[12] Research of the discrete orthogonal transformations received with the use of the dynamics of cellular automata

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Abstract

This paper is aimed at receiving orthogonal bases families from the evolving states of cellular automata. We suggest a comparison technique of the appropriate orthogonal transformations in respect of noises, shown as a result of information losses on the restored data elements.

Keywords: cellular automata, orthogonal transformation, decorrelating, compression.

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Introduction

We have many works currently known, in which a mathematical unit of the theory of cellular automata is used for digital image processing. This unit is applied to improve image quality [1,2], segmentation [3,4], edge detection and test recognition [5-9], and to construct secret separation schemes based on the use of digital images [10], as well as in computer steganography while incorporating digital watermarks into images [11,12].

This paper focuses on the cellular automata approach to construction of orthogonal transformations applied in digital signal processing, particularly, to compress digital images.

It is to be reminded that the most common orthogonal transformations which are used in digital image compression are Karhunen-Loeve transform, Walsh-Hadamard transformation, the discrete cosine transformation, and the discrete wavelet transform (a transformations family) [13-16] .

In addition to the above, there are also other discrete orthogonal transformations. For example, in paper [17] is described the discrete pseudo-cosine transformation as faster operating approximation of the integer discrete cosine transformation.

The paper [18] describes the digital image compression method based on the orthogonal transformation, the bases of which are constructed from the evolving states of cellular automata in Moore-von-Neumann neighborhood.

The author of this paper offers in his works to construct orthogonal bases from the evolving states of one-dimensional cellular automata with the binary alphabet of internal states interpreted as a set of values $\{-1,1\}$, wherefore some special-form local functions are to be defined that shall provide pairwise orthogonality of gradually produced vectors. However, the binary alphabet restricts the space dimension of transformations obtained hereby, which are no more than a particular case of the famous Walsh-Hadamard transformation.

The obtained results were used by other researchers, for example, in computer steganography [12], however the special literature review has shown that any evolvement of the described approach, in terms of increasing the capacity of the internal states alphabet, is currently missing.

In this paper we offer for the construction of orthogonal bases to use cellular automata with the internal states alphabet of the random capacity determined by the value 2^m thus simplifying computations, and to provide research results for the bases constructed for $m=2$ with further increase of this value. In order to formalize the proposed approach a new extension of the classical model of cellular automata – cellular automata with the code set – has been introduced. Besides, the peculiar feature of the proposed approach is not to construct separate orthogonal bases, but their entire families to further select the best of them in the context of the problem here to be studied.

1. Mathematical model of cellular automata

A mathematical model of cellular automata is described by the following set of components: $CA = \langle Z^n, \mathbf{L}, A, \mathbf{Y}, \sigma \rangle$. Its detailed description can be found in [19,20]. Please note that this model corresponds to cellular automata with a finite lattice that seems to be more natural for applied tasks.

Let us introduce an idea of cellular automata with the code set $CA_K = \langle CA, K, \phi \rangle$ as the extension of the classical model of cellular automata, where CA is cellular automata with the internal bases alphabet *A*, *K*— is the ordered set of values, such that $|K| = |A|$, and the transformation $\varphi : A \to K$ shall place elements of the code set *K* in correspondence with characters of the internal bases alphabet *A*.

In this paper we use block cellular automata (partitioning cellular automata) as the base of cellular automata with the code set, which represents one of the existing expansions of the classical model of cellular automata [21]. The main difference between the given cellular automata and the classical cellular automata model is that at any specific time the states of not individual, but block cells are renewed, to which the cellular automata lattice is partitioned, where a block partitioning scheme of the cellular automata lattice varies from step to step. Block cellular automata is described with the set of components $CA_p = \langle Z^n, \mathbf{L}, \mathbf{B}, A, \mathbf{P}, \psi \rangle$, where Z^n, $\mathbf{L}$ and A correspond to similar components of the classical model; $\mathbf{B} = (b_1, \ldots, b_n)$, $b_i > 0$ and $b_i \mid l_i$, $i = \overline{1, n}$ — is the vector which assigns dimensions of the partitioning block;

$\mathbf{P} = (p_1, \ldots, p_m)$,

$p_j \in \{0, \ldots, b_1 - 1\} \times \ldots \times \{0, \ldots, b_n - 1\}$, $j = \overline{1, m}$ — is the vector which assigns the set of block partitioning schemes of the cellular automata lattice and identifies the subsequence of their applications; ψ — is a block transition function which renews the state of each block of the cellular automata lattice at any specific time.

One of the important features of block cellular automata is its reversibility in case when the block transfer function is defined in the form of substitution of the symmetric group $S\left(A^{\prod_{i=1}^{n} b_i} \right)$.

2. Construction of orthogonal bases families using the dynamics of cellular automata

Orthogonal bases are constructed from the evolving states of cellular automata as follows: one-dimensional cellular automata CA is identified, for which $\mathbf{L} = (N)$; cellular automata with the code set $CA_K = \langle CA, K, \phi \rangle$ is defined above it, and for some initial lattice state we analyze the evolving history of cellular automata – the subsequence of the lattice states at time $t = 1, 2, \ldots$ As a result of using the transformation $\phi : A \to K$ at the initial lattice state, we determine the first base vector, after that some more $N - 1$ vectors are selected from the evolving history of cellular automata, so that their pairwise orthogonality might happen to be.

An orthogonal bases algorithm to be built using the dynamics of block cellular automata has been defined in paper [20]. Selection of block cellular automata results from the fact that due to its inherent reversibility, which occurs when the block transfer function is a bijection, its evolving histories do not contain the cycles with an indefinite length.

We shall designate the set of orthogonal bases obtained from the evolving states of cellular automata with the code set $CA_K = \langle CA, K, \phi \rangle$ for all possible initial sates of the lattice as the orthogonal bases family, and shall denote it as $\sum(CA_K)$. We may also speak of the orthogonal transformations families based on the bases from $\sum(CA_K)$.

It was experimentally found that some of the families (for $N = 8$), in general, may contain up to several hundreds of thousands of different bases. This fact has determined the need to partition the bases families to bases subfamilies, consisting of orthogonal bases whose transformations are similar in their properties, and to create an appropriate algorithm which allows assign the given base to a particular subfamily. Such algorithm has been defined in paper [22]. It is aimed at identifying the ability of the orthogonal transform to partition data elements into frequency components. For this purpose, the spatial redundancy vector shall be defined and transformed by formula

$$\mathbf{G} = \mathbf{F}(\mathbf{D} \cdot \mathbf{C}) \qquad (1)$$

where $\mathbf{F} = (f_i)_{i=1}^{N}$ — is a target vector (a row vector);

$$\mathbf{C} = (c_{ij})_{i=1,\, j=1}^{N,\, N},\ c_{ij} \in K,\ i, j = \overline{1, N}$$

is the base of the discrete orthogonal transformaion;

$$\mathbf{D} = (d_{ij})_{i=1,\, j=1}^{N,\, N},\ d_{ij} \begin{cases} \dfrac{1}{\sqrt{\sum_{j=1}^{N} c_{ij}^2}}, & \text{at } i = j, \\ 0, & \text{at } i \neq j, \end{cases}$$

$i, j \quad \overline{1, N}$ – is a diagonal normalization matrix.

After the vector **F** has been transformed, we may determine which of the transformed data elements may be referred to low-frequency components containing the basic (average) information on the initial vector **F**, and which of them – to high-frequency elements which show the variance between separate elements of the vector **F**. For this purpose the parameter λ is introduced identifying the minimum ratio between the element value g_i, $i=\overline{1,N}$, the vector **G** and the value

$$M=\frac{1}{N}\sum_{j=1}^{N} f_j,$$

wherein this element is considered to be a low-frequency component.

We shall note that the term "frequency components" is introduced similarly to the classical orthogonal transformations being used in digital image processing.

B, in the considered algorithm we compute the value of data scattering for low-frequency components by formula

$$v=\frac{\min(\hat{g}_i)}{\max(\hat{g}_i)} \quad (2)$$

where $\hat{g}_i$, $i=\overline{1,r}$, $0\le r\le N$ – are elements of the vector **G** referred to low-frequency components.

Therefore, as the subfamily of the orthogonal bases family $\Sigma(\mathrm{CA}_K)$, we shall call the set of such bases belonging to $\Sigma(\mathrm{CA}_K)$, so that the orthogonal transformations built on their basis have in their frequency spectra r low-frequency components satisfying the specified values of the determining coefficient λ and the dispersion index v, and we shall insymbol as follows $\sum_{r,\lambda,v}(\mathrm{CA}_K)$, $\sum_{r,\lambda,v}(\mathrm{CA}_K)\subseteq\sum(\mathrm{CA}_K)$.

Thus, changing the parameters r, λ, v, we can obtain different kinds of orthogonal transformations including approximations of the known transformations. In particular, we shall note that the foregoing discrete pseudo-cosine transformation may be described in terms of cellular automata as a representative of the bases subfamily $\sum_{1,\lambda}(\mathrm{CA}_{\{-2,-1,1,2\}})$ for some cellular automata CA, where $\lambda\le 0,4$ (the dispersion parameter v is meaningless at $r=1$).

In this paper we construct and analyze the orthogonal transformations similar to the discrete wavelet transform, when $N\equiv 0\,(\mathrm{mod}\,2)$ and $r=\frac{N}{2}$.

In order to build the orthogonal bases we used the dynamics of cellular automata $\mathrm{CA}_K=\langle \mathrm{CA}_\mathrm{p}, K, \phi\rangle$ with the code set $K_s=\{-2^s,\ -2^s+1,\ 2^s-1,\ 2^s\}$, where s – is a true value defined above the block cellular automata $\mathrm{CA}_\mathrm{p}=\langle Z,(8),(2),A,\mathbf{P},\psi\rangle$, where the alphabet of internal states is $A=\{0,1,2,3\}$, a set of partitioning schemes is $\mathbf{P}=[(0)\ \ (1)]$ and the block transition function is $\psi=(0\ \ 8\ \ 4)(3\ \ 5\ \ 15\ \ 12\ \ 11\ \ 14\ \ 6\ \ 7\ \ 9\ \ 13)$. ϕ has been naturally defined as $0\mapsto -2^s$, $1\mapsto -2^s+1$, $2\mapsto 2^s-1$, $3\mapsto 2^s$.

Table 1 presents quantitative characteristics of the constructed families and subfamilies of orthogonal bases for $s=\overline{1,9}$.

Table 1. Number of constructed orthogonal bases

Value s	Family capacity $\sum(\mathrm{CA}_{K_s})$	Subfamily capacity $\sum_{4,0{,}45,0{,}85}(\mathrm{CA}_{K_s})$
1	75052	4
2	226036	142
3	226863	196
4	226863	213
5	226863	223
6	226863	234
7	226863	201
8	226863	188
9	226863	184

You can see that the number of bases in the formed subfamilies is much less than the capacity of corresponding families, however, it still remains quite large and it determines the necessity to choose the best of them in the context of the problem here to be solved. With regard to the foregoing problem of loosy compression of digital images, the best will be those orthogonal bases whose transformations will result in less distortion of the restored data items at equal level of information losses produced under compression.

3. The procedure of selection of the best transformations by restoration error

Suppose that $\mathbf{P}=(p_{ij})_{i=1,\,j=1}^{w,\,h}$, $p_{ij}\in\{0,1,\ldots,255\}$, $i=\overline{1,w}$, $j=\overline{1,h}$ – is a pixel matrix of the digital image (for simplicity we will take a half-tone image); the problem of loose information compression is to be defined then as detecting of this image $\Omega:\mathbf{P}\mapsto l$, where $l\in\{0,1\}^k$ that at specified value of the restoration error $k\to\min$.

The solution to this problem based on the use of discrete orthogonal transformations is to eliminate spatial redundancy from the image which demonstrates relatively equal values of neighboring pixels in local image areas. This is achieved by partitioning the data elements into components containing the basic image information and determining insignificant details. The removal of components of the second type from the transformed data elements followed by entropy coding of the remaining elements shall provide the image compression.

In reference with the above, the efficiency of orthogonal transformations shall be evaluated in terms of the number of zero values obtained as a result of transformation followed by quantization. Information losses herewith occurred shall result to restoration of distorted images which use different methods in their evaluation [23]. In this paper we use the base value which identifies the restoration error of digital image data elements – the root from the root-mean-square-error (RMSE) – computed by formula

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(f_i - g_i\right)^2} \quad (3)$$

Designations used herein correspond to those entered above.

Another classical factor of quality of images restoration is the peak signal-to-noise ratio computed by formula

$$\mathrm{PSNR} = 20\log_{10}\frac{\max\left(f_i\right)}{\mathrm{RMSE}} \quad (4)$$

These values inversely proportional to each other are two equal criteria suitable for the basic evaluation of the quality of image restoration. Within this paper we have identified the root from the root-mean-square-error, which requires less computation in its estimation that is more convenient in processing of many thousands of orthogonal bases subfamilies.

The procedure of selection of the best orthogonal transformations by restoration error shall be determined by the following stage subsequence.

1. Formation of the set (the subfamily or the set of subfamilies) Σ of orthogonal bases of the N – order using the dynamics of cellular automata with the code set.

2. Formation of the set of integer matrices (simple test images) that simulate some basic behavioral features typical for real digital images at local sites: the smooth color transition, the presence of several distinct areas with the smooth color transition, the presence of an object in the course of the smooth color transition, and the presence of lines in the course of the smooth color transition.

The paper offers to use for $N = 8$ the set of 33 halftone digital images with the dimension of 8×8 pixels provided in different combinations of dark, mid and light halftones (Fig. 1).

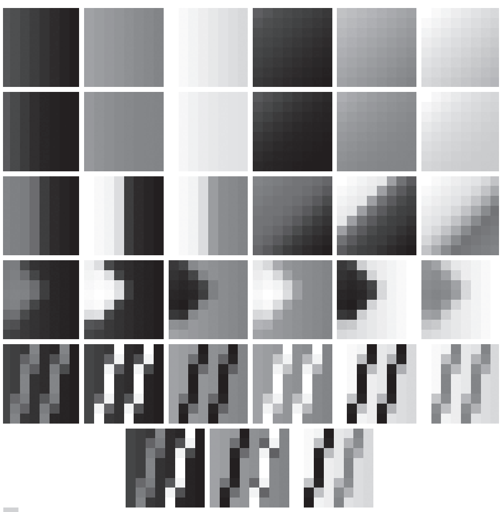

Fig. 1. The set of simple test images

3. The use of orthogonal transformations based on each of the built bases $\mathbf{C}\in\Sigma$ to each of the specified test images followed by quantization of the transformed data elements according to a simple scheme with two coefficients $SQ=(q_L,q_H)$, where q_L– is the quantizer of low-frequency components and q_H– is the quantizer of high-frequency components.

4. The converse of each transformation with computing the RMSE value and the number of zeros among the transformed data elements for each of the images, and the estimation of the following statistical characteristics: the average number of zeros among quantized data elements, the variation coefficient of this value, the average RMSE value and the variation coefficient of this value, which, for some basis $\mathbf{C}\in\Sigma$, shall be designated, respectively, as $\mathrm{z}(\mathbf{C})$, $\mathrm{V_z}(\mathbf{C})$, $\mathrm{RMSE}(\mathbf{C})$, $\mathrm{V_{RMSE}}(\mathbf{C})$.

5. Partitioning of the set Σ into l of noncrossing subsets

$\Sigma = \bigcup_i \Sigma_i$, where Σ_i, $i=\overline{1,l}$,

is determined as follows: a segment of the number line

$$\left[0, \frac{3}{4}N^2\right]$$

which composes the set of possible values of Z characteristic is partitioned into l parts, and if $\mathrm{z}(\mathbf{C})$, $\mathbf{C}\in\Sigma$ enters into the i-segment, then $\mathbf{C}\in\Sigma_i$.

6. Selection of the best orthogonal bases in each subset Σ_i, $i=\overline{1,l}$, for which the mean RMSE value is the minimum value, and their integration into the set $\hat{\Sigma}\subset\Sigma$. When considering the bases $\mathbf{C}_1,\mathbf{C}_2\in\Sigma$, so that $\mathrm{RMSE}(\mathbf{C}_1)\approx\mathrm{RMSE}(\mathbf{C}_2)$, the best one is supposed to be the basic with the smallest scattering of RMSE values.

7. The use of orthogonal transformations based on the bases from $\hat{\Sigma}$ to each of the specified simple test images so that the level of information losses would become equal that is approximately expressed by equal count of zeros among the transformed data elements and is achieved by varying the values of quantization coefficients.

8. The resulting selection of the best orthogonal transformation by restoration error.

Let us consider this procedure to be used in the low-capacity bases subfamily $\sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K_1})$. Values of all entered statistical bases characteristics, which comprise this subfamily obtained when using the quantization scheme $SQ\ (10, 20)$, are given in Table 2. The bases defining matrices are not given herein to reduce the description; instead, the initial state of the cellular automata lattice is pointed out for each base as the code in the alphabet A which defines the first base vector that clearly specifies the total set of base vectors in the known transition function. The length of these codes exceeds N and is defined by formula $N+2(b-1)$, where b – is the length of the partitioning block, since the processing of boundary cells during the cellular automata evolving process is performed not using the wrapping capability, but by means of supplement of the cellular automata lattice.

Table 2. Bases characteristics of the subfamily $\sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K1})$

Lattice initial state	z	V_z	RMSE	V_{RMSE}
2302300312	7.79	70.0	5.046	8.02
3203231002	9.06	60.5	5.040	9.61
2033023303	13.24	72.9	4.780	11.23
0233323033	16.09	58.2	4.617	12.92

It is not hard to note that the bases given in Table 2 possess the following property:

$\forall\, \mathbf{C}_1, \mathbf{C}_2 \in \sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K1})$,

so from that $z(\mathbf{C}_1) > z(\mathbf{C}_2)$ it follows that $\mathrm{RMSE}(\mathbf{C}_1) < \mathrm{RMSE}(\mathbf{C}_2)$.

Therefore, there is no need to use steps 7 and 8 of the proposed procedure, i.e. in $\sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K1})$ the base $\mathbf{C}$ is clearly the best, for which $z(\mathbf{C}) = 16.09$ and $\mathrm{RMSE}(\mathbf{C}) = 4.617$.

We won't discuss in details the use of this procedure in other subfamilies of orthogonal bases having built in this paper because of extensionality of corresponding tables, so we will be restricted with only conclusive results. It should be noted herewith that the value l at $N=8$ shall be naturally assumed as equal to 49, since in this case $0 \le z(\mathbf{C}) \le 48$.

Hence, in each of the subfamilies $\sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K1})$, $s = \overline{1,9}$, the orthogonal bases with the best characteristics have been selected. Table 3 gives, as an example, one representing value for each subfamily. The given bases characteristics have been obtained when using the quantization scheme $SQ = (10, 20)$.

Table 3. Examples of bases with the best characteristics

s	Lattice initial state	z	V_z	RMSE	V_{RMSE}
1	0233323033	16.09	58.2	4.617	12.92
2	2113331322	21.97	46.4	4.337	16.71
3	2000113232	27.97	40.8	4.068	23.65
4	0112232332	27.61	35.4	4.263	21.50
5	2312331012	26.64	33.1	4.074	24.69
6	3333301112	26.70	33.0	3.904	28.13
7	0322032211	26.70	31.8	3.902	28.64
8	3013230311	26.58	32.5	3.932	27.10
9	1313323232	26.55	32.7	3.934	27.46

You can see that when changing the value s from 1 to 7, the improvement in characteristics of the resulting bases can be observed, i.e. the restoration error is reduced simultaneously with the increasing (or comparability) of the number of zeros among the transformed data elements, that will allow to great effect to use orthogonal transformations based on these bases to compress digital images. Please note here that such behavior happens to be not only for the given bases, but also for the respective subfamilies in whole.

Numerical experiments with the built bases were carried out using classical bodies of the test images (half-color and full-color) containing Baboon, Barbara, Boat, Goldhill, Lenna, Peppers, etc. images with the resolution of 512×512 pixels.

The performed experiments have shown that the use of the RMSE value as a criterion for the quality of image restoration enables to correctly identify the best transformations in the investigated subfamilies using the proposed procedure. It is necessary to give some explanations here.

It has been discovered that artifacts appearing on the restored images for different orthogonal transformations, even from the same subfamily, shall diverge in their structure. For example, some typical artifacts are presented in Figure 2, which gives the segment of the classical test image Lenna restored after different orthogonal transformations from the subfamily $\sum_{4,\,0.45,\,0.85}(\mathrm{CA}_{K1})$ when completely removing all high-frequency first-level components from the matrix. You can see that in some cases "a net" appears (Fig. 2*a*); sometimes the artifacts can rather form "a grid" (Fig. 2*b*), and different "blocking effects" may also occur (Fig. 2*c*, *d*). The use of the foregoing quantization scheme $SQ = (10, 20)$ provides the same picture but with less expressed artifacts.

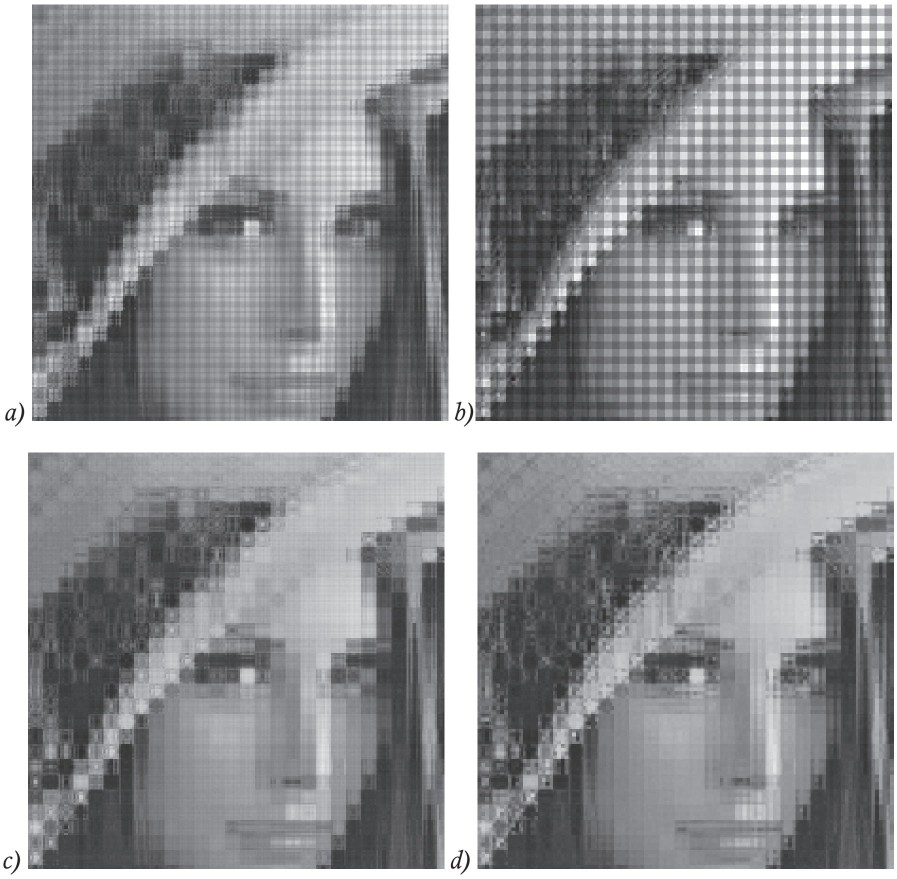

Fig. 2. Examples of artifacts for particular transformations

The root of the root-mean-square-error doesn't allow to measure the difference between these artifacts – the performed experiments have shown no apparent relationship between structural features of the artifacts, which are demonstrated on the restored images, and the proper ranges of RMSE values.

However, if we add the image shown in Fig. 3 to the above images, we can see that it doesn't contain any significant artifacts.

Fig. 3. Image segment restored with no significant artifacts

In addition to the above, for the orthogonal transformations, by means of which the image shown in Fig. 3 was obtained, the RMSE value computed at stage 7 of the above procedure has seemed to be substantially smaller than that for the orthogonal transformations, by which the images shown in Fig. 2 were obtained. These computations were performed after gaining the equal number of zeros among the transformed data elements for all transformations reviewed. Therefore, it might be noted that though the RMSE criterion doesn't allow us to compare various artifacts, it enables to divide the studied transformations into two classes, i.e. those for which detectable artifacts are developed at the given level of information losses on the restored images, and those for which there are no significant distortions observed at the same level of information losses. Thus, the selected criterion allows us to come to a proper decision of the problem on how to select the best transformations in constructed subfamilies.

Conclusion

The paper offers the approach to construction of the bases families of orthogonal transformations with the use of the dynamics of cellular automata and proposes the selection procedure for the best transformations by restoration error using the RMSE value as the quality criterion. The performed experiments have shown that in order to build orthogonal transformations we may use not only classical binary cellular automata, as it was in previous works, but also cellular automata with the internal states alphabet of more capacity. Thus the obtained orthogonal transformations may be used in digital image compression algorithms, e.g. for integer (or rational numbers) approximation of the known transformations, i.e. the discrete cosine transformation and the discrete wavelet transform.

The work will be continued to construct and investigate orthogonal transformations using code sets of the mode differed from the approach discussed in this paper. Besides, the proposed procedure may be supplemented by much more sophisticated criteria of discrepancy evaluation between the original and restored images after the transformation with information losses, than the root of the root-mean-square-error or the peak signal-to-noise ratio.

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References

1. Rosin, P.L. Training cellular automata for image processing / P.L. Rosin // 14th Scandinavian Conference, SCIA 2005, Joensuu, Finland. – 2005. – P. 195-204.

2. Panagiotopoulos, F.K. Quantum–dot cellular automata design for median filtering and mathematical morphology operations on binary images / F.K. Panagiotopoulos, V.A. Mardiris, V. Chatzis // 10th International Conference on Cellular Automata for Research and Industry, ACRI 2012, Santorini Island, Greece. – 2012. – P. 554-564.

3. Kauffmann, C. Seeded ND medical image segmentation by cellular automaton on GPU / C. Kauffmann, N. Piche // International Journal of Computer Assisted Radiology and Surgery. – 2010. – Vol. 5(3). – P. 251-262.

4. Liu, Y. An effective approach of lesion segmentation within the breast ultrasound image based on the cellular automata principle / Y. Liu, H.D. Cheng, J. Huang, Y. Zhang, X. Tang // Journal of Digital Imaging. – 2012. – Vol. 25(5). – P. 580–590.

5. Chen, Y. A cellular automatic method for the edge detection of images / Y. Chen, Z. Yan // 4th International Conference on Intelligent Computing, ICIC 2008 Shanghai, China. – 2008. – P. 935-942.

6. Sahoo, G. Text extraction and enhancement of binary images using cellular automata / G. Sahoo, Tapas Kumar, B.L. Raina, C.M. Bhatia // International Journal of Automation and Computing. – 2009. – Vol. 6(3). – P. 254-260.

7. Belan, S.N. Specialized cellular structures for image contour analysis / S.N. Belan // Cybernetics and Systems Analysis. – 2011. – Vol. 47(5). – 695-704.

8. Zagoris, K. Scene text detection on images using cellular automata / K. Zagoris and I. Pratikakis // 10th International Conference on Cellular Automata for Research and Industry, ACRI 2012, Santorini Island, Greece. – 2012. – P. 514-523.

9. Pena-Cantillana, F. Parallel skeletonizing of digital images by using cellular automata / F. Pena-Cantillana, A. Berciano, D. Diaz-Pernil, M.A. Gutierrez-Naranjo // 4th International Workshop, CTIC 2012, Bertinoro, Italy. – 2012. – P. 39-48.

10. del Rey, A.M. A secret sharing scheme for digital images based on two-dimensional linear cellular automata / A.M. del Rey // 12th International Workshop, IWCIA 2008, Buffalo, NY, USA. – 2008. – P. 318-329.

11. Mankar, V.H. Cellular automata based robust watermarking architecture towards the VLSI realization / V.H. Mankar, T.S. Das, S.K. Sarkar // World Academy of Science, Engineering and Technology. – 2007. – Vol. 7. – P. 20–29.

12. Wu, H. A new JPEG image watermarking algorithm based on cellular automata / H. Wu, J. Zhou, X. Gong, Y. Wen, B. Li // Journal of Information & Computational Science. – 2011. – Vol. 8(12). – P. 2431-2439.

13. Gonzalez, R. Digital image processing / R. Gonzalez, R. Woods. – Moscow: "Technosphera" Publicher. – 2005. – P. 1072. (In Russian).

14. Salomon, D. Compression of data, images, sound / D. Salomon. – Moscow: "Technosphera" Publicher. – 2004. – P. 368. (In Russian).

15. Belov, A.M. Research of the efficiency of algorithms of image compression based on the generalized Haar wavelet transforms / A.M. Belov // Computer Optics. – 2008. – Vol. 32(1). – P. 74–77. (In Russian).

16. Kopenkov, V.N. Efficient algorithms for local discrete wavelet transform with Haar basis / V.N. Kopenkov // Computer Optics. – 2008. – Vol. 32(1). – P. 78-84. (In Russian).

17. Umnyashkin, S.V. Image compression algorithm based on the discrete pseudo-cosine transform / S.V. Umnyashkin, V.V. Kurina // Digital signal processing. – 2009. – 3. – P. 2-7. (In Russian).

18. Lafe, O. Data Compression and Encryption Using Cellular Automata Transforms / O. Lafe // Engineering Applications of Artificial Intelligence. – 1997. – Vol. 10(6). – P. 581-591.

19. Kudrjavtzev, V.B. The foundations of the theory of homogeneous structure / V.B. Kudrjavtzev, A.S. Podkolzin, A.A. Bolotov. – Moscow: "Nauka" Publicher. – 1990. – P. 296. (In Russian).

20. Evsutin, O.O. Designing and testing of the computational approach of decorrelating transformation bases construction with the use of partitioning cellular automata / O.O. Evsutin, S.K. Rososhek // Trudy SPIIRAN. – 2012. – Vol. 4(23). – P. 324-342. (In Russian).

21. Toffoli, T. Cellular automata machines / T. Toffoli, N. Margolus. – Moscow: "Mir" Publicher. – 1991. – P. 280. (In Russian).

22. Evsutin, O.O. Applications of cellular automata in the field of information security and data processing / O.O. Evsutin. A.A. Shelupanov // Doklady TUSUR. – 2012. – Vol. 1-2(25). – P. 119-125. (In Russian).

23. Lapshenkov, E.M. No reference estimation of noise level of digital image is based on harmonic analysis / E.M. Lapshenkov // Computer Optics. – 2012. – Vol. 36(3). – P. 439–447. (In Russian).