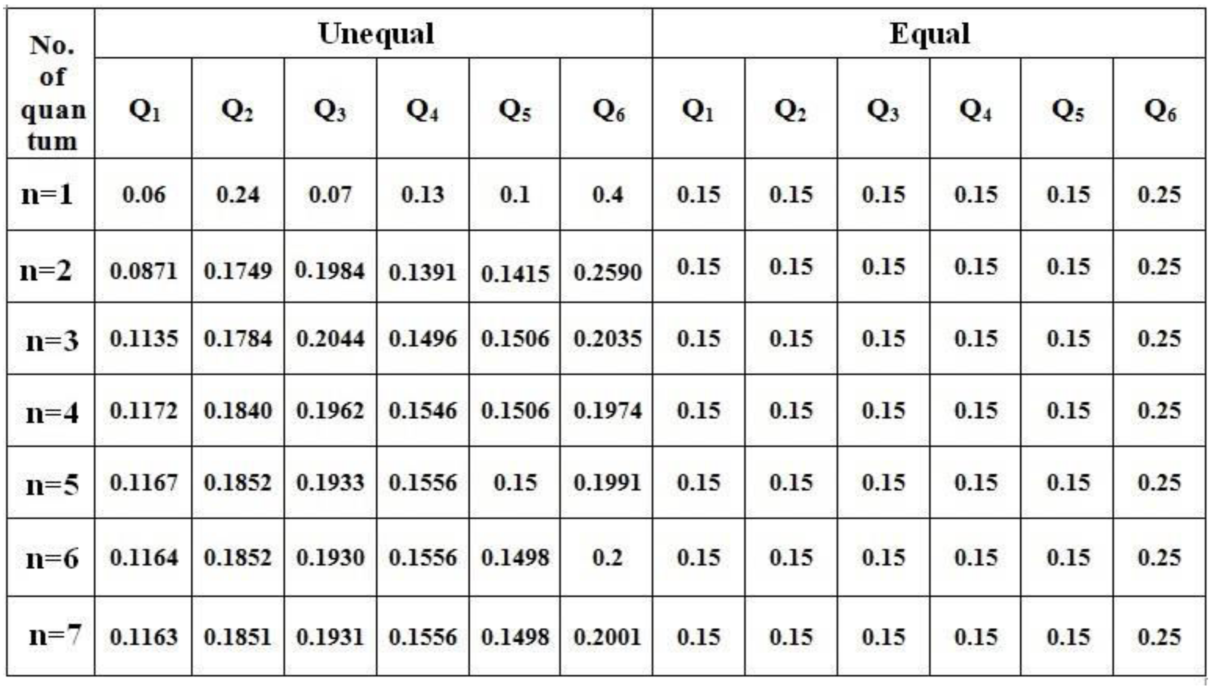
Equal and Unequal probabilities Matrix are follows:

UNEQUAL	EQUAL
	

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_1	0.06	0.24	0.07	0.13	0.1	0.4
Q_2	0.03	0.27	0.05	0.19	0.15	0.31
Q_3	0.20	0.15	0.25	0.17	0.23	0.0
Q_4	0.21	0.14	0.09	0.26	0.18	0.12
Q_5	0.15	0.23	0.37	0.12	0.08	0.05
Q_6	0.05	0.11	0.29	0.07	0.13	0.35

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_1	0.15	0.15	0.15	0.15	0.15	0.75
Q_2	0.15	0.15	0.15	0.15	0.15	0.75
Q_3	0.15	0.15	0.15	0.15	0.15	0.75
Q_4	0.15	0.15	0.15	0.15	0.15	0.75
Q_5	0.15	0.15	0.15	0.15	0.15	0.75
Q_6	0.15	0.15	0.15	0.15	0.15	0.75

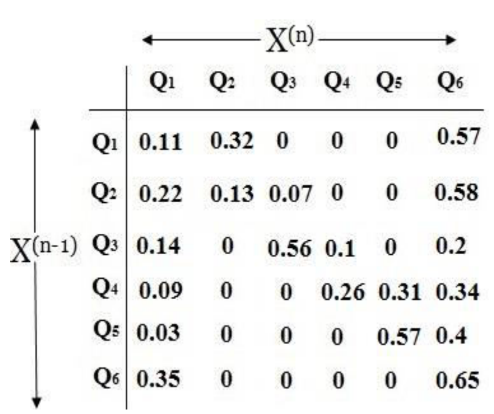
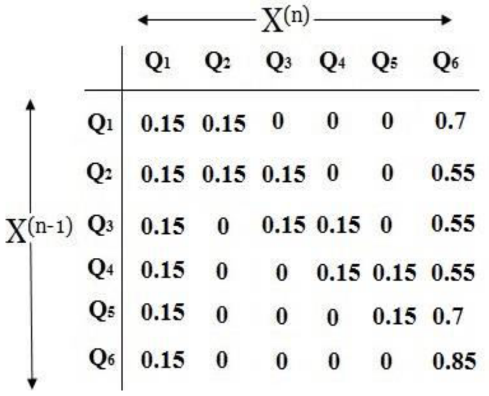
Table6.2.1: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases



No. of quan tum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.06	0.24	0.07	0.13	0.1	0.4	0.15	0.15	0.15	0.15	0.15	0.25
n=2	0.0871	0.1749	0.1984	0.1391	0.1415	0.2590	0.15	0.15	0.15	0.15	0.15	0.25
n=3	0.1135	0.1784	0.2044	0.1496	0.1506	0.2035	0.15	0.15	0.15	0.15	0.15	0.25
n=4	0.1172	0.1840	0.1962	0.1546	0.1506	0.1974	0.15	0.15	0.15	0.15	0.15	0.25
n=5	0.1167	0.1852	0.1933	0.1556	0.15	0.1991	0.15	0.15	0.15	0.15	0.15	0.25
n=6	0.1164	0.1852	0.1930	0.1556	0.1498	0.2	0.15	0.15	0.15	0.15	0.15	0.25
n=7	0.1163	0.1851	0.1931	0.1556	0.1498	0.2001	0.15	0.15	0.15	0.15	0.15	0.25

Scheme II: Let initial probabilities are $pr_1= 1.0$, $pr_2= 0.0$, $pr_3= 0.0$, $pr_4= 0.0$ and $pr_5= 0.0$

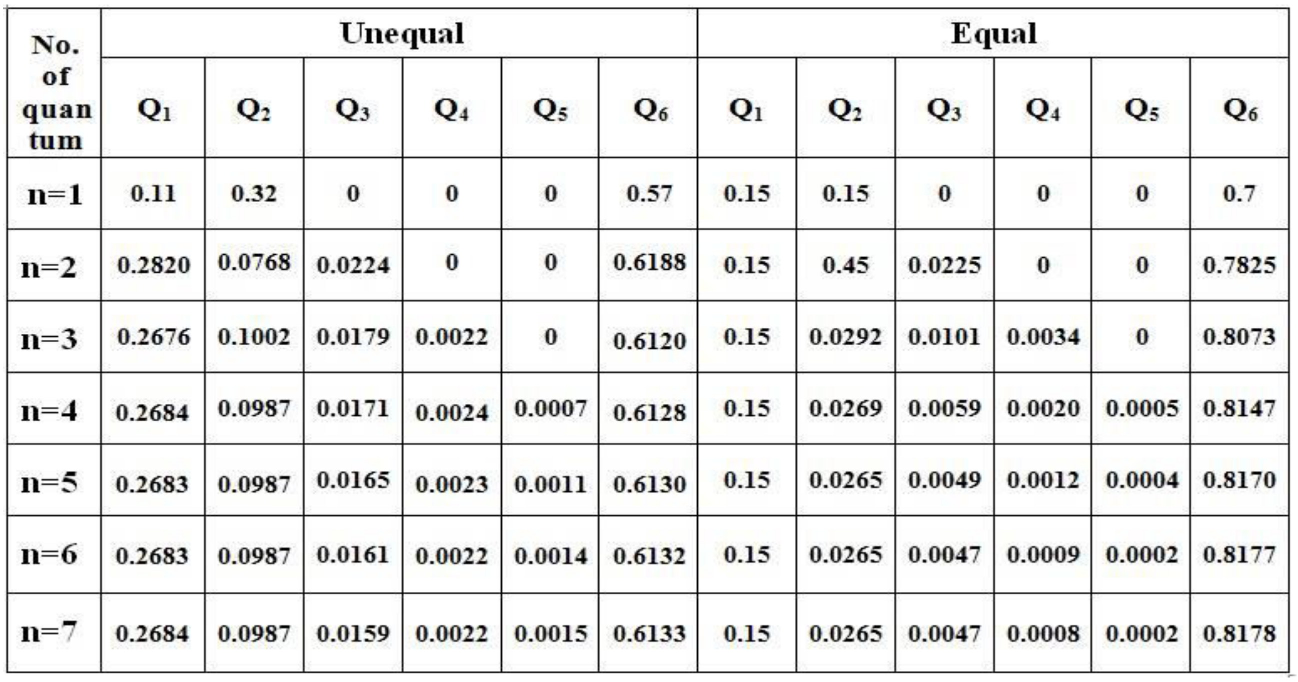
Equal and Unequal probabilities Matrix are follows:

UNEQUAL	EQUAL
	

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline X^{(n-1)}\ Q_1 & 0.11 & 0.32 & 0 & 0 & 0 & 0.57 \\ Q_2 & 0.22 & 0.13 & 0.07 & 0 & 0 & 0.58 \\ Q_3 & 0.14 & 0 & 0.56 & 0.1 & 0 & 0.2 \\ Q_4 & 0.09 & 0 & 0 & 0.26 & 0.31 & 0.34 \\ Q_5 & 0.03 & 0 & 0 & 0 & 0.57 & 0.4 \\ Q_6 & 0.35 & 0 & 0 & 0 & 0 & 0.65 \end{array}$$

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline X^{(n-1)}\ Q_1 & 0.15 & 0.15 & 0 & 0 & 0 & 0.7 \\ Q_2 & 0.15 & 0.15 & 0.15 & 0 & 0 & 0.55 \\ Q_3 & 0.15 & 0 & 0.15 & 0.15 & 0 & 0.55 \\ Q_4 & 0.15 & 0 & 0 & 0.15 & 0.15 & 0.55 \\ Q_5 & 0.15 & 0 & 0 & 0 & 0.15 & 0.7 \\ Q_6 & 0.15 & 0 & 0 & 0 & 0 & 0.85 \end{array}$$

Table 6.2.2: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases



No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.11	0.32	0	0	0	0.57	0.15	0.15	0	0	0	0.7
n=2	0.2820	0.0768	0.0224	0	0	0.6188	0.15	0.45	0.0225	0	0	0.7825
n=3	0.2676	0.1002	0.0179	0.0022	0	0.6120	0.15	0.0292	0.0101	0.0034	0	0.8073
n=4	0.2684	0.0987	0.0171	0.0024	0.0007	0.6128	0.15	0.0269	0.0059	0.0020	0.0005	0.8147
n=5	0.2683	0.0987	0.0165	0.0023	0.0011	0.6130	0.15	0.0265	0.0049	0.0012	0.0004	0.8170
n=6	0.2683	0.0987	0.0161	0.0022	0.0014	0.6132	0.15	0.0265	0.0047	0.0009	0.0002	0.8177
n=7	0.2684	0.0987	0.0159	0.0022	0.0015	0.6133	0.15	0.0265	0.0047	0.0008	0.0002	0.8178

Scheme III: Let initial probabilities are pr_1= 1.0, pr_2= 0.0, pr_3= 0.0, pr_4= 0.0 and pr_5= 0.0

Equal and Unequal probability Matrix are follows:

UNEQUAL	EQUAL
$\begin{array}{cc\|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.32 & 0.68 & 0 & 0 & 0 & 0 \\ & Q_2 & 0.26 & 0.15 & 0.59 & 0 & 0 & 0 \\ \updownarrow X^{(n-1)} & Q_3 & 0.14 & 0 & 0.56 & 0.3 & 0 & 0 \\ & Q_4 & 0.31 & 0 & 0 & 0.24 & 0.45 & 0 \\ & Q_5 & 0.03 & 0 & 0 & 0 & 0.67 & 0.3 \\ & Q_6 & 0.25 & 0 & 0 & 0 & 0 & 0.75 \end{array}$	$\begin{array}{cc\|cccccc} & & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline & Q_1 & 0.15 & 0.85 & 0 & 0 & 0 & 0 \\ & Q_2 & 0.15 & 0.15 & 0.7 & 0 & 0 & 0 \\ \updownarrow X^{(n-1)} & Q_3 & 0.15 & 0 & 0.15 & 0.7 & 0 & 0 \\ & Q_4 & 0.15 & 0 & 0 & 0.15 & 0.7 & 0 \\ & Q_5 & 0.15 & 0 & 0 & 0 & 0.15 & 0.7 \\ & Q_6 & 0.15 & 0 & 0 & 0 & 0 & 0.85 \end{array}$

Table 6.2.3: The transition probabilities $P[X^{(n)} = Q_i]$ **for equal and unequal cases**

No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.32	0.68	0	0	0	0	0.15	0.85	0	0	0	0
n=2	0.2792	0.3196	0.4012	0	0	0	0.15	0.2550	0.5950	0	0	0
n=3	0.2286	0.2378	0.4132	0.1204	0	0	0.15	0.1658	0.2677	0.4165	0	0
n=4	0.2301	0.1911	0.3717	0.1529	0.0542	0	0.15	0.1524	0.1562	0.2499	0.2915	0
n=5	0.2244	0.1852	0.3209	0.1482	0.1051	0.0162	0.15	0.1504	0.1301	0.1468	0.2187	0.2041
n=6	0.2180	0.1804	0.2890	0.1318	0.1371	0.0437	0.15	0.1501	0.1248	0.1131	0.1356	0.3265
n=7	0.2130	0.1753	0.2682	0.1183	0.1512	0.0739	0.15	0.15	0.1248	0.1043	0.0995	0.3725

6.3: Data Set- III

Scheme I: Let initial probabilities are $pr_1=0.3$, $pr_2= 0.1$, $pr_3=0.15$, $pr_4= 0.2$ and $pr_5= 0.25$

Equal and Unequal probability Matrix are follows:

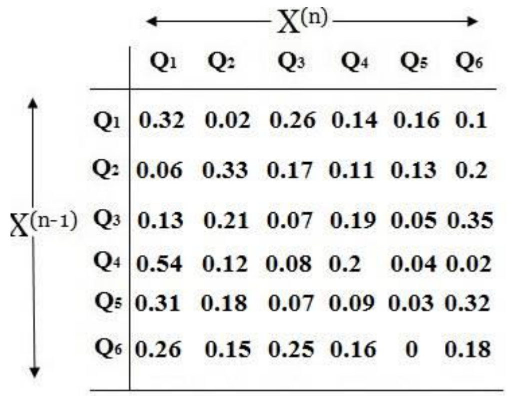
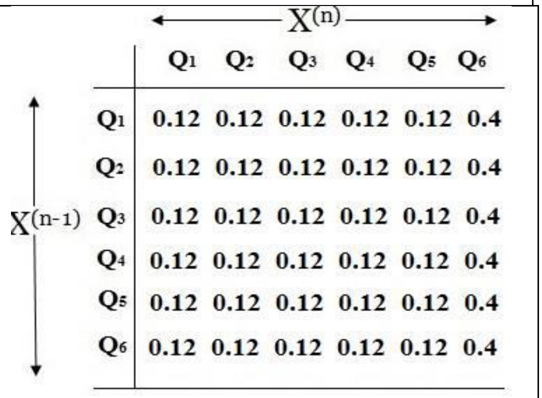
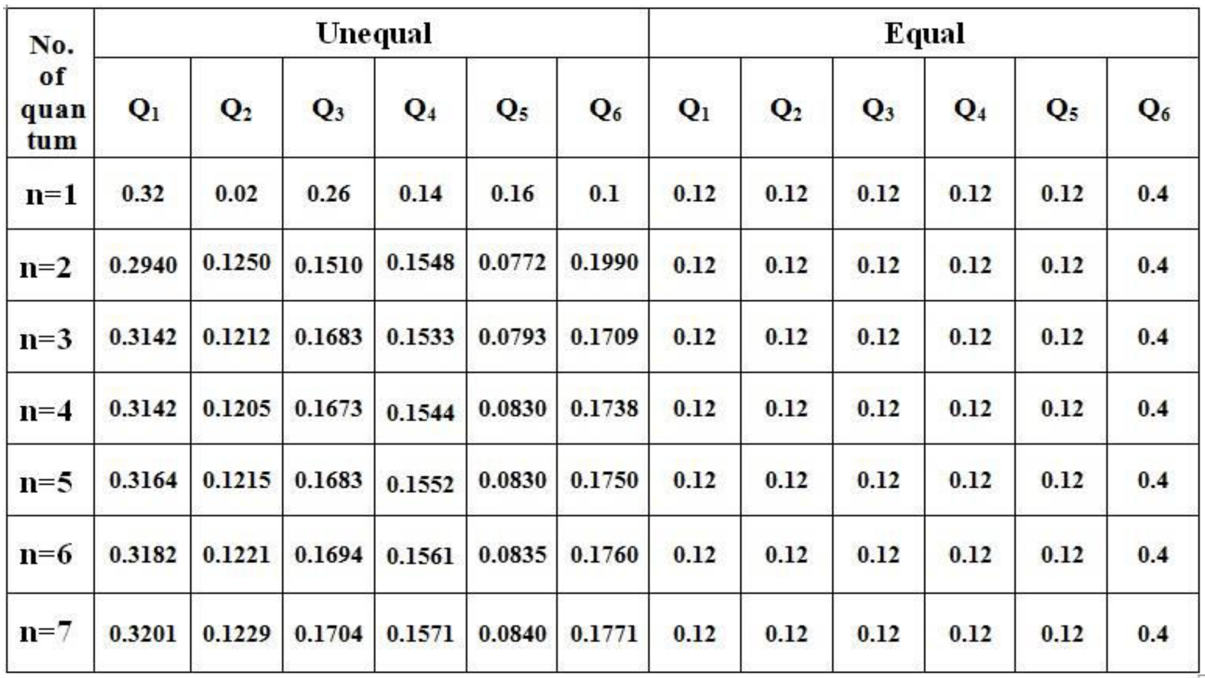
UNEQUAL	EQUAL
 $\begin{array}{c|cccccc} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.32 & 0.02 & 0.26 & 0.14 & 0.16 & 0.1 \\ Q_2 & 0.06 & 0.33 & 0.17 & 0.11 & 0.13 & 0.2 \\ Q_3 & 0.13 & 0.21 & 0.07 & 0.19 & 0.05 & 0.35 \\ Q_4 & 0.54 & 0.12 & 0.08 & 0.2 & 0.04 & 0.02 \\ Q_5 & 0.31 & 0.18 & 0.07 & 0.09 & 0.03 & 0.32 \\ Q_6 & 0.26 & 0.15 & 0.25 & 0.16 & 0 & 0.18 \end{array}$ $X^{(n)}$ $X^{(n-1)}$	 $\begin{array}{c|cccccc} & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \\ Q_2 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \\ Q_3 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \\ Q_4 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \\ Q_5 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \\ Q_6 & 0.12 & 0.12 & 0.12 & 0.12 & 0.12 & 0.4 \end{array}$ $X^{(n)}$ $X^{(n-1)}$

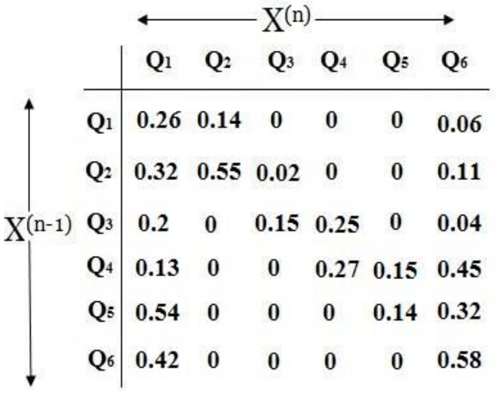
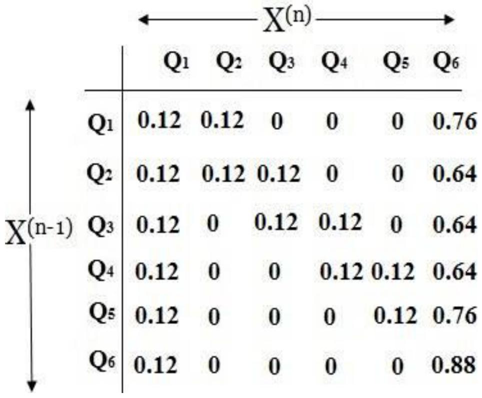
Table 6.3.1: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases



No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.32	0.02	0.26	0.14	0.16	0.1	0.12	0.12	0.12	0.12	0.12	0.4
n=2	0.2940	0.1250	0.1510	0.1548	0.0772	0.1990	0.12	0.12	0.12	0.12	0.12	0.4
n=3	0.3142	0.1212	0.1683	0.1533	0.0793	0.1709	0.12	0.12	0.12	0.12	0.12	0.4
n=4	0.3142	0.1205	0.1673	0.1544	0.0830	0.1738	0.12	0.12	0.12	0.12	0.12	0.4
n=5	0.3164	0.1215	0.1683	0.1552	0.0830	0.1750	0.12	0.12	0.12	0.12	0.12	0.4
n=6	0.3182	0.1221	0.1694	0.1561	0.0835	0.1760	0.12	0.12	0.12	0.12	0.12	0.4
n=7	0.3201	0.1229	0.1704	0.1571	0.0840	0.1771	0.12	0.12	0.12	0.12	0.12	0.4

Scheme II: Let initial probabilities are pr_1= 1.0, pr_2= 0.0, pr_3= 0.0, pr_4= 0.0 and pr_5= 0.0

Equal and Unequal probability Matrix are follows:

UNEQUAL	EQUAL
	

$$X^{(n-1)} \begin{array}{c|cccccc} & \multicolumn{6}{c}{\xleftarrow{\quad} X^{(n)} \xrightarrow{\quad}} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.26 & 0.14 & 0 & 0 & 0 & 0.06 \\ Q_2 & 0.32 & 0.55 & 0.02 & 0 & 0 & 0.11 \\ Q_3 & 0.2 & 0 & 0.15 & 0.25 & 0 & 0.04 \\ Q_4 & 0.13 & 0 & 0 & 0.27 & 0.15 & 0.45 \\ Q_5 & 0.54 & 0 & 0 & 0 & 0.14 & 0.32 \\ Q_6 & 0.42 & 0 & 0 & 0 & 0 & 0.58 \end{array}$$

$$X^{(n-1)} \begin{array}{c|cccccc} & \multicolumn{6}{c}{\xleftarrow{\quad} X^{(n)} \xrightarrow{\quad}} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.12 & 0.12 & 0 & 0 & 0 & 0.76 \\ Q_2 & 0.12 & 0.12 & 0.12 & 0 & 0 & 0.64 \\ Q_3 & 0.12 & 0 & 0.12 & 0.12 & 0 & 0.64 \\ Q_4 & 0.12 & 0 & 0 & 0.12 & 0.12 & 0.64 \\ Q_5 & 0.12 & 0 & 0 & 0 & 0.12 & 0.76 \\ Q_6 & 0.12 & 0 & 0 & 0 & 0 & 0.88 \end{array}$$

Table 6.3.2: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases

No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.26	0.14	0	0	0	0.6	0.12	0.12	0	0	0	0.76
n=2	0.3644	0.1134	0.0028	0	0	0.5194	0.12	0.0288	0.0144	0	0	0.8368
n=3	0.3497	0.1134	0.0027	0.0007	0	0.5335	0.12	0.0179	0.0052	0.0017	0	0.8552
n=4	0.3519	0.1113	0.0027	0.0009	0.0001	0.5331	0.12	0.0165	0.0028	0.0008	0.0002	0.8597
n=5	0.3517	0.1105	0.0026	0.0009	0.0001	0.5341	0.12	0.0164	0.0023	0.0004	0.0001	0.8607
n=6	0.3519	0.11	0.0026	0.0009	0.0002	0.5345	0.12	0.0164	0.0022	0.0003	0.0001	0.8610
n=7	0.3519	0.1098	0.0026	0.0009	0.0002	0.5347	0.12	0.0164	0.0022	0.0003	0	0.8610

Scheme III: Let initial probabilities are $pr_1= 1.0$, $pr_2= 0.0$, $pr_3= 0.0$, $pr_4= 0.0$ and $pr_5= 0.0$

Equal and Unequal probability Matrix are follows:

UNEQUAL	EQUAL
$X^{(n-1)} \updownarrow \begin{array}{c\|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.32 & 0.68 & 0 & 0 & 0 & 0 \\ Q_2 & 0.21 & 0.43 & 0.36 & 0 & 0 & 0 \\ Q_3 & 0.06 & 0 & 0.12 & 0.82 & 0 & 0 \\ Q_4 & 0.42 & 0 & 0 & 0.13 & 0.45 & 0 \\ Q_5 & 0.14 & 0 & 0 & 0 & 0.54 & 0.32 \\ Q_6 & 0.63 & 0 & 0 & 0 & 0 & 0.37 \end{array}$	$X^{(n-1)} \updownarrow \begin{array}{c\|cccccc} & \multicolumn{6}{c}{\longleftarrow X^{(n)} \longrightarrow} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q_1 & 0.12 & 0.88 & 0 & 0 & 0 & 0 \\ Q_2 & 0.12 & 0.12 & 0.76 & 0 & 0 & 0 \\ Q_3 & 0.12 & 0 & 0.12 & 0.76 & 0 & 0 \\ Q_4 & 0.12 & 0 & 0 & 0.12 & 0.76 & 0 \\ Q_5 & 0.12 & 0 & 0 & 0 & 0.12 & 0.76 \\ Q_6 & 0.12 & 0 & 0 & 0 & 0 & 0.88 \end{array}$

Table 6.3.3: The transition probabilities $P[X^{(n)} = Q_i]$ for equal and unequal cases

No. of quantum	Unequal						Equal					
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
n=1	0.32	0.68	0	0	0	0	0.12	0.88	0	0	0	0
n=2	0.2452	0.51	0.2448	0	0	0	0.12	0.2112	0.6688	0	0	0
n=3	0.2003	0.3860	0.2130	0.2007	0	0	0.12	0.1309	0.2408	0.5083	0	0
n=4	0.2422	0.3022	0.1645	0.2007	0.0903	0	0.12	0.1213	0.1284	0.2440	0.3863	0
n=5	0.2478	0.2947	0.1285	0.1610	0.1391	0.0289	0.12	0.1202	0.1076	0.1269	0.2318	0.2936
n=6	0.2542	0.2952	0.1215	0.1263	0.1476	0.0552	0.12	0.12	0.1042	0.097	0.1242	0.4345
n=7	0.2591	0.2998	0.1209	0.1160	0.1365	0.0677	0.12	0.12	0.1037	0.0909	0.0886	0.4768

7. GRAPHICAL ANALYSIS

Graphical Analysis is performed under above mentioned three schemes in section 6.1, 6.2 and 6.3 with different data sets considering Unequal and Equal Probability Matrix to put various quantum values. So this analytical discussion on graphs about the variation $P[X^{(n)} = Q_i]$ over three data sets are as follows

SCHEME I:

Unequal

DATA SET 1

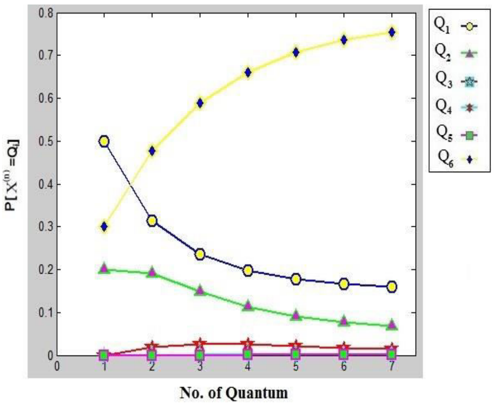


FIG. 7.1

DATA SET 2

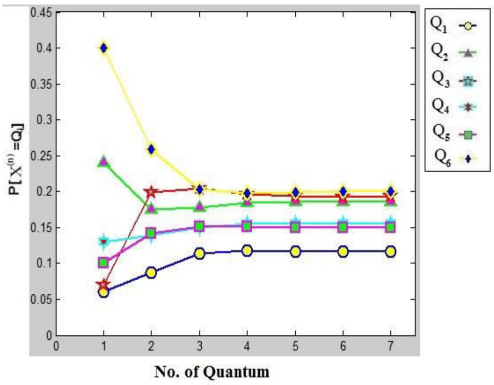


FIG. 7.2

DATA SET 3

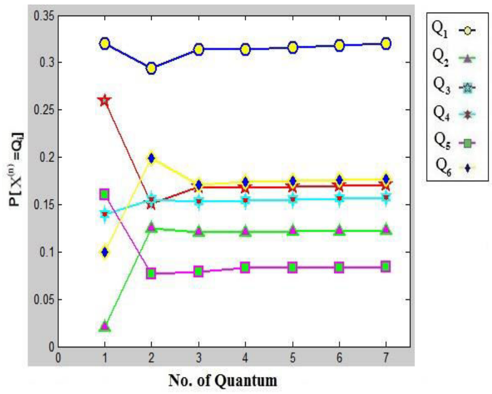


FIG. 7.3

Equal

DATA SET 1

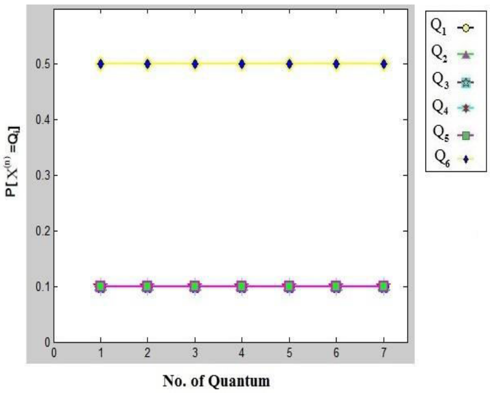


FIG. 7.4

DATA SET 2

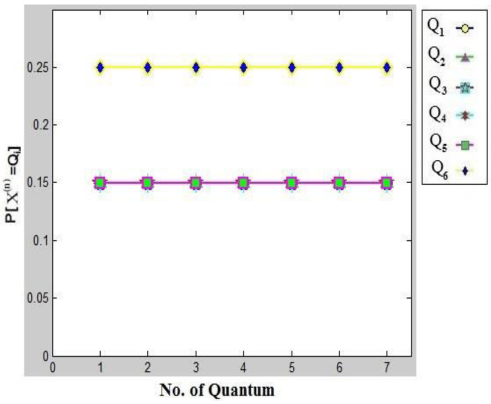


FIG. 7.5

DATA SET 3

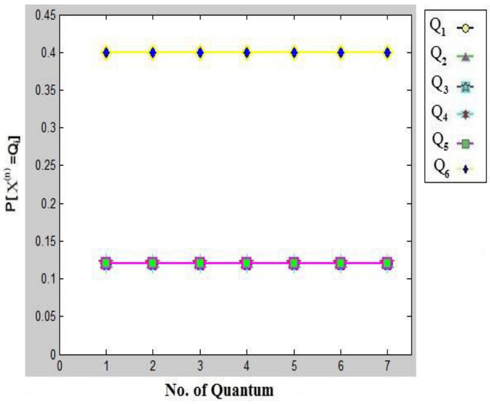


FIG. 7.6

7.2 SCHEME II:

Unequal

DATA SET 1

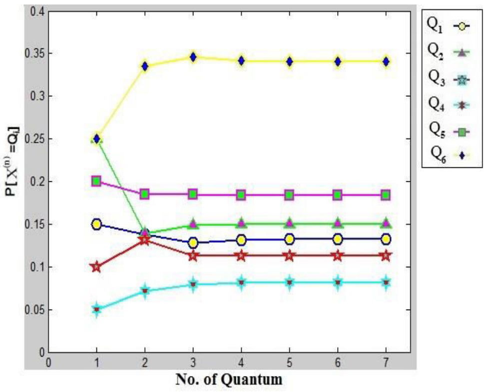


FIG. 7.7

DATA SET 2

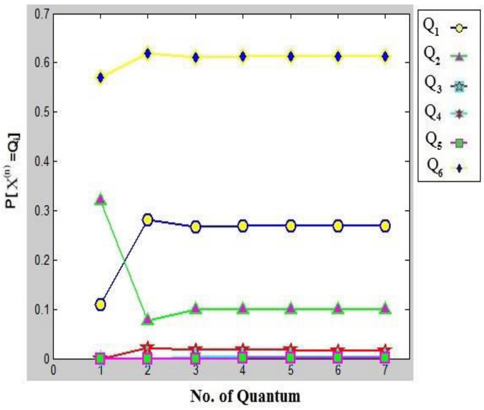


FIG. 7.8

DATA SET 3

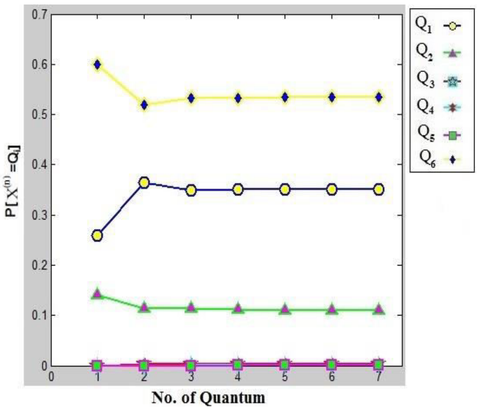


FIG. 7.9

Equal

DATA SET 1

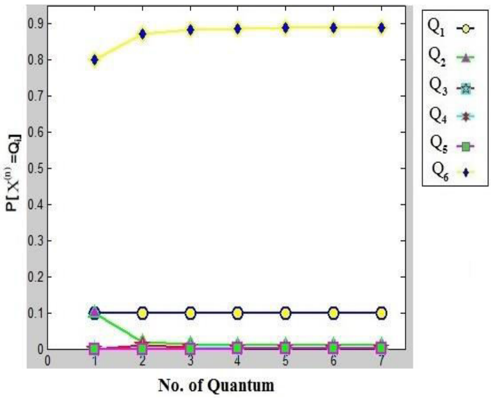


FIG. 7.10

DATA SET 2

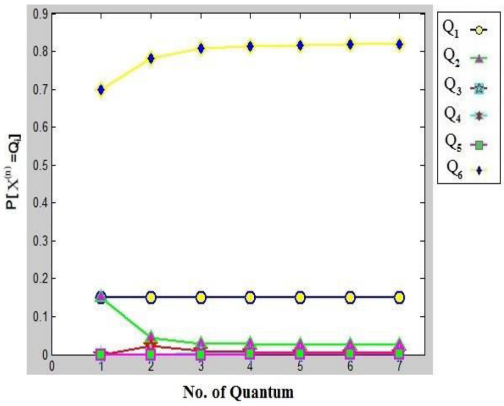


FIG. 7.11

DATA SET 3

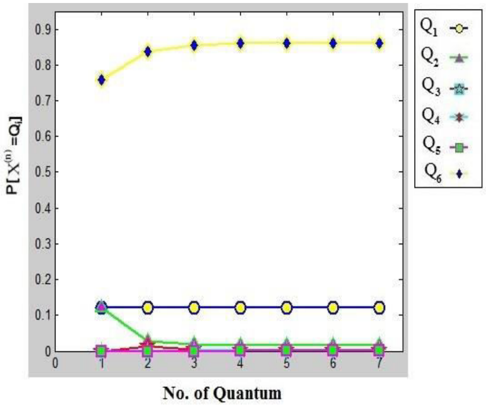


FIG. 7.12

7.3 SCHEME III:

Unequal

DATA SET 1

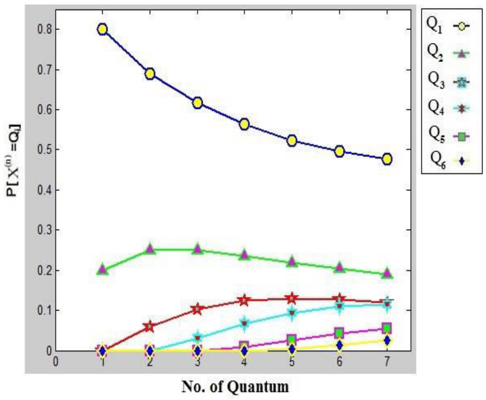


FIG. 7.13

DATA SET 2

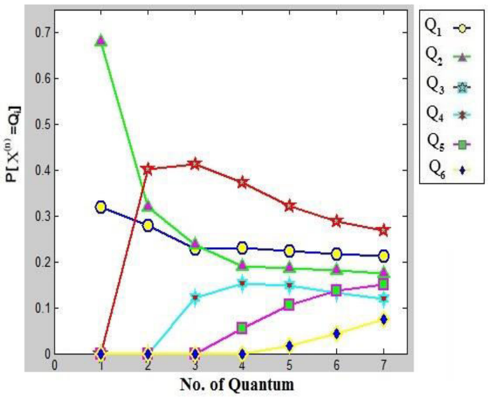


FIG. 7.14

DATA SET 3

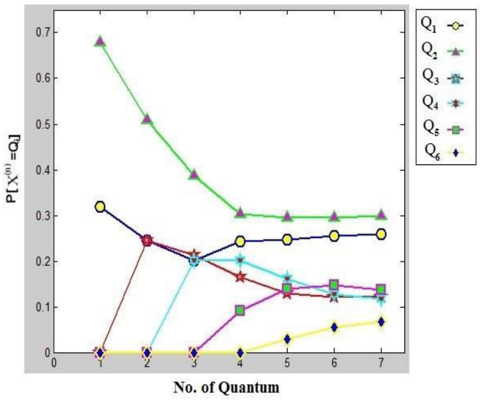


Fig. 7.15

Equal

DATA SET 1

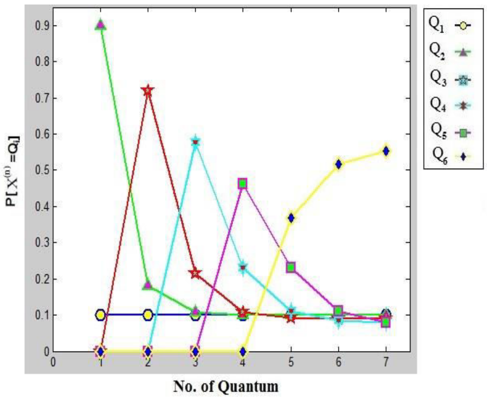


FIG. 7.16

DATA SET 2

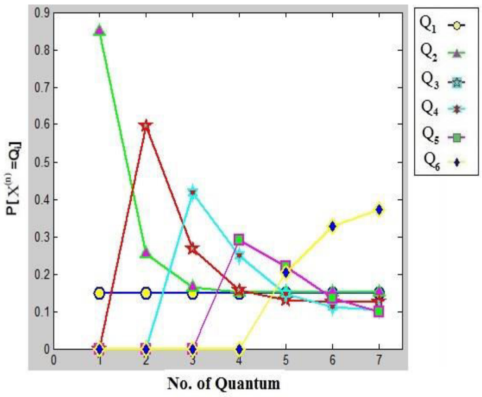


FIG. 7.17

DATA SET 3

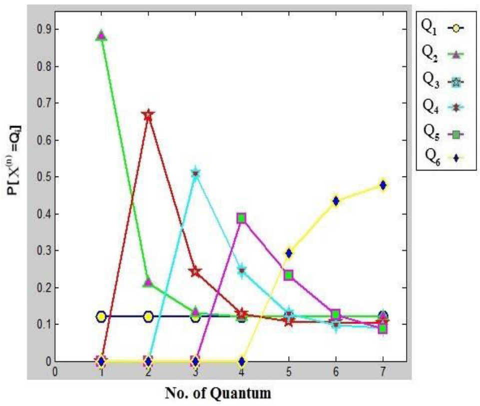


Fig. 7.18

Scheme –I

a) **Unequal:** Although the transition in the states Q_1, Q_2, Q_3, Q_4, Q_5 and Q_6 of the scheduler makes stable pattern when number of quantum $n \geq 2$ but upto $n = 2$ reflects changing in patterns. The remarkable point is that the probability of wait state Q_6 is higher in all data sets than other states especially in fig. 7.1 and fig. 7.2 but state Q_1 is flying high in fig 7.3.This shows a loss of efficiency. So that scheduler spends more time on the wait state than on working states. Therefore, less restricted scheduling scheme leads to a loss of CPU time.

b) **Equal:** The graphical patterns (fig.7.4, fig.7.5 and fig.7.6) reveal static and same in all data sets.

Scheme-II

a) **Unequal:** Graphical patterns (fig.7.7, fig.7.8 and fig.7.9) reveal a higher probability at the wait state than the other states. This again leads to a lack of performance efficiency under these data sets due to more on waiting of the scheduler; Specially probability for the states Q_3, Q_4 and Q_5 is very low as compared to Q_1 and Q_2 in all data sets.

b) Equal: The state probabilities are moved independent of the quantum variation because the pattern of distribution of state probabilities is almost similar in these fig.7.10, fig.7.11 and fig.7.12. So the probability of wait state Q_6 is flying comparatively much high. Therefore it gives degrading in performance and CPU time in scheduling the processes. The special remark is that there are more chance for process contained in Q_1 to be processed than in Q_2, Q_3, Q_4 and Q_5.

Scheme-III

a) Unequal: The probability of scheduler in the wait state is lower than other states probability (for n = 1 to 4, it is almost zero and for n >4, it is slightly high value up to 0.1) over different quantum which is a sign of increase performance efficiency of the MLFQ scheduling in the data sets. The probability of states Q_1 and Q_2 are higher than the previous schemes. Most of the transition probabilities are almost equal in fig 7.14 and fig.7.15 and observed minor variation in fig 7.13 in graphical pattern. The scheme-III provides more chance to job processing than waiting which gives good throughput comparatively to previous schemes.

b) Equal: The transition states pattern in these graphs are identical in fig.7.16, fig.7.17 and fig.7.18, But, the probability of scheduler in wait state is very low (for n =1 to 4, it is zero and for n > 4, it is comparatively high value range from 0.3 to 0.6) which results of good performance of the MLFQ scheduling in these data sets than scheme-I and scheme-II. Other state probability according to quantum variation, Q_2 initiate from higher then moves down but Q_3, Q_4 and Q_5 starts zero in later on shifts up and again going back to down, afterward Q_2, Q_3, Q_4 and Q_5 moves towards almost parallel to Q_1 in all data sets that means gained well being output in this scheme.

8. CONCLUSION

This paper proposes a performance analysis and comparison between three schemes of the multilevel feedback queue scheduling under Markov chain model using equal and unequal probability matrix with various data sets which have features of restriction in terms of some state transition. The equal transition probabilities lead to quantum independency and the information overlapping in scheme-I and Scheme-II which are less restricted scheduling. In the unequal probability matrix, elements make a better picture of transition within states. In these earlier scheduling schemes, the probability towards the waiting state is high enough which indicates for a loss of system efficiency and serious degradation in performance of MLFQ. The graphical pattern does not depend much on quantum variation that is deep effect of equal and unequal probability elements which gives very low chance for processing. Moreover, in these schemes, the different state has less probability which is not a good indication for scheduling. Therefore both schemes are not recommended for further utilization. But in the scheme-III provides a stable pattern of probability variation over quantum almost in all the three data sets. For the variation becomes independent of changes in terms of quantum and wait state probabilities are decreased than other states in both equal and unequal transition matrix. Further, the pattern is having not much variation over changing data. This is an interesting feature which leads to the stability of the whole system that is useful over the earlier two schemes. Therefore, efficiency of this highly imposing restricted scheduling scheme-III in terms of security measures are highly efficient, useful, acceptable and recommendable to light of performance study.

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