[5] Contour analysis and modern optics of gaussian beams

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Abstract

The paper brings forward a theoretical study of the usability of the mathematical apparatus of spiral beams for problems of the contour images identification. The estimation of the approach offered is given in the light of its advantages and shortcomings, numerical simulation results are offered and the method applications aspects are analyzed in the context of actual practical tasks.

Keywords: coherent optics, spiral light beams, contour images.

Citation: Volostnikov V.G. Contour analysis and modern optics of Gaussian beams / Volostnikov V.G., Kishkin S.A., Kotova S.P. // Computer Optics. – 2014. – Vol. 38(3). – P. 476-481.

Introduction

Contour analysis is one of the key elements of the image identification problem which means to study the image as a set of contours [1,2]. This paper describes a new method of image contours identification based on the mathematical apparatus of spiral beams, i.e. light fields maintaining their structure in focusing and propagating.

The principle of the offered approach lies in the fact that all operations are performed not with a flat curve defined by a contour, but with a spiral beam determined herewith. This is because it is more meaningful to consider the spiral beam which possesses a number of favorable properties (its guaranteed description using analytical uniformly convergent functions, many ways to specify its amplitude, roots and expansion coefficients [3]).

It is obvious that the identification result may (and must) be based on the set of solutions for many contours outlined in the image, and it can be easily achieved in the case where there exists a two-contours comparing mechanism. In particular, the purpose of this paper is to identify high-quality contour characteristics and retrieve the similarity information therefrom. The paper clarifies results obtained in [4] and also generalizes the identification approach for deformed/noisy contours.

We shall also note that the image identification area is quite diverse according to the applied methods and significant attention has been lately paid thereto on both theoretical and practical grounds [5,6].

1. Contour description

The first and abiding procedure in the problem of contour images identification is to identify boundaries (contours) of the object. However, it is supposed in the paper that contours have already been identified by one of the existing methods. The next step is to have an appropriate description of the contours obtained using the information, on the basis of which the identification process will be performed, in particular, its characteristics should be precise and invariant with respect to different factors (particularly, they should not depend on the choice of a starting point). Looking ahead, we shall note that expansion coefficients of the spiral beam would play such information role. Fig. 1 shows the image of a ship on which, for simplicity, only one contour, i.e. its boundary, has been identified.

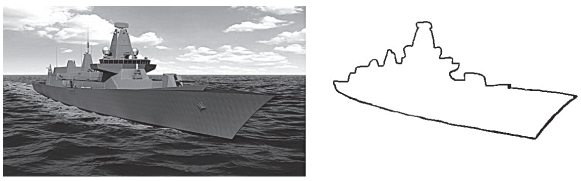

Fig. 1. Image (left) and contour of the object (right)

We shall consider in this paper some particular closed flat curves consisting of an ordered set of points as the mathematical contours representation:

$$\zeta(t) = x(t) + iy(t),\ t \in [0,T]. \qquad (1)$$

It is evident that any closed curve can represent a proper periodic function with period T.

Each contour may be certainly presented as an infinite series in accordance with a system of full orthogonal functions. The task of expansion of the aforesaid functions is described in details in paper [7], which brings forward classical bases applied in image identification tasks.

The problem, however, is as follows. The final set of expansion coefficients for a one-dimensional base is radically dependent on what point we shall "start" the curve from (thus, setting it within the range of $[0,T]$, or $[a,a+T]$). As regards the curve, it certainly makes no difference, but only in case when we use a complete set of basic functions, though in practice calculations are only made using total amounts of these series. It should be noted that if the curve is considered as a two-dimensional object specified within a plane, this problem is eliminated. However, the price to be paid for such choice is a large number of computations and asymptotic growth of the complexity of such algorithms. This approach is particularly considered in this paper and its section 5 explains how to manage with the above said shortcomings.

2. Contour as an initial object to form spiral beams

The analysis of different types of light fields has identified a new type of light beams called spiral [3]. It turned out that the spiral beam is considered to be a light field which maintains its intensity structure up to scale and rotation when propagating and focusing. Furthermore, the structure of such light field can be highly diversified; in particular, it can have a free form of the flat curve including the closed one.

It has been established that the complex amplitude $S(z,\overline{z})$ of this beam for the generating curve $\zeta(t)$ has the following form:

$$S(z,\overline{z}\mid\zeta(t),t\in[0,T]) = \\ = \exp\left\{\frac{-z\overline{z}}{\rho^2}\right\}\int_0^T \exp\left\{-\frac{\zeta(t)\overline{\zeta(t)}}{\rho^2}+\frac{2z\overline{\zeta(t)}}{\rho^2}\right\}\times \\ \times\exp\left\{\frac{1}{\rho^2}\int_0^t\left[\overline{\zeta(\tau)}\zeta'(\tau)-\zeta(\tau)\overline{\zeta'(\tau)}\right]d\tau\right\}|\zeta'(t)|\,dt, \quad (2)$$

where ρ – is Gaussian parameter of the beam, and an over-bar means the complex conjugation. An example of such curve and its respective spiral beam is shown in Fig. 2.

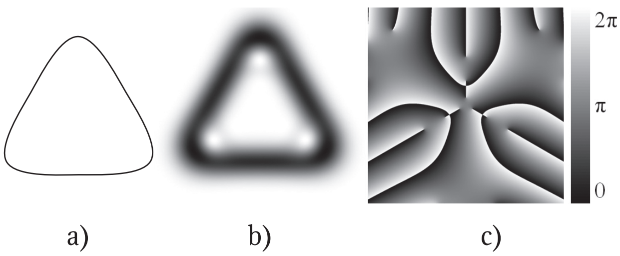

Fig. 2. Generating curve (a) and intensity distributions (b) and phases (c) of the respective spiral beam

The following property of "quantization" of spiral beams in form of closed curves is important. If the condition (quantization) is fulfilled

$$S_{curve} = \frac{1}{2}\pi\rho^2 N_q,\ N_q = 0,\,1,\,2\ldots \quad (3)$$

where S_{curve} is an area under the curve, the beam complex amplitude is independent on the curve starting point. In other words, the spiral beam can't be determined by the contour starting point. Therefore, any total amount of the series $S_N(z,\overline{z}\mid\zeta(t),t\in[a,a+T])$ will not be affected by this starting point up to the total unimodular component depending only on parameter a. Thus, the problem of the starting point in analysis and identification of the input contour is eliminated. This means that, with any desired degree of accuracy and according to the spiral beam $S(z,\overline{z}\mid\zeta(t),t\in[a,a+T])$, we can assign its total series sum as follows:

$$S_N(z,\overline{z}\mid\zeta(t),t\in[a,a+T]) = e^{-\frac{z\overline{z}}{\rho^2}}\sum_{n=0}^{N_c} c_n z^n \quad (4)$$

The equation given below shows that this may solve also the problem of rotation of the analyzed contour since when it rotates by angle α, the total series sum shall vary as follows:

$$S_N(ze^{i\alpha},\overline{z}e^{-i\alpha}\mid\zeta(t),t\in[a,a+T]) = \\ = e^{-\frac{z\overline{z}}{\rho^2}}\sum_{n=0}^{N}\left(c_n e^{i\alpha n}\right)z^n = e^{-\frac{z\overline{z}}{\rho^2}}\sum_{n=0}^{N} c'_n z^n \quad (5)$$

that can prove once again that expansion coefficients are capable to characterize rotation angles.

We should note here another very important aspect. The quantization parameter, as shown in [8], determines a number of zeroes of the complex amplitude within the contour and, in fact, a polynomial degree remaining from the initial analytic function of the spiral beam after getting the total sum with the desired number of expansion coefficients. It is obvious that if the analyzed contour is complicated, the quantization parameter cannot be small: it is impossible to describe complex things simply. Nevertheless, the fact that we have eliminated the problem of dependence on the choice of the starting point and rotation angle is very important and makes the offered method deserving more detailed study.

3. Contour analysis

Suppose we have two contours in the database, the input and control ones, and it is necessary to determine whether they correspond to each other or not.

Before constructing beams we should make the following: we will identify steps of complex grids, on which spiral beams will be calculated so that the areas constrained with the curves will be equal. Reducing them to a common area allows us to solve the problem of an unknown image scale (it will be located by ratio of a step of one grid to the step of another grid), while maintaining independence of the choice of the starting point and the rotation angle.

Let us construct appropriate spiral beams for both contours keeping the required number of members [4]. According to the above scheme we shall assign contours in accordance with two spiral beams; the purpose is to get two sets of complex factors: $\{c_n^{(1)}\}_{n=0}^{N_c}$ and $\{c_n^{(2)}\}_{n=0}^{N_c}$. In the case when the quantization parameter is sufficient enough to distinguish two contours, the aforesaid sets of coefficients shall coincide up to rotation:

$$\forall n \in \overline{1, N_c}, \ \frac{|c_n^{(1)}|}{|c_n^{(2)}|} = 1, \ \varphi_n = \frac{1}{i}\ln\frac{c_n^{(1)} c_{n-1}^{(2)}}{c_n^{(2)} c_{n-1}^{(1)}}, \qquad (6)$$

If $\varphi_n \equiv const$ for all values of n, then φ_n is the contour mutual rotation angle α. This fact can be easily obtained by denoting the ratio of two complex amplitudes based on the spiral beam representation in form of the above total sums (4).

If the condition (6) is not satisfied, we can identify mismatch of these contours.

4. Contour analysis algorithm

Based on the aforesaid, we can summarize a step-by-step sequence of operations (block-diagram, Fig. 3) to determine the similarity between two proposed contours. First, they must be assigned in form of the ordered set of plane points – the curve (step 1). According to the resulting curves we should calculate corresponding spiral beams (step 2) and expand them in accordance with the orthogonal system (step 3) taking the required number of expansion coefficients which is empirically determined within the scope of the problem. And finally, based on comparing two sets of coefficients, we can conclude whether these two contours are identical up to scale and rotation (step 4).

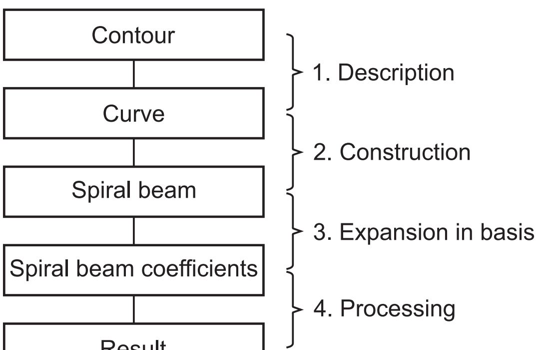

Fig.3. Block-diagram of the identification algorithm

Thus, the identification algorithm possessing the following characteristic properties is to be implemented. First, it relates to independence of the algorithm on selecting the contour starting point and the contour image scale. Second, the contour object may have an arbitrary form; its complexity is only determined by the system resolution, and not by the number of contour elements typical for other methods. Third, all computer calculations carried out at steps 1-3 are used directly in decision making; they don't have total search counting operations which shall be ignored. Thus, the last two characteristic properties look quite attractive against such generally accepted contour identification method as the contour analysis using correlation functions (see. [2]).

We shall further note that in initial consideration of the foregoing method steps 2 and 3 look very resource-intensive since they operate on two-dimensional objects, i.e. the complex on-plane amplitudes and the bases. However, the authors have found and tested the way how to reduce the number of calculations by two or three decimal exponents reducing true calculations to one-dimensional ones. Actually, steps 2 and 3 shall be combined into one step through direct calculation of expansion coefficients using only one-dimensional integrals. Unfortunately, such important in their applied significance details go beyond the scope of this paper suggesting the general concept of contour identification using spiral light beams.

The shortcomings of this approach may include the following. First, the imaging operation which is extremely "hard" for counting, especially on mobile devices, is used in calculations. Second, no effective evaluations of the quantization parameter and the number of series remainder terms sufficient (and necessary) for stability of identification results are currently known. This issue has not yet been sufficiently analyzed and requires more detailed study.

5. Noise accounting

So, the basic principle of the method has been formulated and described as series of steps, however when proceeding from the forgoing theoretical computations to practical implementation of the approach, it is necessary to make some additional comments. It should be noted that the factors which haven't been considered so far in this paper, i.e. noises and distortions, can inevitably occur in real systems. There are a lot of reasons for their presence, i.e. discritization errors in CCD-matrices, information transmission failures from image recording devices to computing hardware implementing the identification process and so on.

The authors have researched how the offered approach can respond to this kind of distortions. The method proved to be very resistant to noises in the identified contour. First of all, this relates to the fact that the equation for the complex amplitude of the spiral beam (2) includes Gaussian exponent which has a "smoothing" effect demonstrated by the fact that small contour deformations can only lead to small changes in the complex amplitude.

Despite the fact that the complex amplitude and its zeros are resistant to the contour deformation, the expansion coefficients (see Vieta formulas, for example, in [9]) may vary quite significantly. This doesn't prevent to use them in identification since the truncated series (4) converge uniformly and very "fast", and the basic information-significant factors have smaller numbers than the quantization parameter.

Besides, it is possible to use an additional and slightly modified global characteristic (in contrast to "local" co-coefficients) which is known in light field optics as an overlap factor, and in functional analysis it is known as a standard scalar product rated in Hilbert space $L_2(\mathbb{R}^2)$. In practice, $K(\theta)$ represents a correlation function according to the mutual rotation angle θ of spiral beams $S^{(1)}(z,\bar{z})$ and $S^{(2)}(z,\bar{z})$, where the scalar product of the numerator in the formula below (7) can serve as a proximity measure. The maximum modulus of this function is achieved at the true mutual rotation angle α of the contours being identified.

$$K(\theta)=\frac{1}{\sqrt{\iint S^{(1)}(z,\bar{z})\overline{S^{(1)}(z,\bar{z})}\,dxdy}}\times \frac{\iint S^{(1)}(z,\bar{z})\overline{S^{(2)}(ze^{i\theta},\bar{z}e^{-i\theta})}\,dxdy}{\sqrt{\iint S^{(2)}(z,\bar{z})\overline{S^{(2)}(z,\bar{z})}\,dxdy}} \quad (7)$$

In contrast to the identification method using correlation functions described, for example, in [2], the equation (7) has the following advantages: the number of summands in each sum is the number of expansion coefficients (approximately dozens, or one or two hundred items), and not the elements defining the contour (over 500). Besides, it is possible to identify the rotation angle with the required accuracy, i.e. if the angle increment θ for identification of the maximum correlation function is small, the greater accuracy may be achieved, and if it is large – the higher speed.

Using the formula (4) and the orthogonality property of Laguerre-Gaussian base, the equation (7) may be easily rewritten in the representation using only expansion coefficients:

$$K(\theta)=\frac{\sum_{n=0}^{N} c_n^{(1)}\overline{c_n^{(2)}}e^{in\alpha}}{\sqrt{\sum_{n=0}^{N} c_n^{(1)}\overline{c_n^{(1)}}}\sqrt{\sum_{n=0}^{N} c_n^{(2)}\overline{c_n^{(2)}}}} \quad (8)$$

Thus, a new tool shall appear which can clarify and confirm, when required, identification results calculated using only the coefficients.

6. Low detailization as express-testing for comparison

The fact of instability of expansion coefficients has stimulated to search additional solutions enabling to neutralize the influence of noises on the considered method. This has led to discussion of the aforementioned correlations, as well as to situation which is called as the "case of ultra low detailization."

The idea is as follows: if in case of a large number of expansion coefficients their accuracy may be worse in the presence of noises, it is necessary to construct spiral beams with a very small quantization parameter (from 2 to 5). Whereby, the intensity of this kind of light field will not superficially resemble the generating curve, as shown in Fig. 2, however in terms of application of the Vieta theorem and energy characteristics of the resulting light field such approach is justified.

As it will be seen below in section "Numerical simulation", this solution gives surprisingly good results though it is not obvious that if we are restricted to only two-five expansion coefficients, a significant part of information on the generating curve would not get lost. Thereby the rate of obtaining results increases manifold. This allows us to consider the case with ultra low detailization as a peculiar kind of express-testing of scale and potential rotation angle, while maintaining all advantages and neutralizing some disadvantages of the original method summarized in section 5 hereof. It bears repeating that the case of ultra low detailization is not the basis to make decision on objects similarity; it is intended to filter out a large number of highly distinguished contours and give a hypothetical rotation angle, which can be tested by the correlation function (7) of highly detailed spiral beams (finding the maximum correlation can be significantly simplified in this case, i.e. searching the mutual rotation angle should be carried out in the vicinity of the hypothetical angle).

7. Numerical simulation

The aircraft image has been selected as a test sample, the boundary contour has been manually outlined and the spiral beam has been constructed thereto. After that the contour has been turned around in graphic editors by 123 degrees, reduced to 75% of the original, and the corresponding spiral beam has been constructed thereof.

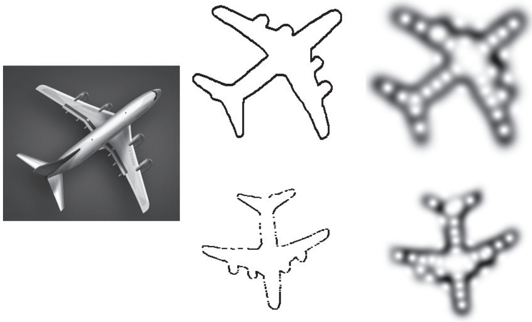

Fig. 4. From left to right: the aircraft initial image, the outlined boundary contour, the intensity of the generated spiral beam

In order to provide visual imaging in construction, the quantization parameter N_q has been equal to 30; the number of expansion coefficients N_c – to 120. The value of Gaussian parameter is $\rho = 1$, and the area under the curve is $S_{curve} = 15\pi$. The correlation function may reach herewith the maximum of its absolute value 0.8 at the required angle of 123 degrees and the scale factor of 1.3. The maximum $K(\theta)$ of its modulus does not inherently exceed 1; in other words, the similarity of upper and lower contours has been identified, but not absolute, that is due to the processing (in Fig. 4 we can observe failures or fallouts on visible parts of the aircraft near its "tail").

Let us proceed to the case with ultra low detailization (Fig. 5). For this purpose the quantization parameter N_q and the number of coefficients N_c have been selected as being equal to 3.

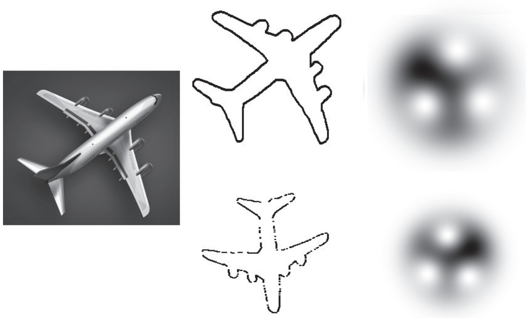

Fig. 5. From left to right: the aircraft initial image, the outlined boundary contour, the intensity of the generated spiral beam

The value of Gaussian parameter is $\rho = 1$, and the area under the curve is $S_{curve} = 1{,}5\pi$. The correlation function may reach herewith the maximum of its absolute value of 0.998 at the angle of 121 degrees and the scale factor of 1.3. The expansion coefficients calculated for the criterion (5) are presented in the table below.

Table. Expansion coefficients and the applied criterion

n	$c_n^{(1)}$	$c_n^{(2)}$	$\frac{\lvert c_n^{(1)} \rvert}{\lvert c_n^{(2)} \rvert}$	φ_n
1	-0.088 – 0.335i	-0.286 + 0.175i	1.034	-98
2	-0.482 + 0.517i	-0.577 + 0.155i	1.182	-139
3	1.175 – 2.049i	-0.365 + 2.131i	1.093	-128
			Rotation angle mean value:	-122

It should be noted that potential rotation angles φ_n in themselves do not completely match the criterion, however a specific feature has been detected, i.e. their arithmetical mean value provides the desired rotation angle with a small error. This fact attracts our attention since it has been confirmed by numerous numerical experiments but has not yet been analyzed in detail. Therefore, numerical experiments prove that the algorithm shown in Fig. 3 can be successfully applied in situations where the identified contour is noisy or distorted. This may be helped with some additional tools, i.e. the correlation $K(\theta)$ and ultra low detailization.

Conclusion

The paper brings forward a new approach to contour analysis based on close interrelationship of coherent optics, the theory of functions and numerical methods. We have shown and theoretically proved the contour-matching algorithm that allows us to determine whether these two contours are identical up to scale and/or rotation. We have demonstrated the dynamics of development of the proposed approach in the presence of noises or deformations in the analyzed image. We have noted the fact of decreasing the number of computer calculations by reducing the problem to calculating one-dimensional integrals. We have introduced additional tools which allow us to confirm identification results, i.e. the correlation and ultra low detailization.

It is assumed to continue researches in the following areas: first, in analyzing performance compared to classical methods of contour analysis; second, in obtaining asymptotic estimates on algorithmic complexity of proposed calculations; third, in developing evaluation criteria for correlation values to be obtained.

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