[6] Phase-locking system modelling based on iteratonal image proceeding methods

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Abstract

Phase-locking system based on the Gerchberg-Saxton algorithm is discussed. A global optimization strategy for retrieving phase information is proposed. The system reduction block-structure method is considered. The system is simulated numerically for different configurations of phased channels.

Keywords: multichannel laser systems, Gerchberg–Saxton algorithm, phase-locking, global optimization, multiaperture wave front sensor.

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Introduction

Development of laser systems with high average power and small beam divergence, particularly with the progress of optical fiber lasers, has recently become one of leading tendencies in the development of laser technologies. However in most cases, increasing the maximum radiation power at near-diffraction output beam divergence shall restrict non-linear and thermooptic processes in active laser medium [1]. One of solutions to this problem is related to implementing the idea of laser beam summation, i.e. creation of multichannel laser radiators. Performance limits of this system may be achieved at coherent summation of laser beams at the output of all channels.

There are over 20 various engineering solutions to the problem of coherent summation of laser beams [2, 3]. Among them of key importance are the methods based on active control of a radiation phase of every laser in the system (active phase-locking methods). In most cases they are implemented by means of a distributed adaptive optical system. Currently, the most widely implemented system is the adaptive aperture sensing system based on the algorithm of parallel stochastic gradient approximation [4, 5]. It is impossible to use the traditional for adaptive optics wave front sensors (WFS), for example, a Shack-Hartmann wave front sensor, in this multiaperture system, because their work is based on the principle of constructing continuous maps of phase aberrations.

In this paper we have considered a phase-locking system for laser radiators with a multiaperture wave front sensor (MWFS) based on the Gerchberg-Saxton algorithm [6, 7]. Numeric analysis and simulation of this algorithm have been performed in the paper, and it has been shown that its characteristic feature for retrieving phase information is the availability of stagnation conditions. A global optimization strategy for retrieving phase information is proposed, and the system reduction block-structure method is considered. Numerical simulation of the system has been performed for different configurations of a multiaperture matrix.

1. Phase-locking system based on the Gerchberg-Saxton algorithm

The algorithm proposed in 1972 by Gerchberg and Saxton enables to retrieve complex fields on the lens aperture and in its focal plane based on their intensity distributions. The mathematical formulation of the problem is to build the complex function $\tilde{E}(\vec{\xi})$ by its module $|\tilde{E}(\vec{\xi})|$ and by the Fourier transform module $|E(\vec{r})|$, where $E(\vec{r})$ is the inverse Fourier transform $\tilde{E}(\vec{\xi})$. A scheme of iterative procedure is given in Fig.1.

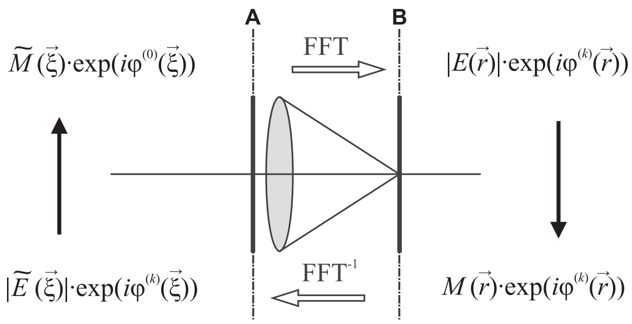


Fig.1. Algorithm scheme for retrieving the field amplitude

For the selected initial phase approximation and the module distribution measured in the aperture plane A, the complex field amplitude is calculated in the focal plane B. Then, the obtained amplitude module is replaced by the measured module. Reversed beam propagation is calculated hereafter. The module is replaced in the aperture plane, and the phase obtained thereat is selected as the next approximation.

The Gerchberg-Saxton algorithm is mathematically written as the following iterative procedure

$$\tilde{E}_0(\vec{\xi}) = \tilde{M}(\vec{\xi})\exp[i\varphi^{(0)}_{\tilde{E}}(\vec{\xi})]; \qquad (1)$$

$$\tilde{E}_k(\vec{\xi}) = P_2 \mathbb{F}\{P_1 \mathbb{F}^{-1}[\tilde{E}_{k-1}(\vec{\xi})]\},\ k=1,2,...$$

where $\varphi^{(0)}_{\tilde{E}}(\vec{\xi})$ is the initial aperture phase estimate (approximation); $\tilde{M}(\vec{\xi}) = |\tilde{E}_{\hat{O}}(\vec{\xi})|$ is a known (measured) module in the aperture plane; $M(\vec{r}) = |E_{\hat{O}}(\vec{r})|$ is the known (measured) module in the focal plane; P_1 and P_2 are operations on module replacement in focal and aperture planes; *FT* and FT^{-1} are direct and inverse Fourier transforms, respectively.

A structural flowchart of the active phase-locking system based on the Gerchberg-Saxton algorithm is given in Fig.2.

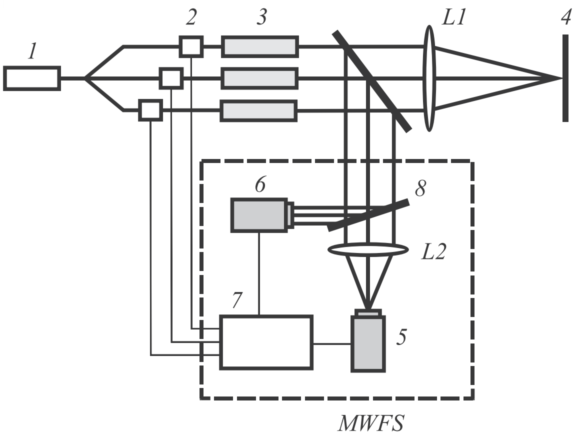


Fig.2. The structural flowchart of the phase-locking system (1 –a driving generator; 2 – phase modulators; 3 – laser amplifiers; 4 – the object plane; 5,6 – CCD-cameras; 7 – a computer; 8 – a beam divider; L1, L2 – lenses).

Optical signal from the integrated driving generator is divided into *N*–number of laser beams. Radiation is divided into two beams after it has passed through a unit consisting of phase modulators and one-mode laser amplifiers. The phase at the output of each amplifier is random due to different optical distances in separate laser channels. The main beam falls on the exit aperture and is focused on the object by the lens L1. The second beam, in turn, is also divided into two sub-beams to register intensity distributions on CCD-cameras in focal and aperture planes of the lens L2. Measured distributions shall go to the computer, where during implementation of the iteration algorithm (1), the phase distribution $\varphi_{\tilde{E}}(\vec{\xi}) = \arg \tilde{E}(\vec{\xi})$ is to be determined, which forms control signals on phase modulators for phase-locking the laser channels.

Thus, the beam divider, CCD-cameras and the computer build up a phase regenerator in the multichannel system, i.e. the multiaperture wave front sensor (MWFS).

2. Numerical analysis of the Gerchberg-Saxton algorithm for the phase-locking system with MWFS

Numeric analysis of the Gerchberg-Saxton algorithm convergence was performed for a hexagonal packing model for laser radiators with the Gaussian amplitude distribution in every channel.

$$A(\vec{\xi}) = A_0 \exp\left[-\frac{|\vec{\xi}|^2}{a_0^2}\right], \qquad (2)$$

where a_0 is the sub-beam equivalent radius; A_0 is the constant value which depends on the defined transmitting power at the input of radiating aperture.

A total number of sub-apertures in this system shall be determined as follows [8]

$$N = 1 + 6\sum_{l=1}^{n} l = 1 + 3n\cdot(n+1), \qquad (3)$$

where *n* is a whole number defining the number of “contours” around a central channel.

The phase in each channel was randomly selected within ± ϖ rad. The quality of retrieving complex functions $\tilde{E}(\vec{\xi})$ and $E(\vec{r})$ was evaluated based on two characteristics. Based on normalized module errors in the aperture plane

$$\tilde{\delta}_k = \frac{1}{\tilde{\delta}_1}\{\int[|\tilde{E}_k(\vec{\xi})| - \tilde{M}(\vec{\xi}]^2 d\vec{\xi}\}^{1/2}, \qquad (4)$$

where $\tilde{\delta}_1$ is the module error at the first iteration, and also based on values of the normalized function of image sharpness [9]

$$S = \frac{1}{S_{PH}}\int I_k^2(\vec{r})d\vec{r} \qquad (5)$$

where S_{PH} is the sharpness function of the phased system (with equal phases in every channel); $I_k(\vec{r}) = |E_k(\vec{r})|^2$.

To understand the Gerchberg-Saxton algorithm behavior for retrieving phase information in MWFS, let us analyze, as an example, the 19-channel laser system. The amplitude distribution in the focal plane of the lens L1 for phased and non-phased systems is given in Fig.3.

Fig. 4 shows the algorithm behavior as starts from two various random initial points $\varphi^{(0)}_{\tilde{E}}(\vec{\xi})$.

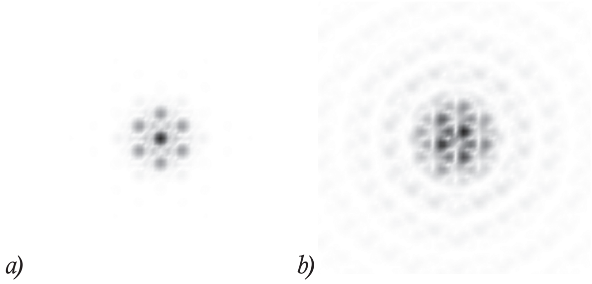


Fig.3. Amplitude distribution in the focal plane of the 19-channel laser system (a – the phased system; b – the non-phased system)

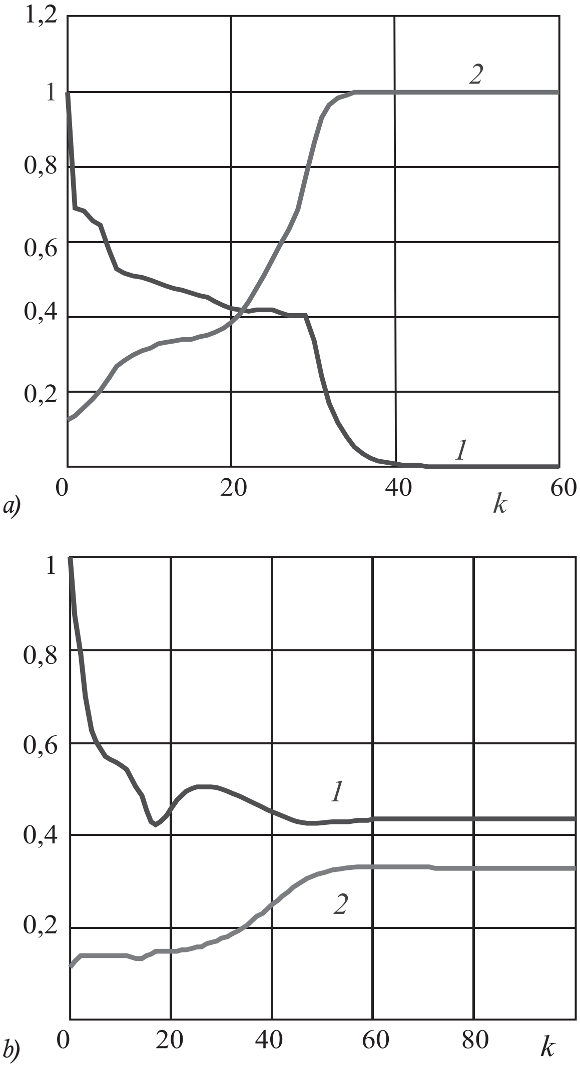


Fig.4. The convergence of iterative procedures (a – the convergence to the true solution; b – the convergence to the local extremum; 1 - $\tilde{\delta}_{mod}$; 2 - $S_{\tilde{E}}$).

It is seen from the figure that in the first case (Fig. 4a) the module error (4) (curve line 1) goes to zero during iterations, and the sharpness function (5) goes to its maximum, i.e. the modules $\left|\tilde{E}(\vec{\xi})\right|$ and $\left|E(\vec{r})\right|$ shall be retrieved here in the only way. Whereas in the second case (Fig. 4b) the algorithm didn’t converge and fall to the stationary state differed from the true state (the so-called ‘stagnation condition’). These conditions are typical for projection image retrieval algorithms and are often associated with local exrema, for example, in the phase problem, where the retrieval algorithm is based on the limited optimization problem which can be solved using the gradient projection method [10]. Therefore, the iteration procedure (1) may be considered as the local optimization of the functional (4).

3. Optimization strategy for retrieving phase information

The existence of stagnation conditions or local extrema in the Gerchberg-Saxton algorithm is considered to be a severe restriction to be used in the phase information retrieval problem in MWFS. The algorithm, being in itself rather simple and fast, is reduced to the true solution $\tilde{E}_{\grave{O}}(\vec{\xi})$ only under certain conditions. For example if the initial approximation $\tilde{E}_0(\vec{\xi})$ is located closely to $\tilde{E}_{\grave{O}}(\vec{\xi})$, when the number of phased channels is small, etc. In these circumstances the guaranteed algorithm convergence may be ensured by means of the global optimization methods.

There are a lot of approaches to solve multi-experimental problems; however, there is no universal standard practice how to solve them. The choice of the optimal method for a particular problem is influenced by various factors caused by specifics of the problem. The specific feature of the Gerchberg-Saxton algorithm is quickness and easiness of the local optimization, when no time-consuming calculation of partial derivatives required. In fact, simplicity of performing the operations P_1 and P_2 (replacement of the obtained module with the known one) ensures a high speed of iterations almost regardless of the dimension of problem. Therefore, when constructing the global optimization procedure, it is reasonable to focus on the approach based on multiple searching for local extrema from different starting points located randomly thorough the whole optimization set, and on further selection of the best one. The algorithm of this type is called a “random multistart” [11]. Its definite advantage, in addition to the speed of iterations, is the possibility for simultaneous searching for local extrema from different starting points that allows one to implement it on a parallel computer consisting of similar processors performing similar operations, the main of which is the fast Fourier transform. In this case, the global extremum search time won’t exceed the performance time for an individual iteration procedure. As it is shown in paper [12], for the phase-locking problem in the multi-channel laser system based on this algorithm, the number of parallel processors linearly increases with increasing the number of phased sources.

However, at the large number of phase-locking channels the increase of the number of parallel processors, which corresponds to the dimension L of the initial retrieval $\{\varphi^{(0)}_{\tilde{E}i}(\vec{\xi})\ i = 1.\,L\}$, doesn't practically influence the result. Fig. 5 gives the dependence of the percentage of the algorithm convergence to the global extremum Conv, depending on the number of laser sources N at L=100 of independent local iteration procedures. While calculating, the number of iterations is k=150, and the maximum module error, at which the algorithm convergence was fixed, is $\tilde{\delta} = 0{,}02$. As is seen from the figure, the average percentage of the algorithm convergence shall decrease depending on the increase of the number of channels in the laser system. In this case, as shown in paper [12], the number of iterations required for the algorithm convergence will increase.

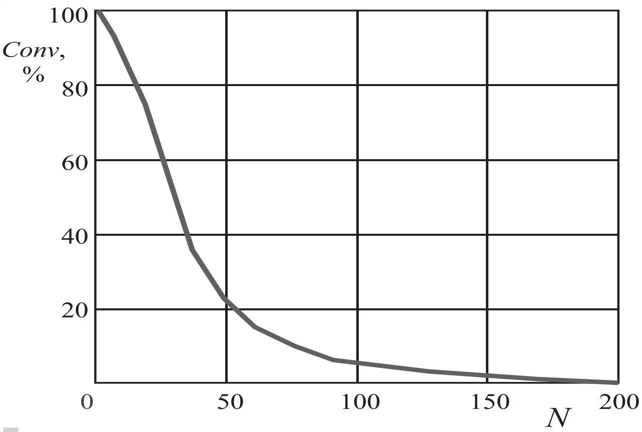


Fig.5. The percentage of the Gerchberg-Saxton algorithm convergence depending on the number of phased channels

From the above figure it is seen that even at *N*=37 the percentage of the algorithm convergence halves (*Conv* ≈ 50%), and at further increasing the number of channels there comes a point, when the algorithm fails to fall into the global extremum from neither of starting points. Therefore, when the number of channels is large, it is necessary to use some methods of reduction (declining) of the dimension of problem, either by means of mathematical methods, or in hardware.

4. Phase-locking block-structure system

One of the possible methods of reduction may be based on the block-structure principle. In this case, the total system of channels is divided into blocks of several channels, in each of which the phase information is parallely retrieved. The base number of channels in the block shall be selected in such a way, that the global extremum is achieved practically during a one-step cycle of iteration procedure. Proceeding from Fig.5, for a hexagonal source arrangement structure with round sub-apertures, the number of channels in the block should not exceed 7 (*Conv* ≈ 98%).

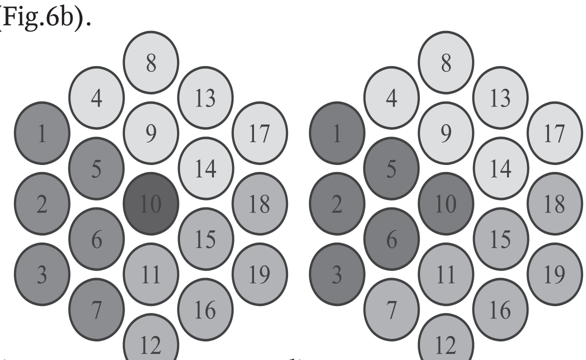
The specific feature of the Gerchberg-Saxton algorithm is characterized by the fact that phase information is retrieved in each channel within an accuracy of general phase shift in the block. However, this phase shift doesn't coincide between individual blocks that result in necessary additional phase "crosslinking" between separate blocks. In this case, it is possible to use the following two options:

1. Dividing the system into blocks with one or several common channels and "crosslinking" the phases with regard to common channels.
2. Dividing the system into blocks without common channels and performing the additional iteration procedure for the general system.

Let's rewrite the formula which defines the total number of sub-apertures in the hexagonal system (3) as follows:

$$N = 1 + 6 \cdot \left[\frac{n(n+1)}{2}\right] = 7 + 6 \cdot \left[\frac{n(n+1)}{2} - 1\right], \qquad (6)$$

where *n* is the whole number (for the block system *n*>*2*). As is seen in the formula (6), in case of hexagonal packing, it is convenient to divide the system either by $\left[\frac{n(n+1)}{2} - 1\right]$ independent blocks six channels each and one block with seven channels, or by $\left[\frac{n(n+1)}{2}\right]$ blocks with seven radiators each with one common channel. Fig.6 shows, as an example, different options of block decomposition of the19-channel system in case of "crosslinking" with regard to the central channel (Fig. 6*a*), and in case of three independent blocks (Fig.6b).



a) *b)*

Fig.6. Options of block decomposition of the 19-channel system (a – the system with the common central channel; b – the system without common channels)

At first glance, the "crosslinking" system with common channels seems to be more attractive, since in this case there is no need to perform any additional iteration procedure for the whole system. However, in this arrangement the system's technical complexity increases, because the problem of optimal radiation decomposition with common channels available hasn't been solved yet. Therefore, let's further consider the Gerchberg-Saxton algorithm convergence for multi-channel laser systems with independent blocks with several channels each.

The results of retrieving the 19-channel block system, when the common central channel available (Fig.6*b*), are given in Fig.7 (curve line 1). In this option in the end of iteration procedure the value of the central channel is deducted from every channel. In this case, the value of the central channel becomes zero in every block; therefore, in order to have a general idea of the field, it is required to simply add the retrieved phase distributions of all three blocks.

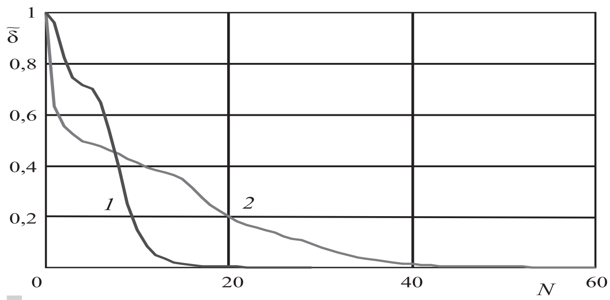


Fig.7. The convergence of iteration procedures (1 – the 6-channel block; 2 – the 19-channel system without blocks)

From the point of view of the algorithm convergence, in case of 19 channels, the difference in iteration quantity for the whole system and for each separate block is not so big. However, when the algorithm converged in the block system practically in each case, for the whole system the algorithm may not go into the global extremum at least six times in sequence.

Similarly, for 37 laser channels the system is divided into six blocks (five blocks by six channels and one block with seven channels). In such decomposition the time required for phase-locking of one block shall remain at the same level as for seven channels (less than 15 iterations), i.e. further increasing the number of phased sources should not result in increasing the phase-locking time; so, only the number of parallel processes may increase (by one or two per each block). Whereas the total number of all necessary iterations for a "non-crosslinking" system shall be composed of the maximum number of iterations for retrieving the 7-channel system and the n-channel system, where n is the number of blocks (Fig. 8).

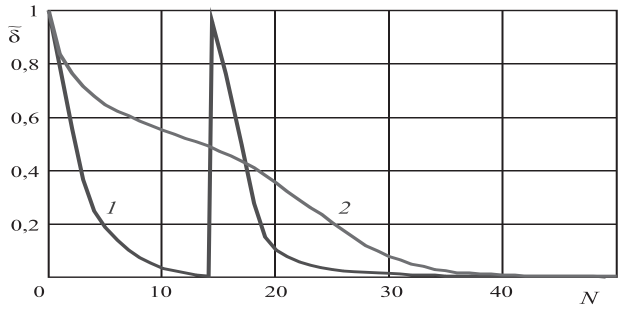


Fig.8. Algorithm convergence at various construction options for the 37-channel system; 1 – a two-stage block system; 2 – a system without blocks

Within this framework, we can construct the dependence of the number of iterations, required for convergence, on the number of channels (Fig.9).

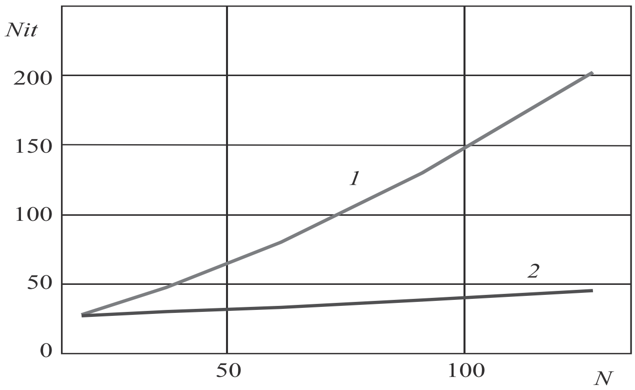


Fig.9. The convergence required iterations depending on the number of channels at various options of the system construction (1 – the system without blocks; 2 – the block-structure system)

Conclusion

The paper describes an approach to the problem of phase-locking the laser radiators based on the iterative image reconstruction algorithms with limitations, particularly, on the Gerchberg-Saxton algorithm. The specifics of these algorithms is the presence of the so-called divergence factor which is characterized by obtaining "successful" and "unsuccessful" solutions, and may be clarified by stagnation conditions available (or by local extrema). The use of global optimization methods allows us to avoid this constraint and to build quite an effective strategy for retrieving phase information, which provides the guaranteed algorithm convergence, and the application of the system reduction block-structure principle makes it possible to coherently add the large (over one hundred) number of channels.

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