ACCURACY OF RELIABILITY CALCULATED BY THE MONTE CARLO SIMULATION METHOD

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Sumary:

Reliability is the main indicator of the quality of special purpose equipment/systems, which are in their service life exposed to extreme operating modes and environmental conditions. Reliability of electronic equipment/systems is difficult to determine analytically, specially for repairabile systems with a large number of elements because of a large number of possible system states, which requires setting up and solving a system of a large number of equations. Therefore, simulation methods are applied to determine the reliability of electronic equipment/systems. This paper examines the accuracy of the point estimate of reliability calculated by the Monte Carlo simulation method depending on the sample size n, and the number of iterations (repetitions of calculation), N_p.

Key words: *electronic system, reliability calculation, accuracy, Monte Carlo method, simulation.*

Introduction

Reliability is one of the important indicators of quality of special purpose equipment/systems which are in their service life exposed to extreme operating modes and environmental conditions. Hence, it is important for the users of such a system to know what its reliability is.

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In this paper, the reliability of electronic equipment/system is defined as a probability, with a certain level of confidence, that it will successfully perform the function for which it was intended, in the time interval from *0* to *t*, under the prescribed operating regimes and environmental conditions.

In general, the reliability of electronic equipment/systems is difficult to determine by analytical methods as they have a large number of elements (usualy in states operational or faulty), and according to that, a large number of possible states of the system, which requires setting up and solving a system of a large number of equations. Simulation methods are, therefore, more often applied to determine the reliability of electronic systems.

For this purpose, the authors of this paper have developed an appropriate mathematical and physical simulation model and a computer program that support the calculation of the reliability of electronic equipment/systems based on the application of the Monte Carlo simulation method. The simulation software package for calculating reliability is first checked on examples of simple electronic systems, for which it was possible to determine the analytical solutions. The comparative review of the obtained results by the simulation and analytical methods for the same electronic systems, subject to an exponential distribution, showed very close proximity, which was a confirmation of the correctness of the application of the developed simulation software package for the calculation of reliability.

In the reliability calculation using the Monte Carlo simulation method it was presumed that the equipment/system is built entirely of electronic elements and the period of normal operation, when failure rates are constant, because failures occur mostly by accident. Then we can use an exponential distribution to approximate the distribution of failures.

Because of the importance to know the reliability of modern military equipment, this paper analyzes the accuracy of the point reliability assessment, obtained using Monte Carlo simulation methods [1], depending on the sample size n and the number of iterations (repeating the calculation), N_p.

Basic assumptions in the Monte Carlo reliability calculation method

It is very difficult to give a precise and complete definition of the Monte Carlo method. However, we can say that this is the numerical method for solving complex mathematical, statistical, physical, telecommunication and other problems with the random selection of samples. The basic idea of the Monte Carlo method is to build a stochastic model that is consistent with the real problem or is the direct simulation of the problem. In both cases, the element of chance is introduced, making a large number of computer "experiments" – iterations and eventually implement their statistical analysis. In other words, the idea of the Monte Carlo method is: instead of describing a random phenomenon by analytical relations, we perform the simulation of this phenomenon in order to obtain its realization. The realization is performed by a simulation of random numbers. As a result of each repetition of such a procedure – iteration, one realization of the studied phenomenon is obtained.

The advantage of the application of simulation methods is that very complicated mathematical problems, which usually have no analytical solution, can easily be solved. The disadvantage of the method is a large number of iterations required to achieve the required accuracy, which means long computing time. However, with the rapid development of computer technology, this problem is becoming less significant and the Monte Carlo method is increasingly used in many fields of science and technology.

The Monte Carlo method is based on the stohastic nature of processes whose states are determined by the laws of probability. Theoretical ideas about the phenomena being treated in the probability distributions of random sizes are the base of the application of the Monte Carlo method for numerical iteration. The Monte Carlo simulation method of the stohastic system leads to solutions to problems which are impossible to obtain analytically. The idea is that taking into account the characteristics of a system, random numbers are assigned to system variables, and by series of simulations and appropriate statistical analysis of the expected results we can draw conclusions (e.g. the expected average value of a variable, reliability of a system, etc.).

For constant failure intensity, reliability of electronic equipment is determined by the following formula [2] (so-called exponential law of reliability):

$$R(t) = \exp\left(-\frac{t}{m}\right) \tag{1}$$

where t – is the required operating time without failure, and m-mean time to/between failures of equipment.

The reliability parameter m is in practice (for example laboratory or field tests) determined by its point estimation by the following relation [3]:

$$\hat{m} = \frac{1}{n}\sum_{i=1}^{n} t_i\,, \tag{2}$$

where t_i is the time to the i -th failure, and n is the total number of failures.

When in the reliability relation m is replaced with $\hat{m}$, we get the estimated reliability or the point estimation of reliability:

$$\hat{R}(t) = \exp\left(-\frac{t}{\hat{m}}\right). \tag{3}$$

For non-repairable/repairable systems, m or $\hat{m}$ represents the mean time to/between failures (MTTF/MTBF). The accuracy of the estimated reliability depends on the accuracy of the estimated mean time to/between failures, and the accuracy of this depends on the number of failures. Therefore, it is necessary to have a large number of times to/between failures to accurately determine the mean time to/between failures. When applying the Monte Carlo simulation method, times to/between failures are not coming from laboratory or field tests of equipment, but are software generated as pseudorandom variables using the relation:

$$t_i = -m \cdot \ln r_i, \tag{4}$$

where

m – adopted theoretical values of the mean time to/between failures and

r_i – pseudorandom number whose value is between zero and one,

$r_i \in (0,1)$.

In the computer program, made for this purpose, pseudorandom numbers r_i are obtained with the computer random generator.

The basic input data for calculating the reliability of electronic equipment/system is the mean time between failures which is given by the formula:

$$MTBF = m = \frac{1}{\lambda}, \tag{5}$$

where λ is the failure intensity rate of the element.

The mean time to/between failures was chosen as the input data because, in practice, this parameter for the electronic elements can be obtained in several ways. One, quite frequent, is the calculation based on standardized manuals for reliability, such as, for example, a military handbook (standard) MIL-HDBK-217 F. Next, manufacturers of electronic equipment/systems with special purpose have to provide information on the mean time between failures for these systems on request.

Dependence of the range of values of the point estimation of reliability of the sample size *n*

For a given operating time without failure of the equipment (t = T_z), adopted theoretical value for m, the number of random variables t_i or the sample size n, and the number of repetitions of calculations N_p, using the computer program developed by the authors, the range of values $\hat{R}_i$ of the estimation of the reliability is defined by the following relation:

$$W(\hat{R}_i) = \hat{R}_{\max} - \hat{R}_{\min}. \qquad (6)$$

The 19 different values of the sample size, provided in this computer program, are obtained using the following relation:

$$n_i = \lfloor n_{\max} / 20 \rfloor \cdot i \, ; \quad i = 1,2,...19 \qquad (7)$$

where n_{max} – is the maximum sample size which is preset (for example n_{max} = 400, n_1 = 20 etc.), the integer value of expression $Q = \dfrac{n_{\max}}{20}$ is taken into account denoted by $\lfloor Q \rfloor$. The ranges of values $\hat{R}_i$ depending on the sample size n, for t = T_z = 24 h (hours), m = 2000 h, N_p = 5000 and n_{max} = 400, determined by this program, are shown in Figure 1.

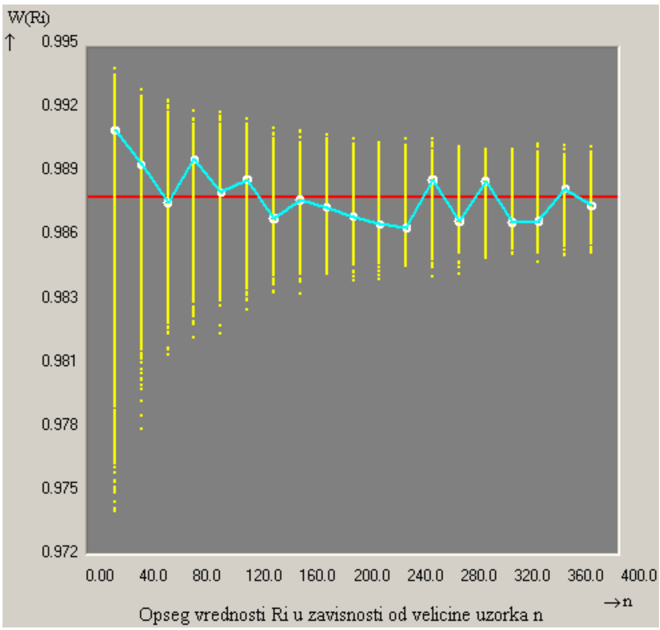

Figure 1 – Dependence of $W(R_i)$ of n
Slika 1 – Zavisnost $W(R_i)$ od n

As shown in Figure 1, when the sample size increases, the range of values for reliability narrows, at first rapidly, and then more and more slowly, as a result of the nature of statistical convergence. The points on the diagram are mean values of the reliability calculated in these ranges.

Probability density function of the point estimation of reliability

Based on the adopted values for the mean time to/between failures *m*, the required operating time without failure $t = T_z$ and the sample size *n,* the n pseudo-random time to/between failures is generated. Based on this, the estimated value $\hat{m}$ is calculated, and then a point estimate of the reliability $\hat{R}(t)$ is calculated based on $\hat{m}$. This procedure is repeated N_p times (N_p should be as large as possible in order to obtain higher accuracy). For m = 2000 h, $t = T_z$ = 24 h (estimated value of the reliability by relation (1) is $R(t)$ = 0.9880717), n = 50 and N_p = 5000, using the developed computer program we got results, shown in Figure 2 in the form of a histogram and a graph of the estimated probability density function of the point estimation of reliability.

Since the values of the point estimation of the reliability $\hat{R}(t)$ are between 0 and 1, we adopted beta distribution for the probability density function in the interval *A*-*B*, where *A* is the begining and *B* is the end of the distribution with shape parameters: *a* and *b*.

The probability density function of the point estimation of reliability is given by the following relation:

$$f(x) = \frac{1}{(B-A)\cdot B(a,b)}\left(\frac{x-A}{B-A}\right)^{a-1}\cdot\left(1-\frac{x-A}{B-A}\right)^{b-1} \tag{8}$$

where is $A \le x \le B$, $a,\ b > 0$ and $B(a,b)$ is the beta function which can be determined using complete gamma functions by the following formula [4]:

$$B(a,b) = \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}. \tag{9}$$

In the function $f(x)$, in the formula (8), *x* should be replaced with $\hat{R}$.

The distribution parameters *A*, *B*, *a* and *b* can be determined using different methods, such as: maximum likelihood method, method of moments, quantila methods, etc. In this computer program, the point estimates of these parameters were determined using the Monte Carlo simulation by the following relations:

$$\begin{aligned} \hat{A}_i &= R_{\min} - 0.01 \cdot h + 0.01 \cdot h \cdot r_i \\ \hat{B}_i &= R_{\max} - 0.01 \cdot h + 0.01 \cdot h \cdot r_i \\ \hat{a}_i &= 0.05 + 29.95 \cdot r_i \\ \hat{b}_i &= 0.05 + 29.95 \cdot r_i \end{aligned} \qquad (10)$$

where r_i – the pseudo-code $r_i \in (0,1)$, h – the class histogram width.

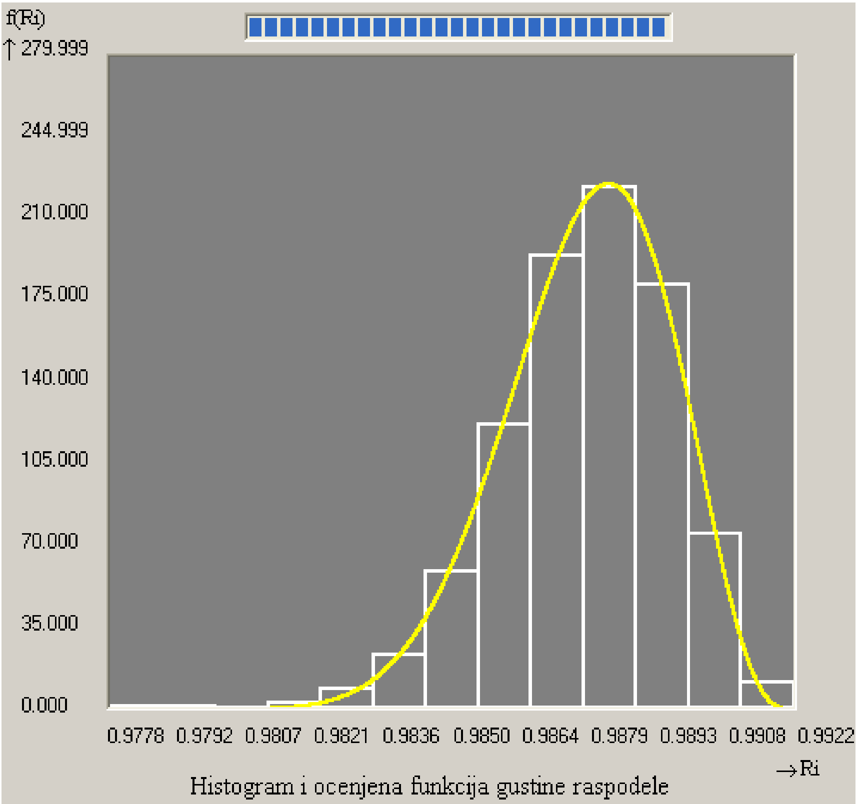

Figure 2– Histogram and graph of the probability density function $f(\hat{R})$

Slika 2 – Histogram i grafik funkcije gustine $f(\hat{R})$

These point estimations of the parameters are put in the relation for the probability density function and then the values of this function in the center of each class histogram can be compared with the height of the histogram class. In other words, the square differences between the value of the function and the class height are found and then these square differences are summarized for all classes of the histogram. Then we repeat the calculation of the parameters and adopt those with the smaller difference. The procedure is repeated for a very large number of times and we adopt the set of parameters which is obtained with the smallest sum of squares of the observed differences. This procedure is mathematically simple, but must be repeated many times, so it requires the use of a computer. The results are satisfactory, as it can be seen in Figure 2, where the estimated probability density function is suited well with the histogram, as well as the corresponding empirical distribution density function.

For m = 2000 h , t = T_z = 24 h (estimated value of the reliability $R(t)$ = 0.988071713), n = 50 and N_p = 5000, using the developed computer program, we obtained the following values of the point estimates of these parameters:

$$\hat{A} = 0.977795$$

$$\hat{B} = 0.992203$$

$$\hat{a} = 9.652830$$

$$\hat{b} = 4.205322.$$

The mean value is $\overline{\hat{R}} = 0.987830808$, the standard deviation is $\hat{s} = 0.020303677$, and the confidence limits are $R_1 = 0.984149310$ and $R_2 = 0.990742220$ with the risks $\alpha_1 = \alpha_2 = 0.025$.

Relative error in determining the reliability

By definition, reliability is the probability that the operating time without failure of the equipment is greater than a required time T_z:

$$R(t) = P(A) = P(t > T_z), \qquad (11)$$

where the event A hapens, under defined conditions, when the operating time without failure is greater than the required time, ie $t > T_z$. If the true (unknown) reliability is very high, then the event A happens more often, and the probability $P(A)$ will be more accurately determined with the same number of data n (sample size or the number of times to/between failures) than when the reliability is low.

For 20 different theoretical (considered as real) values, the point estimation of reliability was carried out, according to the procedure described in the foregoing paragraphs of this work, and the relative errors *RG* were determined using the formula [5]:

$$RG = 100(R - \hat{R})/R \quad [\%]. \tag{12}$$

where R is true, and $\hat{R}$ the corresponding point estimation of reliability.

For, $t = T_z = 24$ h, $n = 2000000$, and the theoretical value R = : 0.050, 0.100, 0.150, ... 0.950, 0.99, R= {0.050, 0.100, 0.150, ... 0.950, 0.99}, using the computer program the relative errors in estimating the reliability were determined, which is graphically shown in Figure 3. We used a sample $n = 2000000$ purposely high, in order to get less deviation of the estimated reliability from the true reliability. As shown in the diagram (Figure 3), the relative error is much smaller for the high values of the true reliability than for the lower values, what was our assumption at the beginning of the paper.

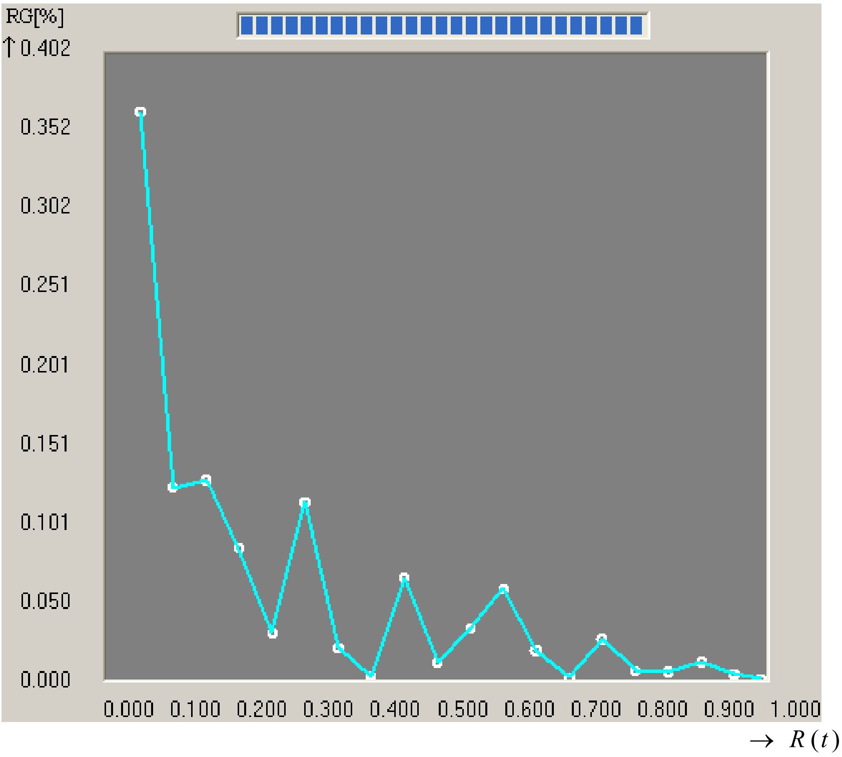

Figure 3 – Relative error RG as a function of R(t)

Figure 3 – Graphical display of the relative error as a function of R(t)

Conclusion

This paper examined the accuracy of the point estimation of reliability depending on the sample size *n*, and the number of iterations (repetitions of calculation), N_p.

It is shown that the sample size *n* has a greater effect on accuracy than the number of iterations N_p, because if a sample is too small, we cannot obtain greater improvement of accuracy with a larger number of iterations. However, the determination of the parameters of the distribution of points estimation requires a very large number of iterations.

The application of the Monte Carlo simulation method [6] is proved to be very suitable in the estimation of distribution parameters, especially when the number of these parameters is greater than 3, because in this case the application of known methods, as the maximum likelihood method and other methods, is very complex. The accuracy of the estimation of the parameters of the distribution using this simulation method is satisfactory, and the application itself is very simple.

It should be noted that the Monte Carlo simulation method requires the application of a computer.

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PROVERA TAČNOSTI PRORAČUNA POUZDANOSTI ODREĐENE PRIMENOM SIMULACIONE METODE MONTE CARLO

OBLAST: elektronika, informacione tehnologije
VRSTA ČLANKA: originalni naučni članak

Sažetak:

Pouzdanost je najvažniji logistički parametar uređaja/sistema specijalne namene obzirom da su isti u svom radnom veku izloženi maksimalnim radnim režimima i ekstremnim klimomehaničkim ulovima okoline.

Otuda je za korisnika sistema specijalne namene važno da zna sa kakvim performansama sistema raspolaže.

Kako je u ovom radu reč o elektronskim uređajima, za koje se može smatrati da u svom radnom veku imaju konstantnu funkciju intenziteta otkaza, to je za proračun pouzdanosti primenjena eksponencijalna raspodela.

Pouzdanost elektronskih uređaja/sistema teško je odrediti analitičkim putem jer obično imaju veliki broj elemenata, a to znači veliki broj mogućih stanja sistema, što zahteva postavljanje i rešavanje sistema sa velikim brojem jednačina. To je razlog da se za određivanje pouzdanosti elektronskih uređaja/sistema sve više primenjuju simulacione metode.

Autori ovog rada razvili su odgovarajući matematičko-fizički simulacioni model, kao i računarski program koji ga podržava, za proračun pouzdanosti elektronskih uređaja/sistema zasnovan na primeni simulacione metode Monte Carlo. Softverski paket proveren je najpre na primerima jednostavnijih elektronskih uređaja/sistema, za koje je bilo moguće odrediti analitička rešenja za pouzdanost. Poređenjem rezultata za pouzdanost dobijenih simulacionom metodom i analitičkim relacijama, za iste elektronske uređaje/sisteme, utvrđena je velika bliskost dobijenih rezultata, što je bila potvrda ispravnosti primene razvijenog simulacionog softverskog paketa za proračun pouzdanosti.

Obzirom na značaj pouzdanosti savremenih vojnih uređaja, u ovom radu analizirana je tačnost proračuna pouzdanosti, dobijene primenom simulacione metode Monte Carlo, u zavisnosti od veličine uzorka n (broja vremena bezotkaznog rada do/između otkaza) i broja iteracija (ponavljanja izračunavanja), N_p.

Na početku rada polazi se od osnovnog ulaznog podatka za proračun pouzdanosti, m – srednje vreme rada do/između otkaza uređaja, za koji u praksi postoji realan problem da se odredi eksperimentalnim putem. Zato se on zamenjuje sa tačkastom vrednošću parametra $\hat{m}$ *koja se generiše kao pseudoslučajno promenljiva po odgovarajućem izrazu. Na osnovu parametra* $\hat{m}$ *proračunava se ocenjena ili tačkasta ocena pouzdanosti R(t)*

Tačnost ocenjene vrednosti pouzdanosti R(t) zavisi od tačnosti ocenjene vrednosti za srednje vreme rada do/između otkaza $\hat{m}$ *, a tačnost ovoga zavisi od broja otkaza. Potrebno je imati što veći broj vremena do/između otkaza da bi se tačnije odredilo srednje vreme do/između otkaza.*

Promenom veličine uzorka elemenata n, za koji se vrši proračun pouzdanosti, menja se opseg vrednosti ocenjene pouzdanosti R_i *i na taj način proverava se tačnost rezultata proračuna pouzdanosti određena primenom simulacione metode Monte Carlo.*

U nastavku rada ispituje se relativna greška u određivanju pouzdanosti simulacionom metodom u zavisnosti od stvarne vrednosti pouzdanosti.

Na kraju ovog rada o ispitivanju zavisnosti tačnosti tačkaste ocene pouzdanosti od veličine uzorka n (broja vremena bezotkaznog rada do/između otkaza) i broja iteracija (ponavljanja izračunavanja), N_p pokazano je da veličina uzorka n ima veći uticaj na tačnost nego broj iteracija N_p, jer kada je uzorak isuviše mali, velikim brojem iteracija ne može se dobiti veće poboljšanje tačnosti.

Međutim, kod određivanja tačkastih ocena parametara raspodele potreban je vrlo veliki broj iteracija. Primenjena simulaciona metoda Monte Carlo pokazala se kao veoma pogodna pri ocenjivanju parametara raspodele, naročito kada je broj ovih parametara veći od 3, jer u ovom slučaju primena poznatih metoda, kao metoda maksimalne verodostojnosti i dr. veoma je složena. Tačnost ocena parametara raspodele primenom ove simulacione metode je zadovoljavajuća, a sama primena je veoma jednostavna. Za simulacionu metodu Monte Carlo neophodna je primena računara.

Ključne reči: *elektronski sistem, proračun pouzdanosti, tačnost, Monte Carlo metod, simulacija.*

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