[11] Image noise removal based on total variation

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Abstract

Today, raster images are created by different modern devices, such as digital cameras, X-Ray scanners, etc. Image noise reduces image quality and has effects in result of processing. Biomedical images, for example, are a type of digital images. It is considered, the noise of such raster images includes two types: Gaussian noise and Poisson noise. In this paper, we propose a method to remove these noises based on total variation of the image brightness function. The proposed model is a combination of two famous denoising models: ROF and modified ROF.

Keywords: *total variation, ROF model, Gaussian noise, Poisson noise, image processing, biomedical image, Euler-Lagrange equation.*

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Introduction

Digital images are a significant type of information in present researches. Raster images are created by different modern digital devices, such as digital cameras, X-Ray scanners, etc. The application of digital equipment in varying conditions may give rise to various effects on raster images, including noises.

The problem of image noise removal is now relevant, too. For the purpose of successful noise removal it is necessary to know its type. There are different types of noise, such as Gaussian noise (most of digital images produced by digital cameras), Poisson noise (X-Ray images), speckle noise (ultrasonograms), etc.

Many noise removal methods have been presently developed for the cases when a noise type is known. For example, total variation [1-13] is a well known and effective approach.

It seems likely that a concept of total variation for noise removal was first applied in paper [12] by Rudin L.I. and his co-authors Osher S. and Fatemi E. (ROF). They proposed to use total variation in image tasks. ROF is intended for Gaussian noise removal [12, 13].

Certainly, it may also be used for removal of some other noise types however it is not really efficient then. Other popular noise is Poisson noise. For example, this noise appears in X-Ray images. ROF cannot efficiently remove such noise. Therefore, Le T. in his paper [14] created another model known as modified ROF.

Both types of noise (Gaussian and Poisson) are popular, and their combination is also important [15]. It often appears in biomedical images, e.g. in electron microscopy imaging [16, 17].

In order to remove both types of noise, we can combine different models. Therefore, this paper offers to apply ROF in combination with modified ROF.

It is expected that this model should efficiently remove a noise combination with regard to a ratio of two types of noise.

A real image was used in experiments after adding some noise thereto. The quality of processing was compared with other noise removal methods, for example, ROF, modified ROF, median filtering [18], Wiener filtering [19], Beltrami regularization [20]. To compare the image quality after restoration, we used such famous criteria as *PSNR* (Peak Signal-to-Noise Ratio), *MSE* (Mean Square Error) and *SSIM* (Structure Similarity) [21, 22]. The most important of them are definitely *PSNR* and *MSE* interrelated criteria, since they are used to estimate the signal reconstruction quality and the image quality.

1. Mixed Gaussian and Poisson noise removal model

Suppose in space R^2 we have specified a restricted region $\Omega \subset \mathrm{R}^2$. We shall call the functions $u(x,y) \in \mathrm{R}^2$ and $v(x,y) \in \mathrm{R}^2$, respectively, as ideal (without noise) and real (noisy) images, where $(x,y) \in \Omega$.

If the function u is a continuously differentiable function, its total variation is as follows:

$$V_T[u] = \int_\Omega |\nabla u| \, dxdy ,$$

where $\nabla u = (u_x, u_y)$ is a gradient, $u_x = \partial u / \partial x$, $u_y = \partial u / \partial y$, $|\nabla u| = \sqrt{u_x^2 + u_y^2}$. We consider in this paper that total

variation of the function u is limited by $V_T[u]<\infty$.
According to papers [1, 2, 12, 13, 23], smoothness of images may be characterized by their total variation. Total variation of noisy images is always greater than total variation of appropriate smooth images.
For the solution of the problem $V_T[u]\to\min$ it is necessary to put limitations on Gaussian noise variation:

$$\int_\Omega (v-u)^2\,dxdy = const.$$

Under these circumstances ROF for removal of Gaussian noise is as follows [12]:

$$u^* = \arg\min_u\left(\int_\Omega |\nabla u|\,dxdy + \frac{\lambda}{2}\int_\Omega (v-u)^2\,dxdy\right)$$

as the solution of the unconditional optimization problem, where $\lambda>0$ is the Lagrange multiplier.
For the purpose of noise removal based on ROF, we have proposed another model [14]. This model can be obtained in the performance of a constrained problem $V_T[u]\to\min$ as follows:

$$\int_\Omega \ln(p(v\,|\,u))dxdy = \int_\Omega (u-v\ln(u))dxdy = const,$$

as the solution of the unconditional optimization problem:

$$u^* = \arg\min_u\left(\int_\Omega |\nabla u|\,dxdy + \beta\int_\Omega (u-v\ln(u))dxdy\right),$$

where $\beta>0$ is a regularization factor. This model is known as modified ROF for Poisson noise removal.
To build a denoising model of mixed noise, we will also solve the noise removal problem based on a tangent property of total variation: $V_T[u]\to\min$. It is expected that noise variation is constant in the given imaging (Poisson noise is not changed, and Gaussian noise depends only on noise dispersion):

$$\int_\Omega \ln(p(v\,|\,u))dxdy = const, \qquad (1)$$

where $p(v\,|\,u)$ is conditional probability of observation of a real image v at the given ideal image u.
Let us consider Gaussian noise. Its distribution density with dispersion σ^2 is determined as follows:

$$p_1(v\,|\,u) = \exp\left(-\frac{(v-u)^2}{2\sigma^2}\right)/(\sigma\sqrt{2\pi}).$$

Poisson noise density is determined as follows:

$$p_2(v\,|\,u) = \exp(-u)u^v/v!.$$

Note that values of the image brightness functions u and v are whole numbers (for example, for eight-bit images a brightness range is determined from 0 to 255).

To eliminate the combination of Gaussian and Poisson noise, let's consider the following linear combination:
$\ln(p(v\,|\,u)) = \lambda_1\ln(p_1(v\,|\,u)) + \lambda_2\ln(p_2(v\,|\,u))$,
where $\lambda_1>0$, $\lambda_2>0$, $\lambda_1+\lambda_2=1$.

According to (1), we shall obtain the following noise removal constrained task:

$$\begin{cases} u^* = \arg\min_u \int_\Omega |\nabla u|\,dxdy \\ \int_\Omega\left(\frac{\lambda_1}{2\sigma^2}(v-u)^2 + \lambda_2(u-v\ln(u))\right)dxdy = \kappa, \end{cases}$$

where κ is a constant value.
Let's reduce this problem to the unconditional optimization task using the Lagrange functional:

$$L(u,\tau) = \int_\Omega |\nabla u|\,dxdy + \tau\left(\frac{\lambda_1}{2\sigma^2}\int_\Omega (v-u)^2\,dxdy + \lambda_2\int_\Omega (u-v\ln(u))dxdy - \kappa\right),$$

so as to come up with the following solution:

$$(u^*,\tau^*) = \arg\min L(u,\tau), \qquad (2)$$

where $\tau>0$ is the Lagrange multiplier.
In this model, if $\lambda_1=0$ and $\lambda_2=1$ then at $\beta=\tau\lambda_2=\tau$ we shall obtain modified ROF for Poisson noise removal. If $\lambda_2=0$ and $\lambda_1=1$ then at $\lambda=\tau\lambda_1/(2\sigma^2)=\tau/(2\sigma^2)$ we shall obtain the ROF model for Gaussian noise removal. If $\lambda_1>0, \lambda_2>0$ then we shall obtain a mixed Gaussian and Poisson noise removal model.

2. Mixed noise discrete model

To solve the above equation (2), we can apply the Lagrange multiplier method [24, 25, 26]. In this paper we shall use the Euler-Lagrange equation [24].
Suppose the function $f(x,y)$ has been defined in the constrained area $\Omega\subset R^2$ and is continuously differentiated up to the second order in x and y at $(x,y)\in\Omega$.
Suppose $F(x,y,f,f_x,f_y)$ is a convex functional where $f_x=\partial f/\partial x$, $f_y=\partial f/\partial y$. The solution of the optimization problem $\int_\Omega F(x,y,f,f_x,f_y)dxdy\to\min$ shall satisfy the Euler-Lagrange equation:

$$F_f(x,y,f,f_x,f_y) - \frac{\partial}{\partial x}F_{f_x}(x,y,f,f_x,f_y) - \frac{\partial}{\partial y}F_{f_y}(x,y,f,f_x,f_y) = 0,$$

where $F_f=\partial F/\partial f$, $F_{f_x}=\partial F/\partial f_x$, $F_{f_y}=\partial F/\partial f_y$.

Then the solution of the task (2) shall satisfy the following Euler-Lagrange equation:

$$-\frac{\lambda_1}{\sigma^2}(v-u)+\lambda_2(1-\frac{v}{u})-$$

$$-\mu\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)-\mu\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)=0, \quad (3)$$

where $\mu = 1/\tau$.

Let us equate the above equation (3) as follows:

$$\frac{\lambda_1}{\sigma^2}(v-u)-\lambda_2(1-\frac{v}{u})+$$

$$+\mu\frac{u_{xx}u_y^2-2u_xu_yu_{xy}+u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}}=0\,, \quad (4)$$

where $u_{xx}=\frac{\partial^2 u}{\partial x^2}$, $u_{yy}=\frac{\partial^2 u}{\partial y^2}$, $u_{xy}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)=u_{yx}$.

In order to obtain a discrete model (4), we shall add an artificial time parameter $u=u(x,y,t)$. The equation (4) corresponds to the diffusion equation:

$$u_t=\frac{\partial u}{\partial t}=\frac{\lambda_1}{\sigma^2}(v-u)-\lambda_2(1-\frac{v}{u})+$$

$$+\mu\frac{u_{xx}u_y^2-2u_xu_yu_{xy}+u_x^2u_{yy}}{(u_x^2+u_y^2)^{3/2}}\,. \quad (5)$$

Let us consider the $N_1\times N_2$ image. Then a discrete form of the above equation (5) shall be as follows:

$$u_{ij}^{k+1}=u_{ij}^k+\xi\left(\frac{\lambda_1}{\sigma^2}(v_{ij}-u_{ij}^k)-\lambda_2(1-\frac{v_{ij}}{u_{ij}^k})+\mu\varphi_{ij}^k\right), \quad (6)$$

$$\varphi_{ij}^k=\frac{\nabla_{xx}(u_{ij}^k)(\nabla_y(u_{ij}^k))^2}{((\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2)^{3/2}}+$$

$$+\frac{-2\nabla_x(u_{ij}^k)\nabla_y(u_{ij}^k)\nabla_{xy}(u_{ij}^k)+(\nabla_x(u_{ij}^k))^2\nabla_{yy}(u_{ij}^k)}{((\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2)^{3/2}},$$

$$\nabla_x(u_{ij}^k)=\frac{u_{i+1,j}^k-u_{i-1,j}^k}{2\Delta x},$$

$$\nabla_y(u_{ij}^k)=\frac{u_{i,j+1}^k-u_{i,j-1}^k}{2\Delta y},\ \nabla_{xx}(u_{ij}^k)=\frac{u_{i+1,j}^k-2u_{ij}^k+u_{i-1,j}^k}{(\Delta x)^2},$$

$$\nabla_{yy}(u_{ij}^k)=\frac{u_{i,j+1}^k-2u_{ij}^k+u_{i,j-1}^k}{(\Delta y)^2},$$

$$\nabla_{xy}(u_{ij}^k)=\frac{u_{i+1,j+1}^k-u_{i+1,j-1}^k-u_{i-1,j+1}^k+u_{i-1,j-1}^k}{4\Delta x\Delta y},$$

$$u_{0j}^k=u_{1j}^k;\ u_{N_1+1,j}^k=u_{N_1,j}^k;\ u_{i0}^k=u_{i1}^k;\ u_{i,N_2+1}^k=u_{i,N_2}^k;$$

$$i=1,\dots,N_1;\ j=1,\dots,N_2;$$

$$k=0,1,\dots,K;\ \Delta x=\Delta y=1;\ 0<\xi<1,$$

where K is a sufficiently large number, $K=500$.

3. Parameters of the mixed noise model

The above procedure (6) may be used for image noise removal if values of the parameters $\lambda_1,\lambda_2,\mu,\sigma$ have been defined. These parameters are often unknown in practice and must be estimated. The parameters λ_1,λ_2,μ in (6) must be represented as $\lambda_1^k,\lambda_2^k,\mu^k$ in each iteration k . In a new procedure these parameters will be computed at each iteration step.

3.1. Optimal parameters λ_1 and λ_2

Suppose (u,τ) is a solution of the above task (2). Then we shall obtain the condition $\partial L(u,\tau)/\partial u=0$.

This condition enables to calculate optimal parameters of the linear combination of noises λ_1,λ_2 :

$$\lambda_1=\frac{\int_\Omega(1-\frac{v}{u})dxdy}{\frac{1}{\sigma^2}\int_\Omega(v-u)dxdy+\int_\Omega(1-\frac{v}{u})dxdy},\ \lambda_2=1-\lambda_1\,.$$

A discrete form for the calculation of parameters is as follows:

$$\lambda_1^k=\frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(1-\frac{v_{ij}}{u_{ij}^k})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(\frac{v_{ij}-u_{ij}^k}{\sigma^2}+1-\frac{v_{ij}}{u_{ij}^k})},\ \lambda_2^k=1-\lambda_1^k\,,$$

where $k=0,1,\dots,K$.

3.2. Optimal parameter m

For the search of the optimal parameter μ , let us multiply (3) by $(v-u)$ and integrate it by parts across the whole region Ω . Finally, we shall obtain the following formula to find the optimal parameter μ :

$$\mu=\frac{\int_\Omega(-\frac{\lambda_1}{\sigma^2}(v-u)^2-\lambda_2\frac{(v-u)^2}{u})dxdy}{\int_\Omega(\sqrt{u_x^2+u_y^2}-\frac{u_xv_x+u_yv_y}{\sqrt{u_x^2+u_y^2}})dxdy}\,.$$

Its discrete form is as follows:

$$\mu^k=\frac{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(-\frac{\lambda_1^k}{\sigma^2}(v_{ij}-u_{ij}^k)^2-\lambda_2^k\frac{(v_{ij}-u_{ij}^k)^2}{u_{ij}^k})}{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\eta_{ij}^k},$$

where

$$\eta_{ij}^k=\sqrt{(\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2}-$$

$$-\frac{\nabla_x(u_{ij}^k)\nabla_x(v_{ij})+\nabla_y(u_{ij}^k)\nabla_y(v_{ij})}{\sqrt{(\nabla_x(u_{ij}^k))^2+(\nabla_y(u_{ij}^k))^2}},$$

$$\nabla_x(u_{ij}^k)=\frac{u_{i+1,j}^k-u_{i-1,j}^k}{2\Delta x},\ \nabla_y(u_{ij}^k)=\frac{u_{i,j+1}^k-u_{i,j-1}^k}{2\Delta y},$$

$$\nabla_x(v_{ij}^k) = \frac{v_{i+1,j}^k - v_{i-1,j}^k}{2\Delta x} v\,,\ \nabla_y(v_{ij}^k) = \frac{v_{i,j+1}^k - v_{i,j-1}^k}{2\Delta y}\,,$$

$$u_{0j}^k = u_{1j}^k;\ u_{N_1+1,j}^k = u_{N_1,j}^k;\ u_{i0}^k = u_{i1}^k;\ u_{i,N_2+1}^k = u_{i,N_2}^k;$$
$$v_{0j} = v_{1j};\ v_{N_1+1,j} = v_{N_1 j};\ v_{i0} = v_{i1};\ v_{i,N_2+1} = v_{i,N_2};$$

$$i = 1,\dots,N_1;\ j = 1,\dots,N_2;\ k = 0,1,\dots,K;\ \Delta x = \Delta y = 1\,.$$

3.3. Optimal parameter s

To calculate the parameter σ we have used the Immerker method [27] herein:

$$\sigma = \frac{\sqrt{\pi/2}}{6(N_1-2)(N_2-2)} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} |u_{ij} * \Lambda|\,, \qquad (7)$$

where $\Lambda = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix}$ is an image mask.

An operator * is the convolution operator where:

$$u_{ij} * \Lambda = u_{i-1,j-1}\Lambda_{33} + u_{i,j-1}\Lambda_{32} + u_{i+1,j-1}\Lambda_{31} + u_{i-1,j}\Lambda_{23} +$$
$$+ u_{ij}\Lambda_{22} + u_{i+1,j}\Lambda_{21} + u_{i-1,j+1}\Lambda_{13} + u_{i,j+1}\Lambda_{12} + u_{i+1,j+1}\Lambda_{11}\,,$$

where $i = 1,\dots,N_1; j = 1,\dots,N_2$; $u_{ij} = 0$, if $i = 0$, or $j = 0$, or $i = N_1 + 1$, or $j = N_2 + 1$.

The parameter σ is calculated in the first iteration.

4. Image quality assessment

To assess the image quality after noise removal, we have used the criteria *PSNR*, *MSE* and *SSIM* [21, 22]:

$$Q_{PSNR} = 10\lg\left(N_1 N_2 L^2 \Big/ \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - u_{ij})^2 \right),$$

$$Q_{MSE} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - u_{ij})^2\,,$$

$$Q_{SSIM} = \frac{(2\bar{u}\,\bar{v} + C_1)(2\sigma_{uv} + C_2)}{(\bar{u}^2 + \bar{v}^2 + C_1)(\sigma_u^2 + \sigma_v^2 + C_2)}\,,$$

where

$$\bar{u} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} u_{ij}\,,\ \bar{v} = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} v_{ij}\,.$$

$$\sigma_u^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (u_{ij} - \bar{u})^2\,,$$

$$\sigma_v^2 = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (v_{ij} - \bar{v})^2\,,$$

$$\sigma_{uv} = \frac{1}{N_1 N_2 - 1} \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} (u_{ij} - \bar{u})(v_{ij} - \bar{v})\,,$$

$$C_1 = (K_1 L)^2,\ C_2 = (K_2 L)^2;\ K_1 \ll 1;\ K_2 \ll 1\,.$$

For example, $K_1 = K_2 = 10^{-6}$, $L = 2^8 - 1 = 255$ is the brightness of 8-bite grayscale image.

The more is Q_{PSNR}, the better is the image quality. If the value Q_{PSNR} is over the range 20 to 25, the image quality is acceptable, for example, for wireless transmission [28].

The value Q_{MSE} has been used to assess the difference between two images, where Q_{MSE} is the mean square error. The lower the value Q_{MSE}, the better the recovery result. The value Q_{MSE} is directly related to the value Q_{PSNR}.

The value Q_{SSIM} has been used to assess the image quality by comparing the similarity of two images. Its value lies within the interval between -1 and 1. The higher is the value Q_{SSIM}, the better is the image quality.

5. Initial solution

It is clear that in the local iteration procedure (6), the result depends on initial values of the parameters $\lambda_1^0, \lambda_2^0, \mu^0$ in general.

If first we set the parameters $\lambda_1^0, \lambda_2^0, \mu^0$, then unsuccessful values will define not good estimates u_{ij} and through them – distribution parameter estimates.

Arbitrary selection of the parameters $\lambda_1^0, \lambda_2^0, \mu^0$ is also unacceptable, since it can really introduce some additional noise in the image.

It is obvious that initial values of the parameters $\lambda_1^0, \lambda_2^0, \mu^0$ should be close enough, when possible, to those values which will be defined. Therefore, let us estimate the parameters $\lambda_1^0, \lambda_2^0, \mu^0$ as the image average by neighbor pixels using, for example, the Immerker method (7).

6. Experiments

The proposed model has been tested on real images. For example, we used a human skull image [29] with 300x300 pixels in size (Fig. 1*a*). The remaining images (Fig. 1*b* – 1*h*) show an enlarged image segment thereof.

To obtain a noisy image, first we added Gaussian noise (Fig. 1*c*) and then – Poisson noise (Fig. 1*e*). Fig. 1*g* shows the noisy image for two mixed noises with the parameters $\lambda_1 = 0{,}8$, $\lambda_2 = 0{,}2$.

The linear combination parameters λ_1 and λ_2 were defined as follows. First we considered Poisson noise with the distribution density $p_2(v\,|\,u)$ and the variation $\sigma_2 = \sqrt{u_{ij}}$, with respect to the mean value u_{ij} in each pixel with the coordinates (i,j), $i = 1,\dots,N_1; j = 1,\dots,N_2$.

The brightness function of this image is designated as $v^{(2)}$. Its values should be within the range from 0 to 255. If the value is beyond this range, it doesn't change $v^{(2)}{}_{ij} = u_{ij}$. Only these five values (0.0056%) happped to be in this image.

Total dispersion of Poisson noise is defined as the mean value $\bar{\sigma}_2 = 10{,}0603$.

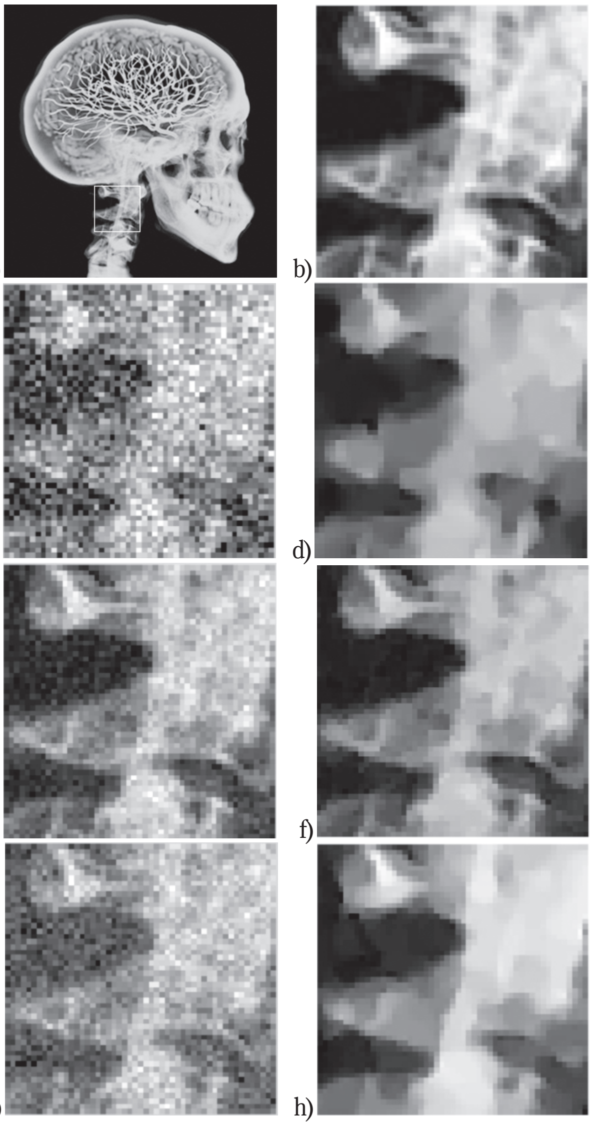

Fig. 1. Noise removal in real images: a) initial image, b) enlarged image segment, c) with Gaussian noise, d) upon Gaussian noise removal, e) with Poisson noise, f) upon Poisson noise removal, g) mixed noise, h) upon mixed noise removal

We further assumed that dispersion of Gaussian noise $\sigma_1 = 40,2412$ was four times more. The brightness function of this image is designated as $v^{(1)}$. As before, values of the brightness function $v^{(1)}$ should be also over the range 0 to 255. In this case, it turned out that 5780 (6.42%) pixels with the brightness value $v_{ij}^{(1)}$ are beyond this range. A resultant image (Fig. 1*g*) has been formed by two noisy images in ratio 0.5 for $v^{(1)}$ and 0.5 for $v^{(2)}$. This means that $v = 0,5v^{(1)} + 0,5v^{(2)}$. Hence,

$$\lambda_1 / \lambda_2 = \frac{40,2412 \cdot 0,5}{10,0603 \cdot 0,5} = 4/1.$$

We shall finally obtain that linear combination coefficients have the following values, respectively: $\lambda_1 = 4/5 = 0,8$ and $\lambda_2 = 1/5 = 0,2$.

For the noisy image the quality criteria have the following values, respectively: Q_{PSNR} =21.4168, Q_{MSE} = 427.9526 and Q_{SSIM} = 0.4246.

Tables 1–3 show the noise removal results in this image for the cases of preset and automatically defined parameters.

Note that in this case the value Q_{PSNR} after noise removal for preset (ideal) parameters is better than for automatically defined estimates, though for the value Q_{SSIM} we can also observe the opposite.

To create the initial image we have used the convolution operator (7). Table 2 shows the dependence of the reconstructed result on the initial solution, where:

(a) the initial parameters $\lambda_1^0 = 0, \lambda_2^0 = 1, \mu = 1$;

(b) the initial parameters $\lambda_1^0 = \lambda_2^0 = 0,5, \mu = 1$;

(c) the initial solution u^0 has been defined as a given sized random matrix;

d) the initial solution u^0 has been defined as the average of neighbor pixels $u^0 = v * A$ by the convolution operator

where $A = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

Table 4 shows that the best result of combined noise removal corresponds to the case (d) of selection of the initial solution by criteria *PSNR* and *MSE*.

Conclusion

This paper offers the method of mixed Gaussian-Poisson noise removal based on the famous variation approach.

The noise removal quality depends on values of the linear combination coefficients λ_1 and λ_2. These values should be preset or automatically defined that is important in the case of processing of real images.

Table 1. Comparison of quality of noise removal methods in real images for mixed noises

Process by	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Without processing	21.4168	0.4246	427.9526
ROF	26.5106	0.8465	145.2183
Modified ROF	26.3153	0.6885	151.8976
Median filter	25.6477	0.7871	177.1364
Wiener filter	24.2657	0.6596	243.5077
Beltrami method	26.8549	0.6678	134.1484
Proposed method with λ_1=0.8, λ_2=0.2, μ = 0.0857, σ = 40.2412	**27.4315**	0.8198	**117.4713**
Proposed method with automatically defined parameters λ_1=0.8095, λ_2=0.1905, μ = 0.0970, σ = 38.2310	27.2567	**0.8383**	122.2941

Table 2. Comparison of quality of noise removal methods in real images with Gaussian noise

Process by	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Without processing	16.5386	0.2516	1442.900
ROF	25.0181	0.7194	204.770
Modified ROF	21.2356	0.4536	489.2402
Median filter	23.1412	0.6314	315.4741
Wiener filter	22.5138	0.5059	364.5051
Beltrami method	20.4575	0.3745	585.2284
Proposed method with λ_1=1, λ_2=0, μ = 0.0978, σ = 40.2412	**25.0200**	**0.7735**	**204.6811**
Proposed method with automatically defined parameters λ_1=0.9738, λ_2=0.0262, μ = 0.0954, σ = 38.9036	24.9681	0.7389	207.1441

For real images this method with automatically defined parameters gives the result, which is close to the ideal, when true parameter values have been predetermined. The method may be used to separately remove either Gaussian or Poisson noise.

Table. 3. Comparison of quality of noise removal methods in real images with Poisson noise

Process by	Q_{PSNR}	Q_{SSIM}	Q_{MSE}
Without processing	27.7349	0.6902	109.5442
ROF	32.0548	0.9355	40.5131
Modified ROF	**33.6101**	**0.9501**	**35.5310**
Median filter	27.7349	0.6902	109.5442
Wiener filter	25.0410	0.8113	203.6962
Beltrami method	31.6356	0.9425	44.6195
Proposed method with λ_1=0, λ_2=1, μ = 0.0853, σ = 0.0001	33.5213	0.9452	36.2235
Proposed method with automatically defined parameters λ_1=0.0045, λ_2=0.9955, μ = 0.0797, σ = 2.7797	32.6244	0.9362	45.3455

Table.4. Dependence of the result of combined noise removal on the initial solution

	(a)	(b)	(c)	(d)
λ_1	0.8095	0.8114	0.9256	0.8069
λ_2	0.1905	0.1886	0.0744	0.1931
μ	0.0970	0.0985	0.1026	0.0965
σ	38.2310			
Q_{PSNR}	27.2567	27.1327	26.4279	**27.2571**
Q_{MSE}	122.2941	125.8371	148.0081	**121.632**
Q_{SSIM}	0.8383	0.8381	**0.8497**	0.8384

The quality of image processing practically equals to the quality of methods dedicated to remove only one type of noise.

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