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VORTEX BEAMS IN TURBULENT MEDIA: REVIEW

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Abstract

The review covers publications concerned with propagation of laser beams through turbulent media described by the Kolmogorov theory and generalizations thereof to describe signal transmission in optical communications and detection systems. In this case, the turbulent medium is interpreted as an optical channel with random parameters. Various optical signals considered include partially coherent beams, non-uniformly polarized vector beams, as well as specifically configured spatial laser beams. Special attention is given to vortex laser beams. The latter are shown to have a number of remarkable properties that give them an advantage over conventional Gaussian beams.

Keywords: turbulent media, optical signals, vortex laser beams, free-space optics communication systems, diffractive optical elements.

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Content

Introduction

The influence of linear random media on propagating optical beams has been a topic of scientific and engineering research for more than a century. The importance of this work has substantially grown because of the wide use of lasers in optical systems operating in the atmosphere, oceans, biological tissues, etc.

One of the most important applications of light beams is wireless or Free-Space Optics (FSO) in which the optical channel is embedded in a random medium. The most advantageous feature of the FSO is the practically unlimited bandwidth at optical frequencies setting it in striking contrast with the classic radio-wave channels. Also, with the appearance of the highly-directional laser radiation the additional advantage of the FSO communication systems is in their secure signal delivery [1 – 3]. Optical signals can also be used in LIDAR systems for remote sensing of material objects or meteorology [4].

Nevertheless, the effects of the natural random media, such as scintillation, beam wander, particulate scattering, on the deterministic (laser) optical beams are substantial setting limits for technical characteristics of the optical systems and their usage. Therefore, enormous efforts have been made in mitigation of natural random media on optical radiation. Particulate scattering effects is a complex subject that must be treated separately and will be omitted in this review.

As of today there exist several methods for tuning of optical channel: the use of partially coherent radiation, laser beams with a specially designed spatial structure (vortex-like, non-diffractional, high-order laser modes), vector beams with non-uniform polarization, and also the sets of several spatially collocated beams at different frequencies. It was also shown that the diversification of the receiving system techniques, such as multiple arrays and computer methods of the received signal, such as compressive sensing, can also improve the optical system performance [5, 6].

A particular advantage for the enhancement of the information channel capacity is delivered by optical beams carrying orbital angular momentum and having an infinite number of possible quantum states [7]. A substantial success of employing such method of channel diversity has been already demonstrated in optical fibers [8] and in free space [9]. Moreover, the stability of vortex beams exposed to turbulent flow was observed in several experimental studies [10–12].

As a rule, separate modes of laser radiation or separate beams of specially designed shape are being considered. However, various superpositions of multi-mode radiation [13, 14], including the ones demonstrating other properties, for example, self-healing [15, 16], can also possess orbital angular momentum. The resistance of such light beams to the optical non-homogeneities of the medium has not been examined so far.

The simplest method for realizing the invariant laser beams of arbitrary complexity relies on using the diffraction optics elements [17, 18]. More involved techniques allowing for the temporal dynamics of the sent information include Spatial Light Modulators (SLM) [19] or coherent fiber arrays [20].

1. Turbulent media

Turbulent media that are very frequently occurring in nature are perhaps the most difficult ones in characterization of their effects on laser radiation and in development of techniques for their mitigation.

Turbulent flows exist in the Earth's atmosphere, air currents, cumulus clouds, star photospheres, and smoke plums, boundary layers around ships and airplanes and in the ocean currents. The dynamics of the turbulent flow characteristics, such as the magnitude and direction of the velocity field, is disordered (chaotic). Such flow possesses vortexes resulting in mixing of individual currents and layers [21]. Instability of the individual motions leads to increasing complexity of the medium globally, that makes it difficult if not impossible to make qualitative and quantitative predictions of the future states of the flow.

Optical turbulence may be well explained by the presence of the inhomogeneities in the index of refraction or, so called, "turbulent eddies" appearing because of the changes in various physical properties of the matter, such as temperature, pressure and the distribution of the chemical composition (such as salt in the oceanic waters). Such eddies are created in various types of matter due to various physical/chemical/biological mechanisms. Splitting and mixing of the inhomogeneous strictures because of the wind in atmosphere, currents in the oceans, cell growth and fluid transfer in the biological tissues result in energy transfer among the eddies of different scales.

1.1. Theory of Reynolds

The transition from the ordered spatio-temporal behavior, known as laminar, to chaotic (turbulent) occurs as nonuniformity of the medium grows. Such characteristics have been introduced by the founder of the turbulence theory O. Reynolds on the basis of experimental results obtained in 1876-1883 can be described by the *Reynolds number*.

Let L, u and v be the characteristic flow size, the characteristic flow velocity and the kinematic viscosity of the medium (fluid, gas, etc.), respectively. Then the Reynolds number is defined as dimensionless quantity $\mathrm{Re}=uL/v$ [22]. As the Reynolds number increases the laminar flow acquires a chaotic component, called vortex. With further increase of the Reynolds number, the number of eddies grow, and as critical value, say Re_{cr}, is reached the flow transitions from the laminar to the turbulent state. Such transition may be sudden or result from a number of subsequent modifications of the motion.

Air flow turbulence, for example, can be represented in the following way. Temperature convection of nonuniformly heated layers, the appearance of the velocity profile with large vertical gradients (due to friction of air flows against the ground surface) and other factors lead to intense air mixing and result in eddies of large scales, with characteristic size L_0, being on the order of the whole flow and known as the outer scale of turbulence. If the Reynolds number $\Delta u_{L_0} L_0/v$, where Δu_{L_0} is the difference in velocities at separation distance L_0, is sufficiently large, then such eddies become less stable. Then, under the influence of the inertial forces, which in this case exceed the viscous forces, the large eddies break down into smaller ones while transferring the energy to them. In this process The Reynolds number decrease in comparison with the maximum value but remains sufficiently large, hence the new eddies are also unstable and break down into even smaller ones. The energy of the larger eddies is being transferred to smaller ones, and the new structures can be very anisotropic and differ from flow to flow.

The eddies' division process without the loss of energy continues until their scales reach the critical value, say l_0, termed as *the inner scale*, which is corresponding to the values of the Reynolds number on the order of one. Eddies of this size become stable and do not break down any more. Starting from this moment the frictional forces dominate inertial forces, and hence the kinetic energy of eddies converts to heat. The interval of scales between L_0 and l_0 is called *the inertial range*, since eddies belonging to this interval are primarily affected by the inertial forces. The larger the Reynolds number, the larger the inertial range is (the ratio L_0/l_0 is on the order of $\mathrm{Re}^{3/4}$).

Eddies with scales $r \le l_0$ belong to the dissipation range. The dissipation of the kinetic energy (a noticeable transition of kinetic energy, needed for mitigation of the frictional forces, into heat) occurs in the smallest eddies. The existence of the inertial range of scales is possible only if $\mathrm{Re} >> \mathrm{Re}_{cr}$, that, in reality corresponds to condition $\mathrm{Re} > 10^5 \div 10^7$ [24]. The structure of turbulence in the inertial range is fully determined by the energy transferred from larger eddies to smaller, and that in the dissipation range depends on the kinematic viscosity of gas (fluid) v. The process of energy transfer as a cascade of eddies in the turbulent flow has be described by L. Richardson in 1922 (c.f. [25]).

1.2. Theory of A. H. Kolmogorov

J. Taylor has introduced the concept of homogeneous and isotropic turbulence in 1935. The basic property of such a medium is its weak dependence (and invariance in the limiting case) on local isotropy (individual characteristics of the flow). The foundation and development of the theory of locally-isotropic turbulence has been developed A.N. Kolmogorov in 1941 [26, 27].

Due to chaotic behavior of the turbulent flow the analysis of its individual fields is practically impossible. O. Reynolds suggested using time averaging over a given interval however such procedure turned out to be not so simple. Later A.N. Kolmogorov introduced averaging in the sense of the flow ensemble, considering the fields of hydro-dynamic characteristics as random functions of spatio-temporal coordinates [28]. The first account of such a statistical approach to the theory of turbulence has been published in an article of M.D. Millionshchikov (Ph. D. student of A.N. Kolmogorov) in 1939.

Many years later Kolmogorov wrote about his papers on these topics: "I became interested in studying turbulent flows in liquids and gases in late 1930s. It immediately became clear to me that the major mathematical apparatus of such investigations must be the theory of random functions of several variables (random fields), that had only been developed at that time."

A.N. Kolmogorov noticed the weakening of the orienting influence of the average value (i.e. the geometry of the whole flow) with each transition from large to small scale eddies . Hence, in spite of the fact that the average flow and the largest eddies, generally speaking, must be inhomogeneous and anisotropic, the statistical regime of sufficiently small eddies in any turbulent medium with very large Reynolds numbers can be treated as homogeneous and isotropic, not depending on characteristics of the large scale eddies. Based on such argument, A.N. Kolmogorov formulated a hypothesis that the statistical regime of the turbulent flow with high Reynolds numbers is universal and is determined only by two dimensional parameters: the mean speed of energy dissipation ε and the coefficient of viscosity v. Averaged flow influences this regime through the energy transferred from the largest eddies through the whole cascade to the smallest ones, at which the mechanical energy converts to heat.

The Kolmogorov's hypothesis about the universal nature of the statistical regime of small-scale turbulence and its dependence only on dissipation rate ε and coefficient of viscosity v, as well as dimensionality restrictions make it ~~is~~ possible to estimate the lower limit of scale, velocity and duration of the eddies taking part in the energy dissipation process. The correctness of the Kolmogorov's theory has been confirmed by a large number of experiments [29].

A.N. Kolmogorov obtained qualitative and quantitative laws, determining the statistical regime of small-scale fluctuations of the well-developed locally isotropic turbulence, on the basis of two famous similarity hypotheses [18]. These hypotheses gave the opportunity to get the fundamental quantitative relations, having the form of new laws of nature. One of these statements, known as the law of two-thirds states that the mean-square value of the velocity difference, measured at separation distance r, is proportional to $r^{2/3}$. Longitudinal and transverse structure functions, introduced by A.N. Kolmogorov in [23] have been measured experimentally and the law of "two-thirds" has been confirmed on a wide interval of r-values.

The law of two-thirds became one of the cornerstones in the subsequent investigations [24, 25, 29–31] of V.I. Tatarskii, M.A. Obukhov and others. The width of the inertial range, determining the range of applicability of this law is intimately related to the meteorological conditions and the height above the ground (for atmospheric processes).

In his speech at an international symposium on turbulence mechanics in Marcel, France, A.N. Kolmogorov has suggested to change the original Obukhov's hypotheses by two new ones relating to normalized velocity differences and to augment them by a third hypothesis postulating that the probability density averaged over a sphere with radius r and energy is log-normal and that the dependence of dispersion $\ln \varepsilon_r$ from $\ln(L/r)$, L being the characteristic size of the flow in the current of interest is linear.

1.3. Work of A.M. Obukhov

In 1941 simultaneously with works of A.H. Kolmogorov, his Ph.D. student A. M. Obukhov showed, with the help of his equation of spectral balance of turbulent energy that

$$E(\kappa) = A\varepsilon^{2/3}\kappa^{-5/3}, \text{ at } 1/L_0 \ll \kappa \ll 1/l_0, \quad (1)$$

where $E(\kappa)$ is the energy spectral density of the turbulent flow (obtained via the 3D Fourier transform of the structure factor tensor), κ is the wave number, defined as a parameter reciprocal to the linear eddy scale and A is the universal constant (c.f. [31]). The law of "five-thirds" for the turbulent energy spectrum and its experimental confirmation was a breakthrough in the statistical theory of turbulence, and played an important part in the development of modern high-efficiency numerical calculations, such as LES and DES [29].

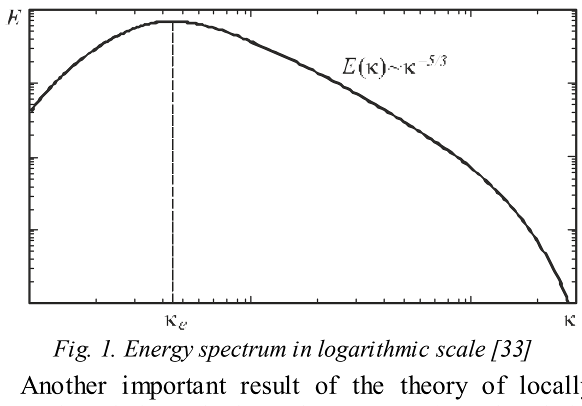


Fig. 1. Energy spectrum in logarithmic scale [33]

Another important result of the theory of locally-isotropic turbulence is the Kolmogorov's formula relating the kinetic turbulent energy, the turbulent viscosity and the energy dissipation rate [32].

1.4. Models of complex random media

The universality of Kolmogorov's spectrum, i.e. its independence from the energy source, is only applicable to the simple turbulent media, for example, the neutral fluids in which the characteristic inner scale is absent. In more complex media, for instance, in plasma, the turbulence results from interaction of different fields and/or disturbances with different characteristic frequencies, scales and absorbance bands. Moreover, nonlinear dissipation mechanisms, collapse of internal waves or surface waves can be of importance. Under such circumstances, simple models based on the inertial range and energy transfer from large to small scales are no longer applicable, and the dimensional analysis alone is not sufficient for obtaining the results in the closed form. Power spectral densities are also possible in such situations, but with certain restrictions, for example if perturbation conditions are satisfied for one type of waves. The example is the Zakharov–Filonenko's spectrum $E(\kappa)\sim\kappa^{-11/4}$, for capillary waves, which also correspond to the inertial range.

Apart from the statistical approach to investigation of the nature of turbulence in the works of O. Reynolds that has led to the theory of isotropic and homogeneous turbulence of Kolmogorov, there also exist structural and dynamic approaches. The foundations of the former, as has been already noted, were given by L. Richardson, who introduced the cascade model of energy transfer from large to small scales.

First attempts to explain the nature of turbulence from the dynamics standpoint as a transition from a nonlinear filed to a disordered, chaotic motion, independent from uncontrollable fluctuations and external noises, have been made by L.D. Landau in 1944 [34] and E. Hopf in 1948 [35]. In their model the flow's complexity grows due to the development of the system of instabilities having incompatible temporal scales. The velocity field becomes more disordered, as a larger number of disturbances with incomparable scales participate in its formation. In such a flow, the autocorrelation function of the velocity field drops fast while it appears possible to find the regular behavior only by monitoring the process during time intervals larger than the Poincare's return time. An image of such turbulence is an attractor representing an open coil on a multi-dimensional torus. A similar attractor, as is shown in a subsequent analysis of D. Ruelle and F. Takens in 1971 [36], is structurally unstable, i.e. it breaks down with a small change of system parameters. This implies that such complex quasi-periodic flow cannot be realized, as a rule.

The conceptual change in the understanding of the nature of turbulence occurred after the discovery of the dynamical chaos phenomenon that is understood as a random behavior of a completely deterministic system. The image of the random behavior of a dynamical system is a so-called strange attractor. A strange attractor is a manifold (set) of trajectories that are all (or almost all) are unstable (saddles) that can appear after small number of bifurcations in the phase space of some very simple flows. The most famous example of it is the convection in a heated torus-shaped cavity located in a vertical plane. The image of chaotic fluctuations of a fluid rotating inside such a cavity is the strange attractor (Lorenz attractor). According to the modern understanding, in the phase space of the Navier-Stokes equations an strange attractor must exist (under certain circumstances) on which the flow's motion corresponds to the regime of the well-defined turbulence.

The theory of turbulent boundary layer which also aims at studying the averaged statistical characteristics of flows in stationary conditions was developed by A.M. Obukhov and A.C. Monin [25, 37–43]. A.M. Obukhov obtained the analogous power laws for a passive fluid [41], under the assumption that the air is uncompressible. The works of A.C. Monin and A.M. Obukhov became the foundation for the air flow in a non-uniform but stationary ground boundary layer. As it stands it is the first theory of anisotropic turbulent air flow.

The effects of the random media on the propagating electromagnetic signals in random media were described by V.I. Tatarskii [24] with the help of the Kolmogorov-Obukhov's turbulence spectrum.

Theoretical and applied problems of turbulence have been investigated in the papers of A. M. Yaglom, another student of A.N. Kolmogorov. In particular, the invariance of frequency spectrum of Lagrange accelerations of the liquid particle in the turbulent flow was established in Ref. [44], as well as its dependence on the dissipation rate of the turbulent kinetic energy. In other words, in the inertial range of the well-developed turbulence this spectrum plays a part of a "white" noise. This result made it possible to relate the results of the Brownian motion theory with the turbulence theory of Kolmogorov-Obukhov. In another early paper by A.M. Yaglom [45] the second exact result in the turbulence theory (the first one was the Kolmogorov's "four-fifths" law of the dynamic equation for the velocity field) has been obtained for the temperature field in the passive flow.

It is also of importance to recall the semi-empiric theory of turbulence (c.f. [46]), being a combination of important, from the applied point of view, methods of calculation of the turbulent flow characteristics, with the help of some empiric relations. The foundation of this theory is due to L. Prandtl, J. Taylor and T. Karman (1915-1935). With the help of this theory, in Ref. [32] A.N. Kolmogorov has obtained the complete system of equations for the turbulent flow, in which turbulence, besides the mean velocity field, is characterized by two more functions of coordinates and time: turbulent energy and its scale. The analogous idea was somewhat later formulated by L. Prandtl [47]. Such an approach has allowed for description of many properties of the turbulent flows. The ideas of semi-empiric theory of turbulence have been applied to the problem of finding of the mean wind field in the ground boundary layer of the atmosphere and to the theory of transfer of suspended particles by the turbulent flow in the papers by Kolmogorov's Ph.D. students A.C. Monin [48] and G.I. Barenblatt [49, 50].

G.I. Barenblatt has developed the theory of turbulence in stratified fluids, has investigated the mechanism of dust winds and tropical storms, has suggested mathematical models of heavy particles transfer in turbulent flows and basic model of turbulent dynamics in spilled mixed fluid in stably stratified fluid, mathematical model of non-stationary heat- and mass transfer in a stably stratified turbulent flow, the model for mitigation of the turbulent friction with the help of polymer additive, mathematical models of turbulent explosion, mathematical model of the flow transfer from laminar state to turbulent with the account of the previously existing turbulence. Also the scaling laws for the well-developed turbulent sheared flows, in particular for tubes, boundary layers and near the wall regions (see for example [51 – 54] and references wherein) have been obtained.

As we have already noted, the theory of turbulence is far from being completed. New experimental results adjust its contents, sometimes modifying its major postulates. For example, there currently exists an opinion that the law of "five-thirds" takes place not for all turbulent flows, and that each turbulent flow has its own energy cascade, depending on its structure [55].

In Ref. [56] it is shown that the universal logarithmic Karman-Prandle law (independent from the Reynold's number) for distribution of velocities in the major internal region of the sheared turbulent flow, whose derivation was suggested by L.D. Landau, is based on a not very relevant assumption. Hence, the logarithmic law cannot be correct and does not confirm the experiment. In [56] an alternative scaling (power) law is suggested that does depend on the Reynolds' number, and also the corresponding resistance law is obtained.

As is mentioned in Ref. [57], in the theory of turbulence the mathematical apparatus of the theory of random functions makes it possible to do a deep analysis of kinematic properties of hydrodynamic fields. The most significant restrictions, from a physical point of view, are coming from the dynamics equations. The investigations of Kolmogorov in this field are not concerned with mathematical part of the theory, i.e. kinematics, but rather with the dynamics of turbulent flows. In the very beginning he became interested in the dynamics of the isotropic turbulence, and already in 1939 his Ph.D. student M. D. Millionshchikov has published an interesting paper [58], relating to the final stage of development of isotropic turbulence, in which, according to the ideas of Fridman and Keller, one may neglect the third moments of the velocity field in comparison with the second moments. In a latter paper of M.D. Millionshchikov [59] a method of statistical approximation was suggested, that makes it possible to obtain an approximate closed system of equations for the second and third order moments from the chain of equations for all the moments. This approximation, which is called from that time the "Millionshchikov's hypothesis", allows neglecting semi-invariants of the forth order and to approximate the forth moments of hydrodynamic fields through the second moments according to the rule valid in the case of multi-dimensional normal distribution of probabilities:

$$\overline{\omega_1\omega_2\omega_3\omega_4} = \overline{\omega_1\omega_2}\cdot\overline{\omega_3\omega_4} + \overline{\omega_1\omega_3}\cdot\overline{\omega_2\omega_4} + \overline{\omega_1\omega_4}\cdot\overline{\omega_2\omega_3}\,. \tag{2}$$

We also note here that the approximate methods of nonlinear system analysis, similar to the "Millionshchikov's hypothesis," play important part in other branches of modern physics.

2. Propagation of light beams in natural turbulent media

Since the invention of the laser in 1950s the influence of natural random media, such as atmospheric turbulence, oceanic turbulence and some soft biological tissues, on propagating optical waves became of a problem of substantial scientific and engineering interest in a number of countries. It became apparent that if an optical wave is deterministic, i.e. it has well defined constant spatial intensity profile and phase, the natural media affect it in two fundamentally different ways: light is scattered by discrete particulate structures [4] and is gradually modified by continuous changes in the refractive index, caused by either thermodynamic changes in the medium, i.e. temperature or pressure in the atmosphere, or the changes in the chemical composition, i.e. salinity in the oceans [4 – 6, 24, 30]. The latter effect is broadly known as optical turbulence. The major practically important outcomes of the laser optical waves interacting with turbulent atmosphere are the additional (to free-space) diffraction, scintillations, beam wander, and changes in the wave's coherence state. In 1970-2000s a number of techniques have been proposed for mitigation of such effects relating to structuring and randomizing the source [19], applying multiple wavelengths [20], and capturing the light by aperture arrays [60].

In this section we will overview some effects of natural random media on the properties of several recently introduced classes of stochastic (random) beams. It has been shown theoretically and experimentally that that if a beam is pre-randomized at the transmitter it can be optimized in terms of spread, spatial intensity distribution and scintillation. However, compared to deterministic laser beams, stochastic beams experience much more complicated effects, such as random source induced additional diffraction, non-monotonic changes in the degree of coherence, possible changes in the degree and state of polarization, and, for quasi-monochromatic beams possible changes in spectral composition. Among the optical technologies that are benefiting from the source randomization are the FSO communication systems and the LIDARs (radars at optical frequencies) [1 – 3]. Since the deterministic beams can be treated as limiting cases of the corresponding stochastic beams, i.e. which preserve the spatial intensity distribution at the source but attain the maximum value of the degree of coherence, we will discuss the general cases of random beams and point to the corresponding deterministic beams as the special cases.

2.1. Models of spatial power spectra of turbulence

We begin by discussing a general approach for predicting second-order statistics of stochastic beams propagating in continuous random media with known correlation functions (or theirs Fourier transforms, power spectra).

According to the classification of random processes [61], most optically turbulent media can be characterized as processes with stationary increments. In such cases the mean value of the refractive index at a position vector $\mathbf{r}=(x, y, z)$ in the three-dimensional space,

$$n_0(\mathbf{r}) = \langle n(\mathbf{r}) \rangle_M , \tag{3}$$

is not a constant but may slowly vary with time of the day, meteorological conditions, etc. Therefore, it is convenient to consider correlation functions of the refractive index adjusted by its mean value. For example, the correlation function at two positions in space, $\mathbf{r}_1$ and $\mathbf{r}_2$, has form:

$$B_n(\mathbf{r}_1;\mathbf{r}_2) = \left\langle \left[n(\mathbf{r}_1) - n_0(\mathbf{r}_1)\right]\left[n(\mathbf{r}_2) - n_0(\mathbf{r}_2)\right]\right\rangle_M , \tag{4}$$

In Eqs. (3) – (4) the angular brackets with subscript M denote the average taken over ensemble of turbulent realizations.

In the case of the well-developed turbulence its statistics are assumed to be homogeneous, i.e. translation-invariant in the inertial range of scales. In such cases the correlation function $B_n(\mathbf{r}_1; \mathbf{r}_2)$ may be represented via its Fourier power spectrum $\Phi_n(\boldsymbol{\kappa})$, on the basis of the famous Wiener-Kintchine's theorem:

$$\begin{aligned} B_n(\mathbf{r}) &= \iiint \exp(i\boldsymbol{\kappa}\cdot\mathbf{r})\Phi_n(\boldsymbol{\kappa})\,\mathrm{d}^3\boldsymbol{\kappa}, \\ \Phi_n(\boldsymbol{\kappa}) &= \left(\frac{1}{2\pi}\right)^3 \iiint \exp(i\boldsymbol{\kappa}\cdot\mathbf{r})B_n(\mathbf{r})\,\mathrm{d}^3\mathbf{r}. \end{aligned} \tag{5}$$

Here $\boldsymbol{\kappa}=(\kappa_x, \kappa_y, \kappa_z)$ is the three-dimensional vector representing spatial frequencies (with units 1/m) along the three Cartesian axes.

The assumption that in the inertial range of scales the turbulence is also isotropic, i.e. rotationally-invariant, implies the following one-dimensional spectral representation.

$$\begin{aligned} B_n(r) &= \frac{4\pi}{r}\int_0^\infty \sin(\kappa r)\Phi_n(\kappa)\,\mathrm{d}\kappa, \\ \Phi_n(\kappa) &= \frac{1}{2\pi^2\kappa}\int_0^\infty \sin(\kappa r) B_n(r)\,\mathrm{d}r, \end{aligned} \tag{6}$$

where r and κ are the magnitudes of vectors $\mathbf{r}$ and $\boldsymbol{\kappa}$, respectively.

For a variety of natural media in which optical turbulence is driven by a single thermodynamic quantity, chemical compound or mechanical process its power spectrum in the inertial range is represented by a power law. In Fig. 2 the one-dimensional power spectrum is shown in the logarithmic scale, hence being a line with a negative slope.

The basic analytic power spectrum model belongs to Kolmogorov [26]:

$$\Phi_n(\kappa) = 0{,}033\, C_n^2 \kappa^{-11/3}, \quad 1/L_0 << \kappa << 1/l_0, \tag{7}$$

where C_n^2 is the refractive index structure, which can have large range of values, depending on the medium. For the atmospheric turbulence near the ground the typical values are $10^{-15} \div 10^{-13}\ \mathrm{m}^{-2/3}$ and go down to zero with growing height. The most popular model for the vertical profile of C_n^2 is Hafnagel-Valley (HV) model [6].

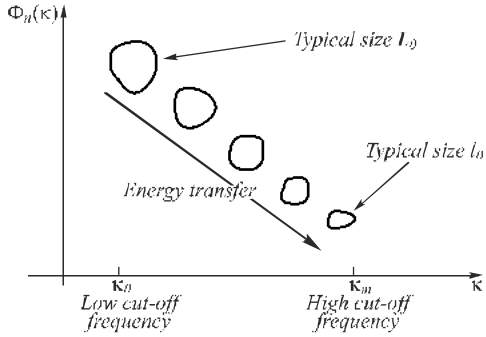


Fig. 2. One-dimensional spatial power spectrum of isotropic turbulence (logarithmic scale)

Kolmogorov spectrum does not take into account the effects of the inner and outer scales of turbulence and, more importantly, leads to divergent integrals for some of the statistical moments of light. More elaborated spectra were suggested by Tatarskii [24] and von Karman [2] which addressed these issues.

Perhaps the most elaborate mathematical model based on the Kolmogorov power law has been obtained by Andrews [62] on the basis of the experimental data of Hill [63]. Hill showed that there exists a bump at high spatial frequencies (driven by the smallest turbulent eddies) which deviates the spectrum from the Kolmogorov's -11/3 line. If $\kappa_m = 5.92/l_0$, and $\kappa_0 = 1/L_0$ are the cut-off spatial frequencies corresponding to the inner and outer scales, and $\kappa_l = 3.3/l_0$ then the spectrum has the form:

$$\begin{aligned} \Phi_n(\kappa) &= 0.033\, C_n^2 \kappa^{-11/3} \frac{\exp\left[-\left(\kappa^2/\kappa_m^2\right)\right]}{\left(\kappa^2+\kappa_0^2\right)^{11/6}} \times \\ &\times\left[1 + 1.802(\kappa/\kappa_l) - 0.254(\kappa/\kappa_l)^{7/6}\right], \quad 0 \le \kappa < \infty. \end{aligned} \tag{8}$$

Classic Kolmogorov power law has been experimentally shown to be violated in a number of cases [64 – 68]. For instance, at high altitudes above the ground the power constant of the atmospheric turbulence has been found to have either smaller or larger values than –11/3. For instance, low values of the power constant imply that smaller scales carry more energy and hence on average turbulent realizations have more refined structure. On the contrary, for higher values of the power constant account for more energy belonging to larger scales and the turbulence has more of the crude irregularities. The deflections in the power constant from the Kolmogorov value result in the qualitative and quantitative changes in the optical beam statistics [69].

Recent measurements of the atmospheric power spectrum at different altitudes above the ground revealed that the atmosphere is of the Kolmogorov type only in the Earth's boundary layer (within the first 2 km) and has three-layered structure (see Fig. 3.)

Analytical models have been developed for the power law spectrum with power constant α varying in the interval $3<\alpha<4$ [71 – 73]. For example, such a model, which also takes into account the inner and the outer scales has form:

$$\Phi_n(\kappa) = A(\alpha)\,\tilde{C}_n^2 \frac{\exp\left[-\left(\kappa^2/\kappa_m^2\right)\right]}{\left(\kappa^2+\kappa_0^2\right)^{\alpha/2}}, \quad (9)$$

$$0 \le \kappa < \infty, \quad 3<\alpha<4,$$

$$\kappa_0 = 2\pi/L_0, \quad \kappa_m = c(\alpha)/l_0,$$

$$A(\alpha) = (1/4\pi^2)\Gamma\left(\alpha-1\right)\cos(\alpha\pi/2),$$

$$c(\alpha) = \left[(2\pi/3)\Gamma\left((5-\alpha)/2\right)A(\alpha)\right]^{1/(\alpha-5)}.$$

Here $\Gamma(\cdot)$ is the Gamma function, and $\tilde{C}_n^2$ is the generalized refractive index structure parameter with units $m^{3-\alpha}$.

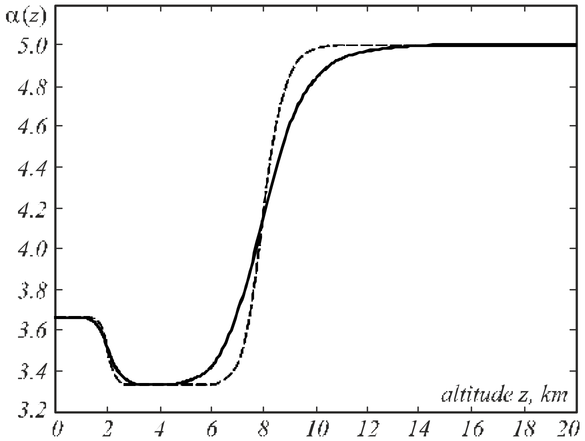


Fig. 3. Three-layer models of non-Kolmogorov turbulence. Two curves correspond to different measurement campaigns [70]

In the close proximity of the ground, i.e. within the first several meters above the ground atmospheric turbulence may exhibit anisotropic characteristics. Namely, it was shown [74] that the refractive index correlation function has smaller width in the vertical direction than in the horizontal. This implies that a typical turbulent eddy is of the ellipsoidal shape rather than spherical (see Fig. 4)

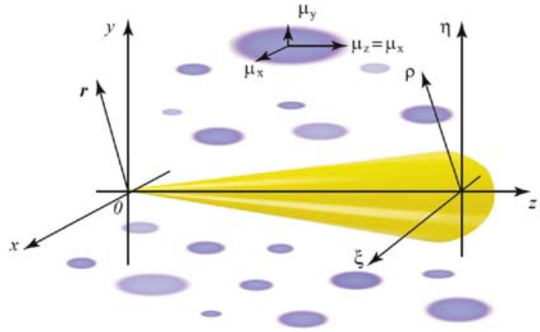


Fig. 4. Anisotropic ellipse of average refractive index fluctuations in atmospheric turbulence

The mathematical model for the anisotropic power spectrum that takes into account turbulent anisotropy, in addition to the inner, outer scales and the power law slope has form [77]

$$\Phi_n\left(\kappa_x,\kappa_y,0\right) = \frac{\mu_x\mu_y\mu_z A(\alpha)\tilde{C}_n^2\exp\left(-\kappa_x^2/\kappa_{mx}^2-\kappa_y^2/\kappa_{my}^2\right)}{\left(\mu_x^2\kappa_x^2+\mu_y^2\kappa_y^2+\kappa_0^2\right)^{\alpha/2}}, \quad (10)$$

$$3<\alpha<4,$$

where μ_x and μ_y are the anisotropic factors in two transverse directions, μ_z is the anisotropic factor in direction of propagation (see Fig. 4), $\kappa_0^2=\kappa_{x0}^2+\kappa_{y0}^2+\kappa_{z0}^2$; $\kappa_{x0}=2\pi/\sqrt{3}L_x$, $\kappa_{z0}=2\pi/\sqrt{3}L_z$ and $\kappa_{y0}=2\pi/\sqrt{3}L_y$ with $L_x=L_z$ and L_y being the outer scales in the $x(z)$ and y directions, respectively, $\kappa_{mx}=c(\alpha)/l_x$, $\kappa_{mz}=c(\alpha)/l_z$ and $\kappa_{my}=c(\alpha)/l_y$ with $l_x=l_z$ and l_y being the inner scales in the $x(z)$ and y directions, respectively.

The optical properties of various soft biological tissues are determined by the local statistical properties of the refractive index. For the majority of the bio-tissues, the average value of the index is 1.34 – 1.36 (with variation within 0.01 – 0.04). More importantly, it was measured by Schmitt and Kumar [87, 88] that the tissues have a well defined power spectrum of the form

$$\Phi_n(\kappa) = \frac{4\pi C_n^2 L_0^2\left(\alpha-1\right)}{\left(1+\kappa^2 L_0^2\right)^{\alpha}}, \quad (11)$$

where $1.28<\alpha<1.41$ and C_n^2 is on the order of $10^{-3}m^{3-\alpha}$.

Oceanic turbulence is due to fluctuating temperature and salinity [89]. The analytical model for the power spectrum of the homogeneous and isotropic oceanic turbulence has been developed by Nikishovs [85] and has the form:

$$\Phi_n(\kappa) = 0.388\times10^{-8}\varepsilon_K^{-1/3}\kappa^{-11/3}\times \times\left[1+2.35(\kappa\eta_K)^{2/3}\right]f(\kappa,w_K,\chi_T), \quad (12)$$

where ε_K dissipation rate of the turbulent kinetic energy per unit mass ($10^{-4}\div10^{-10}$ m²/sec³), $\eta_K=10^{-3}$ m is the inner scale of turbulence,

$$f(\kappa,w_K,\chi_T) = =(\chi_T/w_K^2)\left(w_K^2 e^{-A_T\delta_K}+e^{-A_S\delta_K}-2w_K e^{-A_{TS}\delta_K}\right), \quad (13)$$

where χ_T is the dissipation rate of root-mean square temperature, $A_S=1.9\cdot10^{-4}$, $A_{TS}=19.41\cdot10^{-3}$, $\delta_K=8.284(\kappa\eta_K)^{4/3}+ +12.978(\kappa\eta_K)^2$, w_K – is the temperature-salinity balance parameter varying in the range [–5, 0], 0 corresponding to the temperature-only contributions, –5 corresponding to the maximum possible salinity contribution, in addition to temperature fluctuations. Unlike χ_T и ε_K, that affect the strength of the spectrum, w_K affects its shape (Fig. 5).

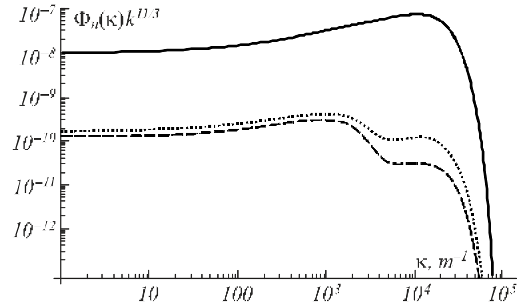


Fig. 5. Oceanic spectrum [84] normalized by Kolmogorov spectrum in the logarithmic scale: $w_K=-0.1$ solid curve, $w_K=-2.5$ dotted curve, $w_K=-4.9$ dashed curve

2.2. Effects of random media on propagating optical beam

Just like for deterministic beams the classic propagation techniques: Rytov approximation, parabolic equations and the extended Huygens-Fresnel integral [6] can be applied for stochastic beams if the moments greater or equal than two must be evaluated.

More recently two other methods have been developed based on the plane-wave decomposition [79] and on the coherent mode decomposition [80].The former approach relies on calculation of the change that the random medium produces on any two tilted plane waves, initially correlated in the source plane. Once such paired plane wave modes are known after propagation in the medium the moments of any random (and deterministic, as the special case) can be determined by summing up the results with suitable weights. For a variety of random media it is fairly simple to calculate the results for the pair of plane waves, tabulate them and then apply the data for any source distributions. The latter method is based on the coherent mode decomposition [90] of random fields being a two-dimensional veriosn of the Mercer's expansion [91]. Indeed, it is well known that any random beam can be represented as the infinite series of coherent modes each being the product of eigenfunctions at a pair of positions, depending on the source correlation function and weighted by the eigenvalues, depending on the source intensity distribution. Hence, if the propagation characteristics of the coherent modes are known in a given medium then the result for any random beam can be found by superposing the propagated modes. In practice, truncated versions of both plane wave series and coherent mode series, can be used with retention of the sufficiently many terms. Such beams are sometimes termed as pseudo-partially coherent.

Let us adopt the following notations for the random beam propagation analysis. Let the random source situated in the plane $z=0$ be characterized by vectors $\mathbf{r}_1'$ and $\mathbf{r}_2'$, where $\mathbf{r}'=(x',y',z')=(\boldsymbol{\rho}',\mathbf{z})$. The beam then propagates into the pozitive half-space $z>0$ close to the positive z-axis (see Fig. 6.). In any plane $z>0$ transverse to the direction of propagation the beam is characterized by vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, where $\mathbf{r}=(x,y,z)=(\boldsymbol{\rho},\mathbf{z})$.

Application of the extended Huygens-Fresnel integral to a beam generated by a random source with second-order correlation function (cross-spectral density) $W(\rho'_1, 0, \rho'_2, 0;\lambda)$, λ being the wavelength, and propagating in a homogeneous and isotropic turbulence with power spectrum $\Phi_n(\kappa)$ results in the following expression for the resulting correlation function $W(\rho_1, z, \rho_2, z;\lambda)$ in the plane $z>0$:

$$\begin{aligned}W(\boldsymbol{\rho}_1,z,\boldsymbol{\rho}_2,z;\lambda)&=\left(\frac{1}{\lambda z}\right)^2\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2;\lambda)\times\\&\times\exp\left\{-i\pi\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}'_1)^2-(\boldsymbol{\rho}_2-\boldsymbol{\rho}'_2)^2}{\lambda z}\right\}\times\\&\times\exp\left\{-\frac{4\pi^4 z}{3\lambda^2}\left[(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2+(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)+\right.\right.\\&\left.\left.+(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2\int\limits_0^{\infty}\kappa^3\Phi_n(\kappa)\,\mathrm{d}\kappa\right]\right\}\mathrm{d}\boldsymbol{\rho}'_1\,\mathrm{d}\boldsymbol{\rho}'_2,\end{aligned}\tag{14}$$

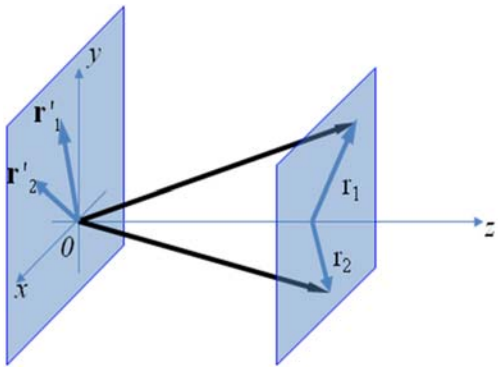


Fig. 6. Schematic diagram illustrating beam propagation in random media

Let us now overview several existing models for optical beams and effects that random media produce on them on propagation. We note that while practically arbitrary intensity distribution can be considered the profile of the correlation function must satisfy certain conditions such as non-negativity and quasi-Hermiticity.

The second-order correlation properties of the Gaussian Schell-model [GSM] being the basic random model-source has the form

$$W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2;\lambda)==I(\lambda)\exp\left[-\frac{\rho'^2_1+\rho'^2_2}{4\sigma^2}\right]\exp\left[-\frac{(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2}{2\delta^2}\right],\tag{15}$$

where the first term defines the spectral composition, that can be of Gaussian form,

$$I(\lambda)=I_0\exp\left[-\frac{(\lambda-\lambda_I)^2}{2\Lambda_I^2}\right].\tag{16}$$

the second term gives the spatial distribution of the average intensity and the last term defines the coherence state of the source.

Parameters σ, δ and Λ_I are the r.m.s. widths of the corresponding distributions whose choice defines the spread of the average intensity, typical width of the degree of coherence and the width of the spectral line. In the limiting case as σ tends to zero or infinity, one obtains spherical or plane wave, as δ approaches zero or infinity an incoherent or coherent wave (lowest order Gaussian beam) is generated and when Λ_I is close to zero or infinity a single-line spectrum or white spectrum is produced.

In order to obtain a beam-like field the parameters of the GSM source must satisfy the inequality:

$$\frac{1}{4\sigma^2}+\frac{1}{\delta^2}<<\frac{2\pi^2}{\lambda^2}.\tag{17}$$

For the general set of parameters of the GSM beam the expression for its second-order correlation function $W(\rho_1, z, \rho_2, z;\lambda)$ can be found in the closed form and analyzed for the beam spreading, as well as its coherence changes and the spectral changes. It has been found (cf. [2]) that the spread term has three components: coherent Gaussian beam spread as in free space, coherent Gaussian beam spread in the turbulent medium and the GSM beam spread as if it propagates in free space. The first and the last terms have quadratic dependence on propagation distance and the second term has cubic dependence. Hence close to the source the intensity and coherence properties of the source play the crucial part in determining the spread while at larger distances the turbulence starts to dominate. The degree of coherence of the GSM beam generally grows (widens) in the beginning of the path, reaches its maximum and then gradually decreases to zero at larger propagation distances. Only in the limit of the coherent GSM source (Gaussian beam) the degree of coherence starts from unity and monotonically decreases with distance. Also the GSM beams with nontrivial spectral composition are shown to exhibit a turn in the shift of the spectral line. At small distances the spectrum is red-shifted but at sufficiently large distances it is gradually blue-shifted back to the original composition. Other effects in the GSM beams like scintillations and beam-wander have also been calculated (cf. [19]) and it was found that the GSM beams are more resilient to turbulence than corresponding Gaussian beams, i.e. they have lower scintillation index and lower beam-wander centroid.

Unlike in the GSM case [94, 95] where the spatial beam distribution and its correlation properties are independent from the spectral composition the following model makes it possible to couple them [101]:

$$W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2;\lambda)= =I(\lambda)\exp\left[-\frac{\rho'^2_1+\rho'^2_2}{4\sigma^2(\lambda)}\right]\exp\left[-\frac{(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2}{2\delta^2(\lambda)}\right], \quad (18)$$

$$I(\lambda)=I_0\exp\left[-\frac{(\lambda-\lambda_I)^2}{2\Lambda_I^2}\right], \quad (19)$$

$$\sigma(\lambda)=\sigma_0\exp\left[-\frac{(\lambda-\lambda_\sigma)^2}{2\Lambda_\sigma^2}\right], \quad (20)$$

$$\delta(\lambda)=\delta_0\exp\left[-\frac{(\lambda-\lambda_\delta)^2}{2\Lambda_\delta^2}\right]. \quad (21)$$

As we see, in addition to $I(\lambda)$ the r.m.s. widths σ and δ might also have the spectral dependence.

Different combinations of the spectral composition of the source parameters have been shown to enable both types of controlled spectral shift (blue and red), being either on- or off-axis. Besides, depending on the combination of parameters, the spectral shifts due to short-distance source correlations can be either suppressed or not suppressed by turbulence effects as the beam propagates over large distances. In other words, in contrast to the earlier reported findings according to which the atmospheric turbulence always tends to suppress the original (on-axis) spectral shift, the results reported in [100] showed that in some situations such a shift can actually be enhanced by the atmosphere. The said effect occurs when the intensity I_0 and root-mean-square width of the beam $\sigma=\sigma(\lambda)$ are modulated by a Gaussian function (see (16)) and δ is a constant. On the other hand, when δ is a Gaussian function and σ is a constant, the choice of the central wavelength λ_δ defines the magnitude and direction of the shift due to turbulence. The spectral composition of both quantities, $\sigma(\lambda)$ and $\delta(\lambda)$, can also produce a number of non-trivial results. In particular, in the situation when the central wavelengths, λ_σ and λ_δ, are located on different sides of λ_I the spectrum of the propagating beam may coincide with the original spectrum at any propagation distance for a given turbulent channel, which was deemed impossible according to previous studies.

A new, so-called multi-Gaussian Schell-model beam (MGSM) proposed in [102] was shown to generate far fields with flat intensity profiles. Its correlation function has form:

$$W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2;\omega)=\exp\left[-\frac{\rho'^2_1+\rho'^2_2}{4\sigma^2}\right]\times \times\frac{1}{C_0}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}\exp\left[-\frac{|\boldsymbol{\rho}'_2-\boldsymbol{\rho}'_1|^2}{2m\delta^2}\right]. \quad (22)$$

The authors in [103] derived analytic formulas for the cross-spectral density function of the MGSM beam under the paraxial approximation at arbitrary distances from the source in vacuum, also deducing expressions for the spectral density and the degree of coherence. In addition, using an extended Huygens-Fresnel principle, the following results for propagation of such beams in linear random media and, in particular, atmospheric turbulence were obtained: while the beams propagating in free space retain the flat intensity profiles near the optical axis, this is not the case in the presence of a random medium, when at sufficiently large propagation distances the intensity returns to a Gaussian-like profile. Thus, the key feature of the MGSM beams, the ability to form far-field flat profiles, can be used either for unlimited propagation distances in vacuum or for limited distances in natural random media, where the propagating beam was shown to retain its profile at a certain distance defined by the proper choice of the beam size σ, correlation width δ, total number of terms M, and the wavelength λ. The ability of the new beams to form the robust flat intensity profiles near the beam axis can find uses for communications, sensing, and in other situations where the presence of a medium between the source and the object is inevitable.

Other two models of Schell-type where the degree of coherence is a product of a Gaussian function with either Bessel function or Laguerre polynomial was shown to generate rings in the far field on propagation in free space [92].

$$W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2)=\exp\left(-\frac{|\boldsymbol{\rho}'_1|^2+|\boldsymbol{\rho}'_2|^2}{4\sigma^2}-\frac{|\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2|^2}{2\delta^2}\right)\times \times J_0\left[\beta\frac{(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)}{\delta}\right], \quad (23)$$

$$W^{(0)}(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2)=\exp\left(-\frac{|\boldsymbol{\rho}'_1|^2+|\boldsymbol{\rho}'_2|^2}{4\sigma^2}-\frac{|\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2|^2}{2\delta^2}\right)\times \times L_n\left[\beta\frac{|\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2|^2}{2\delta^2}\right]. \quad (24)$$

In papers [93, 94] the beams generated by Laguerre–Gaussian-correlated and Bessel–Gaussian-correlated sources were studied on propagation in the atmosphere. It is shown that after forming the intensity rings at some distances from the source the beams fill the central part leading to a 3D bottle beam. Such structures are convenient for circumventing obstacles and trapping of particles.

Schell-like sources leading to more complex intensity distributions in free space and random media were suggested and experimentally realized in Refs. [95, 104 – 111]. They include rectangular profiles, frames and cages, arrays, concentric rings, azimuthally varying profiles, combs, etc. Propagation of these beams in various random media has been extensively studied (cf. [112 – 115])

Only few model sources are known beyond the Schell-type. For instance, in Ref. [36] a source was introduced whose degree of coherence does not depend on the difference between two locations. A beam produced by such a non-uniformly correlated source is able to self-focus either at positions on or off axis at a prescribed distance from the source. For the off-axis case such a self-focused beam retains its maximum at the formed transverse location but shifts its maximum back to the axis at sufficiently large distances [37].

3. Generation of vortex beams and application in optical communication systems

With the advent of diverse laser beams characterized by a desired spatial profile, polarization state, and phase front form, several hundreds of articles have been published over the recent 15 years, handling the propagation of laser beams through atmospheric turbulence. Definite types of laser beams have been shown to be less prone to turbulence-related degradation. Vortex beams are among such beams.

In particular, vortex beams stable to the medium turbulence were discussed in Ref. [10], in which the beams were found to “split, deviate, and wander” from the detector region, while never vanishing. The fifth-order vortex beam was shown to be preserved upon propagation in a turbulent medium over a 2-km distance, before being split into first-order beams that preserve their shape over more than 10 km. The broadening of a vortex beam upon propagation in a turbulent medium was theoretically and experimentally studied in [11]. The vortex beam was shown to be less affected by turbulence when compared with non-vortex beams. A similar result was reported in Ref. [32].

In these studies, it was assumed that the vortex beams can be effectively generated by elements of singular optics [7], which are a particular case of diffractive optical elements (DOEs) [13 – 18].

The Gaussian beam with initial size of 3 – 5 cm can transmit information in a free space over 5 – 10 km [10, 11]. DOE of this size or more (up to 200 mm in diameter) can be produced using CLWS-300 IAE [121]. In the next section, the requirements for DOE resolution are justified and is shown that they are satisfied with said installation.

3.1. Generation of vortex beams using singular optics elements

J.F. Nye and M.V. Berry were the first to introduce vortex beams with phase singularity in 1974 [160], with the beams associated with the peculiarities of randomly scattered fields. The light field has the intensity null at the singularity point, while the phase is indeterminate. Sharp phase changes occur in the vicinity of the singular point.

Laser vortex beams were discussed in numerous studies published by Russian and international researchers. An area of optics that studies optical beams with helical phase singularities (i.e. vortex laser beams) is called “singular optics” [7]. In recent years, properties of the vortex beams based on Bessel and Laguerre-Gauss modes, Zernike functions, hypergeometric modes, and other types of distributions that can be defined as structured laser radiation have been actively studied [161].

A vortex laser beam that carries the orbital angular momentum is described by a phase $\exp(im\varphi)$, where φ is the azimuthal angle relative to the optical axis, m is the topological charge – an integer that shows how many times the phase changes by 2π while circulating around the optical axis.

Simplest elements of singular optics are a spiral phase plate (SPP), a spiral axicon, a spiral zone plate, and a binary grating with a «fork» (Fig. 7). Required resolution for elements with carrier frequencies (Fig. 7*c* – *d*) is not more than 20 lines per mm, which corresponds to line width of 25 micrometers at possibility of CLWS-300 IAE installation to perform recording with a line width 1 micron.

For the first time, the potential use of a SPP as an element of Bessel optics was theoretically discussed in 1984 in the Proceedings of the USSR Academy of Sciences [162]. In 1992 such a SPP was fabricated and experimentally studied [163]. Later on, diffraction of plane and Gaussian beams by a SPP (Fig. 8) was studied in mode detail [164].

Considering that the fabrication of conventional multi-level SPPs is rather challenging, binary DOEs to generate vortex beams that are easier to fabricate were simultaneously proposed. In 1992, an amplitude grating with screw dislocation [165] and a radial helical amplitude grating [166] were fabricated. In both optical elements the vortex phase structure was synthesized via binary encoding with a spatial carrier frequency added to a linear or quadratic relation.

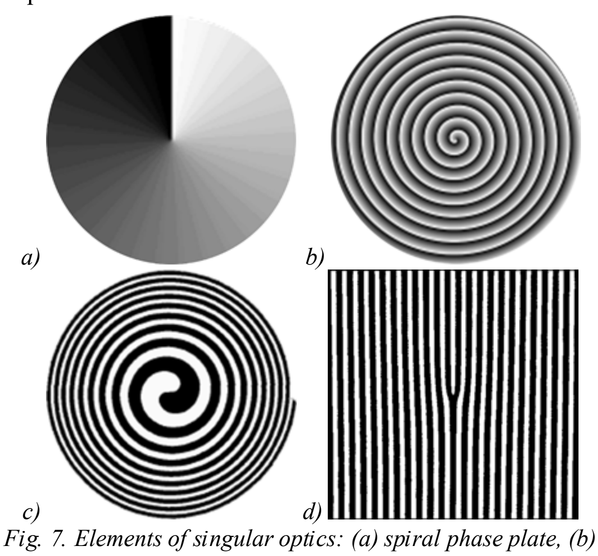


Fig. 7. Elements of singular optics: (a) spiral phase plate, (b) spiral axicon, (c) spiral zone plate, (d) binary “fork”-grating

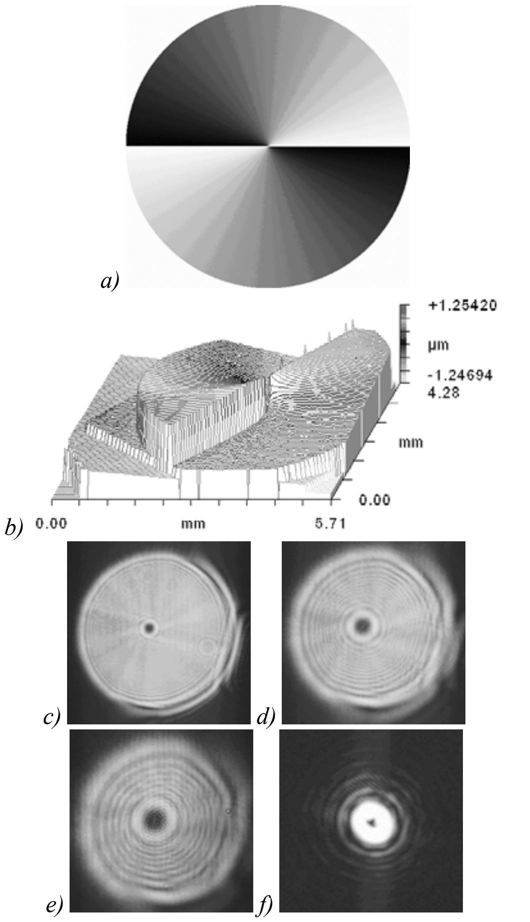


Fig. 8. Experimental generation of laser beam with vortex singular phase of second order [164]: phase distribution (a), central part of spiral phase plate (b) and intensity pictures recorded by CCD-camera at different distances (c) – (e) and at focal plane (f)

Note that in this case a number of spurious diffraction orders inevitably appear, resulting in the corresponding decrease of the diffraction efficiency of the operating diffraction order.

Using more complicated DOEs (Fig. 9) designed based on the above principle, it is theoretically possible to attain a 80% efficiency in generating in the ±1-st diffraction orders of vortex beam superpositions with periodical self-reproduction, including a rotating intensity pattern [167].

To generate a multi-mode laser beam with a predetermined mode content it is required to calculate the phase function of a DOE matched with a finite linear combination of modes with known weights:

$$\tau(x,y) = A(x,y)\exp[i\varphi(x,y)] \approx \approx \sum_{n,m\in\Omega} C_{nm}\Psi_{nm}(x,y), \quad (25)$$

where $\varphi(x, y)$ is the DOE phase, $A(x, y)$ is the DOE amplitude (for phase DOEs, it is related to the amplitude of the incident beam), $\Psi_{nm}(x, y)$ is a set of mode functions, Ω is the set of indices with non-zero coefficients, and C_{nm} are complex coefficients. Squared modules of the coefficients define the weight of the corresponding mode in the beam, while phases of the coefficients are free parameters enabling the DOE complex transmittance to be optimized [17, 154].

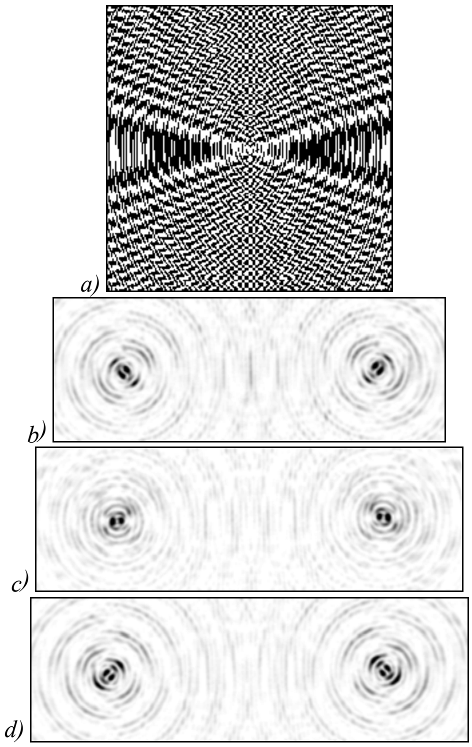


Fig. 9. Experimental generation of rotating laser beams at ±1-st diffractive orders by binary DOE (a) [167]: intensity distributions recorded by CCD-camera at different distances

Note that multi-mode vortex beams can also carry a fractional or even zero AOM [168]. In such beams, micro- and nano-objects get involved in a progressive spiral motion not due to the transfer of OAM but due to a photon energy flux (Fig. 10).

A SPP with fractional topological charge was discussed in [169]. In [170] the illumination of a sectional SPP of integer m^{th} order by light of different wavelengths was shown to result in the generation of m shifted singular intensity distributions with unit topological charge. Symmetry of the intensity pattern was shown to be of the m^{th} order.

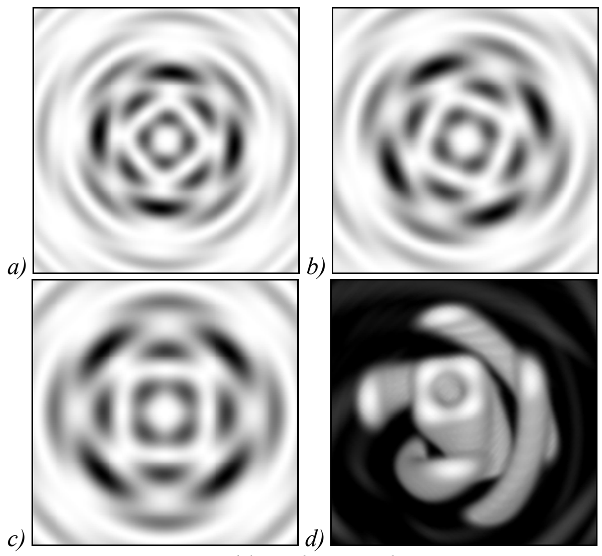
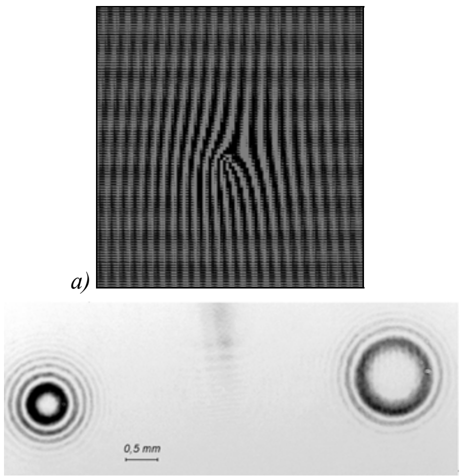


Fig. 10. Rotating Bessel laser beam with zero OAM: intensity distributions at different distances: z = 1.5 m (a), z = 2 m (b), z = 2.5 m (c) and picture of energy transportation (d)

A refractive conical axicon has been known since 1954 [171]. When illuminated from bottom in parallel to the cone axis it can be used to generate a narrow diffraction-free laser beam or, when combined with a lens, a narrow doughnut intensity pattern, as well as enhancing the depth of focus in microscopes. By matching the axicon with a spiral phase plate, one obtains an optical element called a spiral axicon. The first diffractive spiral axicon was fabricated by photolithography and experimentally used for generating higher-order Bessel beams in 1992. A spiral axicon combined with a spherical lens was fabricated somewhat earlier in gelatin by bleaching and then utilized to focus light into a ring with the eliminated central intensity peak [173].

In [174] it was demonstrated that a spiral axicon could be used for obtaining a smaller focal spot from circularly polarized light thanks to the efficient generation of the longitudinal on-axis E-field component. Generation of a photonic nanohelix in the near field with the aid of a binary spiral diffractive axicon was demonstrated in [175].

To generate new types of beams, for example, hypergeometric modes [176], capable of diffraction-free propagation over larger distances when compared with Bessel modes, more sophisticated optical elements need to be used [177] (Fig. 11).



a) b)

Fig. 11. Experimental generation of vortex hypergeometric beams by coded binary DOE [177]: central part of coded element (a), intensity distributions for convergent and divergent beams with vortex phase of third order (b)

3.2. Application of vortex beams in optical communication systems

A definite stability of vortex beams under turbulent perturbations is due to the fundamental principle of energy conservation: in a closed system, conservation laws are applicable to the sum of all components, meaning that while constituent components can exchange the energy, linear and angular momentum, the total sum remains unchanged. Being characterized by a peculiar structure and phase, an optical laser vortex is able to “wander” from the detector region, while not entirely vanishing [10]. When a vortex beam acquires random spatial fluctuations (due to an opaque screen or the propagation through an atmospheric turbulence) it becomes a 'partially coherent vortex beam'. The use of a “wandering beam” model enables a partially coherent beam to be constructed based on a coherent vortex beam [122, 123].

Characteristics of partially coherent Bessel-Gauss beams propagating through an atmospheric turbulence were studied in [124]. The extended Huygens-Fresnel principle was used to find in which way the topological charge (order) of the optical vortex and coherence of the initial field affected the beam's intensity and coherence in the sensor plane. The study has shown that as the beam propagates through a turbulent medium the Bessel-Gauss distribution is eventually transformed into a Gaussian one. The conversion rate depends on the topological charge and coherence of the initial field, as well as the strength of turbulence. It is worth noting that the higher order is the optical vortex, the longer it takes for the conversion to occur. Vortex beams with the large topological charge have been found to be more robust [125, 126].

In Ref. [127], a partially coherent vortex beam propagating in the horizontal direction was shown to broaden in a larger degree when compared with the oblique propagation.

Properties of scalar and vector vortex beams propagating through an atmospheric turbulence were studied and compared in Ref. [128]. The effect of a turbulent medium was modeled using a spectral von Karman model. The dispersion was compared for a fundamental Gaussian beam, a scalar vortex beam with the topological charge +1, and a radially polarized beam at different distances under the same turbulence conditions. The comparison results have shown the vector vortex beam to have an advantage in terms of turbulence-related robustness. The stability of elliptically polarized vortex beams in turbulent conditions has also been studied [129].

Vortex beams are peculiar not only due to their enhanced stability under random fluctuations of an optical medium but also due to their practical importance in channel multiplexing for data transmission. Vortex beams carrying OAM (OAM) have an infinite number of possible quantum states [7], prompting their successful use for mode division multiplexing in optical communication systems [130 – 132].

In particular, an experimental implementation of a free-space 11-dimensional communication system using OAM-carrying modes has been described [12]. The implemented optical communication system was shown to have a maximum measured OAM channel capacity of 2.12 bit per photon. To off-set the negative effects of turbulence, leading to the reduced capacity, it was proposed that the distance between the vortex modes under detection should be increased.

A free-space multi-channel communication system based on OAM-carrying laser beams was discussed in Ref. [133]. The impact of an atmospheric turbulence on the communication system was analyzed, with the turbulence shown to result in a deteriorated capacity and inter-channel crosstalk. Optimal OAM combinations and the multi-channel system capacity were identified for different turbulence levels. The OAM-carrying free-space multiplex telecommunication systems were shown to operate fairly well at low turbulence levels.

As a rule, the multi-channel optical communication systems with mode division multiplexing are realized using diffractive optics components [17, 18, 132, 134]. In more detail, the diffractive optical elements used to generate and select laser modes, including the vortex ones, are discussed in the next section.

The state-of-the art technology of optical communication systems based on the conventional time/frequency division multiplexing is on the brink of reaching the capacity limits. One way to achieve a manifold increase in the communication line capacity is through the use of mode division multiplexing (MDM), which was proposed for optical fibers relatively long ago [135]. Using the MDM, different transverse modes of the same optical fiber are used as a data carrier of a communication system. The data transmitted can be contained either in the mode content or in a proportion of energy carried by a particular laser mode.

The MDM idea has not yet found practical uses because, first, a specified superposition of modes with desired mode energy content is difficult to excite and, second, the energy is being redistributed between the modes in a non-perfect fiber upon long-range data transmission. However it has been theoretically and experimentally shown [136, 137] that in 1-2-m long optical fibers, like those used in endoscopy, characterized by minor bends of radius smaller than the fiber's core radius, modes are not mixed (meaning there is no inter-mode energy redistribution), only acquiring a phase delay due to bend's curvature. It has been experimentally shown [138] that the highest mode (with azimuthal index 9) is insensitive to bends of a 4-m long graded-index fiber, provided that their radius is larger than 5 cm. From [139], the 'mixture' coefficient of adjacent modes has been known to be predominant. Thus, by the excitation of distant modes their mixing can be minimized. In this regard, showing special promise are modes with different azimuthal indices. Recent years have seen a revival of interest in the MDM technology due to the development and ever increasing use of local networks [132, 140 – 142].

When using the MDM, the excitation of a specified superposition of modes with desired inter-mode energy distribution is a major challenge. When low-order modes – e.g. linearly polarized first-order modes – are to be excited, the fiber is often subjected to a periodic deformation (squeezing or bending) [147, 148]. Such modes can also be excited using a tilted grating recorded in a fiber containing a photosensitive medium, by interfering two laser beams [149]. However, a simpler approach in this case is the use of phase modulation. For instance, in [150] the phase modulation was carried out by shifting dielectric plates placed in the wavefront of a Gaussian beam. The excitation of helical modes by coupling laser light into a fiber end at an angle and with an off-axis shift was implemented in [151, 152]. In [153], linearly polarized higher-order modes were generated using amplitude optical masks containing encoded complex mode distribution functions [153]. In this case, a separate mask was utilized for each mode.

All aforesaid methods either are difficult to realize or do not allow the excitation of a mode content with a predetermined energy contribution. The said challenges can be efficiently addressed using diffractive optics techniques. With the aid of DOEs any combination of modes with desired weights can be efficiently excited and selected [132, 134, 154, 155].

Application of DOEs allowed to experimentally implement MDM in the industrial step-index fibers at few-mode regime [156, 157], where the transmission of optical signals, matched with the Hermite-Gaussian modes was carried out on 50 m. However, in this case, application of vortex beams also gives an advantage as it removed the need to coordinate modes scale with the parameters of the optical fiber. The efficiency of this idea was experimentally confirmed in subsequent studies [158, 159].

DOEs that generate a variety of mode beams in different diffraction orders are employed as spatial filters to analyze the transverse mode content of light and perform the simultaneous coupling of light into a set of fibers [132, 134, 154, 178].

If a DOE whose transmittance is represented as a linear combination of a finite number of basis functions $\Psi_{nm}(x, y)$ with different spatial carrier frequencies:

$$\tau(x, y) = \sum_{n=0}^{N}\sum_{m=0}^{M} \Psi^{*}_{nm}\left(x, y\right) \exp\left[i\left(\alpha_{nm}x + \beta_{nm}y\right)\right], \quad (26)$$

is combined with a spherical lens (Fig. 12) and illuminated by a light wave $w(x, y)$, the intensity of light at given points in the focal plane with spatial frequencies $(\alpha_{nm}, \beta_{nm})$ is approximately proportional to the squared modules of the coefficients w_{nm} in the field expansion in terms of the said basis:

$$w(x, y) = \sum_{n=0}^{N}\sum_{m=0}^{M} w_{nm} \Psi_{nm}\left(x, y\right). \quad (27)$$

Optical vortices with integer topological charge are defined by mathematically orthogonal functions:

$$\Psi_{m}(\phi) = \frac{1}{\sqrt{2\pi}} \exp(im\phi). \quad (28)$$

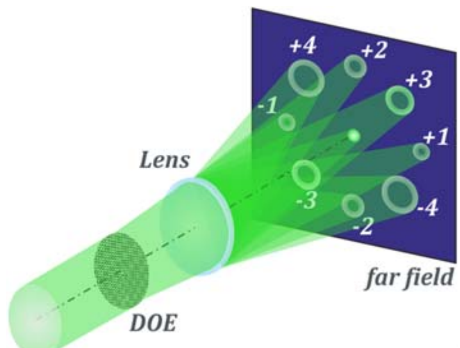


Fig. 12. Action of multi-order DOE with added lens: the coefficients of expansion of the incident field are proportional to the intensity of the corresponding points of the focal plane

Thus, the complex amplitude of an arbitrary light field can be expanded into a series in cylindrical coordinates in terms of optical vortices (or angular harmonics) [179]:

$$f(r,\phi)=\sum_{m=-\infty}^{\infty} f_m(r)\exp(im\phi)\,. \qquad (29)$$

where $f_m(r)=\dfrac{1}{2\pi}\int\limits_0^{2\pi} f(r,\phi)\exp(-im\phi)\,\mathrm{d}\,\phi\,.$ (30)

Assume a multi-order DOE (Fig. 12) matched with a desired set of optical vortices (26):

$$\tau(r,\phi)=\sum_{m=-M}^{M}\exp\left[-im\phi+i\frac{2\pi}{\lambda f}r\rho_m\cos(\varphi-\theta_m)\right], \qquad (31)$$

where (ρ_m, θ_m) are spatial frequencies in polar coordinates, f is the focal length of a lens that performs a Fourier transform, and λ is the wavelength of laser light.

Using a DOE in (31), different-order vortex singularities can be detected simultaneously in the beam under analysis (Fig. 13), performing an optical expansion of the incident beam into a series in terms of an optical vortex basis. The said approach was used to measure the OAM of laser light [180], also showing promise for mode division multiplexing in optical communication systems [132].

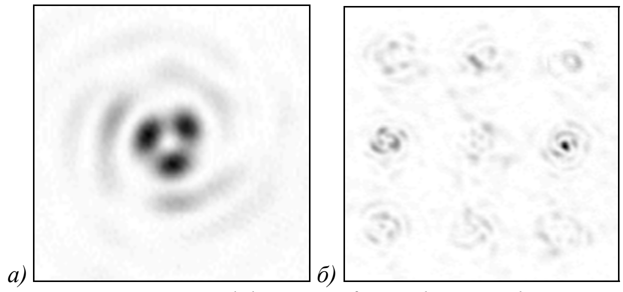


Fig. 13. Experimental detection of optical vortices by means of 8–channel DOE: (a) analyzed beam containing superposition of optical vortices with different orders, (b) focal picture with correlation peaks in according diffractive orders

Note that multi-order DOEs matched with angular harmonics (28) can be used as detectors of the polarization state of laser beams [181], providing an opportunity for the further increase of the number of data transmission channels.

Furthermore, diffractive optical technique allows arbitrarily modulating the states of polarization by means of spatial light modulators (SLM) [182-184], so not only amplitude and phase of a complex light field, but also the transverse states of polarization may be controlled. Polarization state manipulation/encryption [185-187] provides additional flexibility in the key encryption design, complementing amplitude and phase encryption.

The use of MDM technology in free-space optical communication systems also has high potential. Note that the MDM approach utilizing the OAM-related vortex basis seems to be most promising [7, 143, 144]. A representative example is the use of the MDM principle in local wireless networks in different wavelength ranges, including the radio frequency and millimeter range [145, 146].

Conclusion

In this review, we have briefly described the history of development of the theory of turbulent media, discussing, among various models, the non-Kolmogorov ones.

The main bulk of the article discusses the studies of the propagation of light beams in natural media, having shown that definite types of structured beams propagate through random optical medium perturbations with less distortion than a fundamental Gaussian beam. The candidates include partially coherent beams, laser beams with a specific spatial structure (vortex and non-diffracting beams, higher-order modes), non-uniformly polarized vector beams, as well as beam's array.

Of special interest for data channel multiplexing are vortex beams that carry orbital angular momentum and have an infinite number of possible quantum states. The review of articles handling the propagation of vortex beams in turbulent media has shown that as a rule individual modes of laser light are studied. In the meantime, beams composed of multi-mode superpositions can also carry OAM, while also showing self-reproduction properties. Note that the propagation of such composite beams in random media has not yet been studied, although they are of considerable practical interest. It is also worth noting that in fact the only way to generate such beams is by means of diffractive optics elements. This has prompted us in the final part of the article to discuss DOEs enabling an arbitrary superposition of light beams with specified weights to be generated and recognized (detected).

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