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UNIVERSITY STUDENTS' CONCEPTIONS OF THE RELATIONSHIP BETWEEN MATHEMATICS AND PHYSICS AND THE RELATIONSHIP BETWEEN MATHEMATICS AND PHYSICS LEARNING

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Abstract. *The present study identified university students' conceptions of mathematics and physics as well as students' conceptions of mathematics and physics learning. Furthermore, the study elicited university students' conceptions of the relationship between mathematics and physics and the relationship between mathematics and physics learning. As a final, the study compared university students' conceptions of mathematics and physics as well as students' conceptions of mathematics and physics learning. A purposeful sample of university students (pre-service elementary science and mathematics teachers) who had taken required mathematics and physics courses participated in the study. Open-ended questionnaires and interviews were used to collect data. The analysis found that most students believed that mathematics was closely related to physics and that it was difficult to succeed in physics without understanding mathematics. In addition, more students exhibited coherent conceptions of physics and higher-level conceptions of physics learning compared to the number of the students with coherent conceptions of mathematics and higher-level conceptions of mathematics learning. How the nature of the disciplines of mathematics and physics affects the development of student conceptions is discussed.*

Key words: *conceptions, conceptions of learning, mathematics, physics, qualitative study.*

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Introduction

Learner conceptions are important due to their far-reaching influence on thought and behaviour (Ponte, 1994). For example, learners' conceptions – in particular, their conceptions of learning – play a key role in their desire to learn mathematics (Andrews & Hatch, 1999) and science (Tsai, 2004). Thompson (1992) defines conceptions as "conscious or subconscious beliefs, concepts, meanings, rules, mental images and preferences" (p. 132), and conceptions of learning are defined as "the ways in which the learners make sense of learning in general" (Bowden & Marton, 2003, p. 69).

Theoretical Framework

Conceptions are cognitive in nature and part of knowledge (Ponte, 1994). They reflect individuals' belief systems (Ernest, 1989). Andrews and Hatch (1999) claim that individuals' conceptions are framed by their beliefs. There are some classifications for conceptions and conceptions of learning in the literature. For example, Ernest (1989) classified mathematics teachers' philosophical conceptions of mathematics as *instrumentalist* (mathematics is a set of rules and skills acquired for a particular purpose), *Platonist* (mathematics is a static, unified body of knowledge) and *problem solving* (mathematics is continually in flux and evolving). Crawford, Gordon, Nicholas and Prosser (1994) claimed that university students' conceptions of mathematics could be dichotomised as *fragmented* (mathematics involves numbers, rules and formulas) and *cohesive* (mathematics involves a complex logical system and way of thinking). In addition, some university teachers viewed mathematics as problem solving that involves a set of notations and symbols used to analyse models abstracted from reality that serves as a tool for other sciences (Mura, 1993, 1995). Wood, Petocz and Reid's (2012) recent classification of mathematics conceptions extended the Crawford et al.'s (1994) classification and identified three conceptions of mathematics: *mathematics is about components* (viewing mathematics as the study of numbers, calculation and disparate mathematical activities), *mathematics is about models* (mathematics is related to models that represent real-life situations), and *mathematics is about life* (mathematics is a way of thinking that is related to everyday life).

Conceptions of learning also exert considerable influence on approaches to learning. Purdie and Hattie (2002) argued that the literature presented two prominent approaches to classifying conceptions of learning: a *surface understanding of learning* that involves the attainment, reproduction and use of knowledge and a *deep understanding of learning* that involves the construction of meaning and personal change. Similarly, Crawford et al. (1994) presented two types of university students' conceptions of mathematics learning: *reproduction,* which involves learning to reproduce knowledge and procedures through memorisation, and *understanding*, which involve learning by doing to acquire a relational understanding of theory and concepts.

In a prominent study of conceptions of learning conducted by Saljo in 1979 (cited in Tsai, 2004; Purdie & Hattie, 2002), individuals exhibited five different conceptions of learning, conceiving of learning as: (1) *an increase in knowledge;* (2) *memorisation;* (3) *the acquisition of facts and procedures to be remembered and/or utilised in practice;* (4) *the abstraction of meaning;* and (5) *an interpretative process aimed at understanding reality*. Following Saljo's (1979) study, some researchers (e.g., Marshall, Summer, & Woolnough, 1999; Marton, Dall'Alba & Beaty, 1993; Tsai, 2004) also categorised conceptions of learning. For example, Marton et al.'s (1993) categorisation of conceptions of learning included the novel concept of *personal change*. Marshall et al. (1999) categorised conceptions of learning as: (1) *memorising definitions, equations and procedures;* (2) *applying equations and procedures;* (3) *making sense of physical concepts and procedures;* (4) *seeing phenomena in the world in a new way; and* (5) *personal change.* Tsai (2004) investigated high school students' conceptions of learning in a specific scientific domain and identified seven dimensions of conceptions of scientific learning in Chinese culture that were consistent with the findings of previous studies (e.g., Marshall et al., 1999; Marton et al., 1993; Saljo, 1979). Lee, Johanson and Tsai (2008) presented a new categorisation of conceptions of science learning by developing a questionnaire that reduced the seven dimensions of conceptions of learning science identified by Tsai (2004) to six dimensions. Tsai, Ho, Liang and Lin (2011) identified three of these six dimensions as *lower-level conceptions of science learning*, in which science learning involves practicing problems and memorising facts and rules, and three dimensions identified as *higher-level conceptions of learning science*, in which science learning involves an increase in knowledge and knowledge application.

Related Literature

The following review of the literature primarily focuses on conceptions of mathematics and science and conceptions of mathematics and science learning and teaching.

Conceptions of Mathematics and Mathematics Learning and Teaching

Some researchers (e.g., Andrews & Hatch, 2000; Pehkonen & Tompa, 1994; Petocz et al., 2007) have compared learner conceptions of mathematics or mathematics learning and teaching across different countries. For example, Andrews and Hatch (2000) noted that Hungarian and English teachers' conceptions of mathematics and mathematics teaching were similar in certain respects. For example, both groups believed that mathematics teaching included teaching mathematical skills, encouraging problem solving and explanation of the life. A study of elementary school students revealed that Finnish students' conceptions were more calculation-centred and emphasised teacher control over student learning compared to Hungarian students' conceptions (Pehkonen & Tompa, 1994). A more recent study that compared conceptions of mathematics learning internationally (Petocz et al., 2007) focused on undergraduate conceptions of mathematics. They found that there were significant differences in the mathematics conceptions of students from different universities and that more senior students exhibited broader conceptions of mathematics and viewed mathematics as an approach toward life and a way of thinking.

Some researchers (e.g., Gordon & Nicholas, 2013; Lloyd, 2013) have tested the effectiveness of pre-service education programmes on the development of mathematics conceptions. Gordon and Nicholas (2013) investigated university students' conceptions of a mathematics bridging course designed to "provide students with a way forward with their chosen degree programme, and to ameliorate students' difficulties with mathematics" (p. 110). They found that students perceived the course as helpful in improving their approach to learning mathematics and extending their thinking skills. Lloyd (2013) found that many practices and mathematics conceptions of pre-service mathematics teachers in pre-service education were observed in their in-service activities.

Another important research area related to conceptions of mathematics has focused on conceptions of mathematical proof. Basturk (2010) investigated mathematics student teachers' conceptions of mathematical proof and found that students considered mathematical proof necessary for identifying mathematics connections. Ko's (2010) study reviewed conceptions of mathematical proof and focused on the importance of proof in mathematics. She found that mathematics teachers with an inadequate understanding of proofs experienced difficulty in teaching proofs.

Conceptions of Science and of Science Learning and Teaching

In the field of science, studies primarily have focused on conceptions of science learning and teaching, such as the relation between conceptions of science learning and epistemological beliefs (Chan & Elliot, 2004). Chan and Elliot (2004) found significant correlations between the epistemological beliefs of teacher education students and the extent to which they exhibited traditional or constructivist conceptions of teaching and learning. For example, beliefs in the simplicity, certainty and unchanging nature of knowledge were related to traditional conceptions of teaching and learning, whereas beliefs that learning involved understanding how to learn rather than knowledge acquisition were related to constructivist conceptions of teaching and learning (Chan & Elliot, 2004). Consistent with the findings of Chan and Elliot (2004), Otting, Zwaal, Tempelaar and Gijselaers (2010) found that hard work and valuing how to learn were positively related to constructivist conceptions of teaching and learning but negatively related to traditional conceptions of teaching and learning.

Some researchers (e.g., Chiou, Liang & Tsai, 2012; Lee et al., 2008) have found that certain dimensions of conceptions of learning are related to approaches to learning. Lee et al. (2008) found that lower-level conceptions of science learning (e.g., viewing science as involving memorisation of definitions and success on exams) were positively related to surface approaches to science learning (e.g., rote learning). In contrast, they found that higher-level conceptions of science learning (e.g., learning how to apply science knowledge) were positively related to deep approaches to science learning (e.g., being highly motivated to learn science). Chiou et al. (2012) found similar results for conceptions of learning and approaches to biology.

Finally, learners' conceptions of science learning exhibit cultural differences (Purdie & Hattie, 2002). For example, Purdie and Hattie (2002) found that the mean scores for conceptions of science learning were significantly different for high school students from different cultures.

Significance of the Study

Mathematics and physics are interrelated subjects (Basson, 2002; Martinez-Torregrosa, Lopez-Gay & Gras-Marti, 2006). Some researchers (e.g., Basson, 2002; Quale, 2011) claim that it is difficult to succeed in physics without knowing basic mathematical operations. Thus, mathematics might be viewed as a prerequisite for learning physics at first glance. However, McDermott (1991; 1993) claims that conceptual understanding is important, particularly for learning physics, and Hewitt's (2006) textbook "Conceptual Physics" exemplifies this claim. Believing that it is difficult to learn physics without having the appropriate background in mathematics might lead students to have negative attitudes towards physics and physics learning. This perceived dependence of physics on mathematics might also prevent students from appropriately applying their cognitive and creative thinking skills to learning physics. Therefore, exploring the relationships between students' conceptions of mathematics and physics as well as their conceptions of mathematics and physics learning might enable us to better understand how two disciplines are related to each other.

In the literature presented above, researchers primarily focused on conceptions of learning or teaching in a single domain (mathematics, biology, or science). The results of these studies indicated that learners exhibited different types of conceptions. The above studies also discussed the reasons why learners held such conceptions and primarily related them to learning and teaching processes. However, the nature of the discipline itself might also influence student conceptions. Students might not exhibit identical or similar conceptions regarding different disciplines. The present study investigating student conceptions of the relationships between the separate disciplines of mathematics and physics can enable researchers to better understand how the nature of the disciplines can influence student conceptions. In this regard, researchers developing education programmes or curricula to modify student conceptions should attend to the nature of these disciplines.

Finally, the results of the TIMMS and PISA exams (see Martin, Mullis & Foy, 2008; Mullis, Martin & Foy, 2008; PISA 2012 Results in Focus, 2012), indicate that Turkish students generally exhibit low math and science scores. In the past decade, curriculum developers have become aware of this problem and have begun to modify elementary and secondary science and mathematics curricula in Turkey. One goal has been to facilitate student learning by adopting a constructivist approach that focuses more on student skills and the relevance of mathematics and physics to daily life (Turkish High School Mathematics Curriculum, 2011; Turkish High School Physics Curriculum, 2011). However, student understanding of mathematics, physics and mathematics and physics learning following the implementation of the new curricula in Turkey has yet to be comprehensively examined. Exploring the extent to which students adopt a constructivist perspective to mathematics, physics and mathematics and physics learning can determine the effectiveness of the new curricula.

The Purpose and Research Questions of the Study

The purpose of the current study was to compare university students' (first and second year pre-service science and mathematics teachers) conceptions of mathematics and physics as well as conceptions of mathematics and physics learning by investigating the relationships among these conceptions. The following research questions were addressed:

- What are university students' conceptions of mathematics?
- What are university students' conceptions of physics?
- What are university students' conceptions of the relationship between mathematics and physics?
- What are university students' conceptions of mathematics learning?
- What are university students' conceptions of physics learning?
- What are university students' conceptions of the relationship between mathematics learning and physics learning?

Methodology of Research

Sample

The present study was conducted in a university in Turkey. A purposeful sample of pre-service elementary science (N=79) and mathematics (N=108) teachers from the department of elementary education was selected. There were 91 male and 96 female participants. All participants had been educated with high school curricula that emphasised active student involvement in learning and skill development. In Turkey, the curricula incorporating a constructivist perspective were first implemented during the 2008-2009 academic year. Therefore, only first and second year pre-service elementary science and mathematics teachers who had been exposed to the new curricula participated in the study. Because it was assumed that the new curricula would modify the belief that "being successful in physics depends on mathematics achievement", it was hypothesised that few of the study participants would exhibit this belief. In addition, data from the sample were expected to provide reliable and in-depth information because participating students had majored in mathematics and physics.

Data Collection

Data were obtained using qualitative data collection methods that involved both open-ended questionnaires and interviews. Data primarily consisted of responses to an open-ended questionnaire that asked participants to (1) define mathematics and physics; (2) explain the relationship between mathematics and physics; (3) describe how mathematics and physics might be learned; and (4) explain the relationship between mathematics learning and physics learning. To obtain in-depth information rather than simple lists of items without any details, participants were instructed to restrict their responses to the single theme that best represented their ideas and were required to explain their answers. This approach identified students' strongest conceptions. Eleven of the study participants were also interviewed; these students were chosen randomly from the larger sample based on the number assigned to each open-ended questionnaire. Interviews were conducted to confirm the analysis of the responses to the open-ended questionnaire (Bogdan & Biklen, 1998). Following Fraenkel and Wallen (2005), interviews were semi-structured and included follow-up questions, such as "why do you think so?" and "could you give me a specific example?" to probe participant responses.

To address ethical issues, guidelines provided by Fraenkel and Wallen (2005) were followed. The names of participants and the study location have not been identified to maintain confidentiality. All participants were informed of the purpose of the study and participated voluntarily. Interviews were conducted during participants' free time and were audio-recorded after obtaining their permission. Finally, study participants were informed of the study results after completion of the data analysis.

Data Analysis

First, questionnaire responses were analysed. Categories and codes were constructed based on participant responses (Bogdan & Biklen, 1998; Miles & Huberman, 1994) because identifying regularities and patterns in the data in addition to the research topics is useful for generating research categories. Then, words and phrases (codes) that represented these categories were identified (Bogdan & Biklen, 1998). Specifically, words and phrases related to the research questions in the questionnaire responses were identified, and labels for relevant conceptualisations such as "mathematics includes numbers, etc." and "emphasis on real-life context" were chosen. The coded words and phrases were then matched with the primary study categories that they were related to.

Merriam (1998) notes that category names can be taken from the literature or created by the researcher. Previously identified categories in the literature on conceptions and conceptions of learning were used to name the categories in the present study. For example, participants' conceptions of mathematics and physics were categorised as *fragmented* or *cohesive* (Crawford et al., 1994) and their conceptions of mathematics and physics learning were categorised as *lower-level* or *higher-level conceptions of learning* (Tsai et al., 2011). Although adopting a single classification scheme for the distinct disciplines of mathematics and physics might seem problematic, it enables us to compare participant conceptions of the disciplines and conceptions of learning and provides us with an overview of the relationships among these conceptions. Moreover, although Crawford et al.'s (1994) categorisation has been elaborated by Wood et al. (2012), I used the Crawford et al.'s (1994) classification scheme because it more closely corresponds to the findings of the current study. In addition, Tsai et al.'s (2011) categorisation of conceptions of learning were chosen from among other classification schemes (e.g., Crawford et al., 1994; Purdie & Hattie, 2002; Tsai, 2004) because it better represented the results of the current study. Table 1 presents the conceptualisation of each category, and examples of participant conceptions of the disciplines of mathematics and physics and conceptions of mathematics and physics learning.

Table 1. Conceptualisation of each category representing conceptions of mathematics and physics and of mathematics and physics learning with examples of each.

Category	Conceptualisation	Specific examples from questionnaire responses
Fragmented conceptions	Includes numbers, rules, formulas etc. Includes laws and theories to be memorised Includes problem solving	Mathematics is a discipline that includes number and symbols I think that physics means laws and theories Both mathematics and physics include numbers Physics cannot exist without mathematics
Cohesive conceptions	Is a part of life Includes logical thinking	Physics explains life by considering cause-effect relationships Physics is life Both mathematics and physics are based on logic
Lower-level conceptions of learning	Learning by problem-solving Learning by memorising	Mathematics can be learned by practicing more problems Both mathematics and physics include memorising rules and formulas
Higher-level conceptions of learning	Learning through relation to daily life Learning by relating subjects Learning by observing and performing	Physics can be learned better when it is related to daily life Using visual materials to learn mathematics helps us learn better It is necessary to relate both physics and mathematics to daily life

Miles and Huberman (1994) argue that techniques such as "clustering", "counting", and "triangulation" are important for drawing conclusions and confirming results. Thus, the codes identified in questionnaire responses were clustered into primary categories and then the frequency of occurrence of each code was tabulated. Finally, audiotapes of the interviews were transcribed and the data obtained from each participant's interview were compared to the participant's questionnaire responses to confirm analysis results. In addition, considering the suggestions of Silverman and Marvasti (2008) on reliability of findings in a qualitative study, agreement regarding the number of categories and codes identified by one researcher having a PhD degree was obtained. We also examined the extent to which questionnaire and interview responses were consistent with the proposed study categories and codes. Disagreements were resolved through discussion.

Results of Research

The frequency of occurrence of each code in the questionnaire responses was tabulated. It is presented in parentheses in the tables presenting study results. Excerpts from questionnaire and interview responses are used to illustrate the conceptions found in the study. The source of the excerpt is indicated; for example, O18 refers to an excerpt from Participant 18's questionnaire responses and I173 refers to an excerpt from Participant 173's interview.

Conceptions of Mathematics and Physics and of the Relationship between Mathematics and Physics

In the questionnaire, students were instructed to define mathematics and physics and to explain the relationship between the two disciplines. Table 2 presents the student conceptions identified by the analysis.

Table 2. Conceptions of mathematics and physics and of the relationship between the two disciplines.

Conceptions of mathematics	Conceptions of physics	Conceptions of the relationship between mathematics and physics
Fragmented conceptions • Set of numbers, rules and symbols (N=61) • Calculations (N=21) • Used by other disciplines such as physics, chemistry and geography (N=16)	Fragmented conceptions • Set of formulas (N=25) • Set of laws and theories (N=9)	Fragmented conceptions • Mathematics as a prerequisite for physics (N=82) • Calculations (N=25) • Abstractness (N=14)
Cohesive conceptions • Part of daily life (N=43) • Explanation of life and natural events through numbers and calculations (N=23) • Puzzles encourage thinking (N=14)	Cohesive Conceptions • Explains life, matter, space and natural phenomena (N=115) • An aspect of daily life (N=23) • Explain cause and effect relationships in nature (N=12)	Cohesive Conceptions • Part of daily life (N=38) • Helpful for understanding the world (N=14) • Develop logical thinking skills (N=11)

As Table 2 indicates, student conceptions of mathematics were more likely to be fragmented rather than cohesive. In contrast, student conceptions of physics were more likely to be cohesive rather than fragmented. When student conceptions of relationship between mathematics and physics were compared with student conceptions of physics, the number of fragmented conceptions increased dramatically.

Fragmented Conceptions of Mathematics

Most students described mathematics as a set of numbers, rules and symbols. They also viewed it as involving calculations and reported that they always performed calculations while engaging in mathematics. Frequent use of arithmetical operations might produce this conception. Moreover, some students reported that mathematics was an instrument employed by other disciplines and thus formed the basis for the development of other disciplines. For example, they thought that certain innovations in other disciplines required mathematical calculations. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O7: A collection of numbers. Without numbers, mathematics cannot exist.
> O165: It is a discipline of numbers and symbols. There are no verbal explanations in mathematics. It only involves numbers.
> O17: Mathematics is a discipline that involves only calculations. We always add or subtract something from something else.
> O25: It is the language of other disciplines. That is, if there were no mathematics, we could not calculate the time differences between two meridians. Elements would have no atomic number.
> O173: Mathematics is more important compared to other disciplines. I think other disciplines are based on mathematics.
> I173: All disciplines are based on mathematics. For example, we say that there are seven colours in the rainbow. In addition, when the scientists do science, they combine ingredients based on certain measurements. That is, they weigh something or measure the length of something. These activities are related to numbers, that is, mathematics.

Cohesive Conceptions of Mathematics

Some students described mathematics as an aspect of life due to its frequent use in daily life. In particular, they stated that numbers and basic calculations such as addition and subtraction made daily life easier. In addition, numbers and calculations were necessary to explain events such as ocean temperature or the rotation of the earth and the moon. They reported that it would be difficult to understand and make sense of fundamental events without mathematics. They also believed that it would be difficult to function in everyday life without mathematics. Finally, some students viewed mathematics as a thought-provoking instrument or puzzle that was useful for developing individual thinking skills and that thinking about how to solve equations and problems could improve their neural functioning and accelerate their thought processes. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O77: It is an aspect of life. Without it, people are unable to live. They could not buy anything at the market without knowing it.
> I77: Mathematics is part of living. It improves people's lives in almost every area. Everyone needs to use mathematics. [The interviewer asks why everyone needs to use mathematics.] For example, people use it while shopping. People use mathematics in building apartments and architectural structures, which enhance people's lives. He/she finds a location more easily, for example, when describing which floor someone lives on …
> O5: If there is no water, life is impossible. The same is true for mathematics. For example, you cannot be aware of time if you do not know mathematics.
> O18: It explains nature and life. As a result, we can make sense of what happens. The rotation periods of the earth and the moon cannot be understood without mathematics.

Fragmented Conceptions of Physics

Two fragmented conceptions of physics as a "set of formulas" and "set of laws and theories" were identified. Some students viewed physics as merely involving a set of formulas that were needed to explain physical events in the world, and reported that the formulas used in physics formed its core. In addition, based on their previous learning experiences, a few students viewed physics as a collection of laws and theories and referred to Newton's Laws of Motion and Einstein's Theory of General Relativity based on their prior learning. For these students, physics represented certain well-known laws and theories. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O24: Explaining complex events easily and quickly with the help of formulas and quantitative data.
> I24: I think that physics explains some events that are difficult to understand with the help of numbers and formulas. For example, we do not think of calculations when using an elevator. However, with certain formulas we can understand the operation of an elevator better. If there were no formulas, we could not

> understand its operation. That is, it helps explain some complex events easily. [The interviewer asks what "complex events" means]. Events that we cannot easily understand.
> O41: Physics means formulas and formulas mean physics. Physics involves many formulas and they have to be memorised if we want to learn physics.
> O2: A collection of laws and theories.
> O101: Newton's and Einstein's contributions to physics. $E=mc^2$. Einstein changed the world.

Cohesive Conceptions of Physics

Most students perceived physics as essential for understanding nature. They reported that physics could explain natural phenomena, such as rain and earthquakes, and natural events in life such as object motion and the principles underlying technology. In addition, they stated that physics focused on aspects of space and matter such as nuclear fusion, meteor impacts, and planetary rotation. Some also noted that physics was an aspect of life due to its contribution to the development of everyday technology such as cell phones and computers. Students with this conception thought that everything used in everyday life was a product of physics. Another conception exhibited by students involved the explanation of natural phenomena in terms of cause-and-effect relationships. For example, they reported that people were able to walk due to the force of friction and that vehicles were able move due to the action of external forces, revealing the belief that causes produced effects. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O81: It is a discipline examining the actions of all the objects in the cosmos. It is involved even in chemistry and biology. Physics definitely answers all of the questions people ask about the cosmos.
> I81: Basically we can think of physics as a discipline examining all of the properties of objects. If you investigate the answers to these questions, you have to focus on physics. I guess you can find the answers to all of your questions from physics … For example, if you are curious about how rain forms, you can learn about it from physics. Or, you can examine why people do not fall off the world when it rotates.
> O74: Physics tries to explain everything that we see in our lives. For example, meteor impacts and solar and lunar eclipses occur following the principles of physics.
> O56: If you want to move in the world, first you begin with physics. You can use physics to do everything in the world … Thus, physics allows us to analyse the world and then to achieve results that are beneficial to the human endeavour.
> O124: It is a part of our lives because everything that we use - for example, cell phones and computers - works based on the principles of physics.
> O5: It is a discipline explaining natural events based on cause-effect relationships. For example, for a vehicle to move, there must be a force. If one does not exist, the vehicle cannot move.

Fragmented Conceptions of the Relationship between Mathematics and Physics

The most frequent conception of the relationship between mathematics and physics was that physics depended on mathematics. Over fifty percent of the students stated that it was difficult to achieve anything in physics without mathematics. In this view, physics was primarily based on calculations, which made it difficult to solve physics problems without mathematics. The students who perceived physics as a mathematics-dependent discipline also viewed both mathematics and physics as primarily consisting of calculations. Like mathematics, physics required calculations and the use of formulas. Students also regarded both mathematics and physics as abstract in nature and based on abstract concepts. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O7: Physics and mathematics are similar to each other due to numbers. You can see all of the operations of mathematics in physics.
> I7: Both are quantitative. For example, numbers. Both include equations. We use equations in both. If mathematics were unknown, physics problems could not be solved. In fact, mathematics is a tool for solving physics problems. Let us imagine how to calculate the velocity of vehicles without mathematics.
> O56: Without mathematics, there is no physics or it cannot be fully understood. How can we solve physics

> questions without mathematics?
> O74: For example, in a linear equation system, assume that y = ax + b. If we compare it with x = v.t in linear motion, y refers to x, a refers to v, x refers to t and b = 0. That is, equations in mathematics represent physics formulas.
> O140: Both of them examine abstract concepts. Everything is abstract in mathematics. You cannot feel and touch numbers. Similarly, you cannot feel electric fields.

Cohesive Conceptions of the Relationship between Mathematics and Physics

For some students, the contribution of mathematics and physics to the development of technology enables people to live more comfortably. The tools and devices that enhance their lives, such as photocopy machines and cell phones were invented as a result of this contribution. Therefore, they believed that both mathematics and physics improve people's lives. In addition, both mathematics and physics help people to understand their world. Knowledge of mathematics and physics allows people to make sense of object properties. For example, mathematical knowledge enables people to understand the dimensions of objects and mentally represent them. Similarly, knowledge of physics allows people to relate physical concepts such as velocity and temperature to their world. Students also stated that both mathematics and physics encourage people to improve their mental processes and help to develop logical thinking skills. The following excerpts from a questionnaire and interview responses illustrate these conceptions:

> O88: We can describe something; however, we cannot explain it to someone without going into detail. For example, when we talk about a table, we have to mention its dimensions to understand it better. Similarly, people can understand what it means when we say that a turbo jet reached a speed of 900 km/h.
> O65: Both physics and mathematics help us to understand our environment and world. Without them, we cannot understand many events. Temperature is meaningless without mathematics and physics.
> I65: Physics and mathematics are similar. They both explain the events in the world. For example, let us consider temperature, which is a topic studied by physics. However, it [temperature] is meaningless without mathematics … we say 30^{0}C and -10^{0}C. If we did not provide numbers in front of the centigrade symbol, it would not mean anything.
> O94: Both of them are based on mental processes. They can help us think more analytically.
> I94: Mathematics and physics primarily focus on mental processes. To understand them, more effort is required. Because of this effort, people's logical ability develops. As a result, they think more analytically.

Conceptions of Mathematics and Physics Learning and the Relationship between Learning Mathematics and Learning Physics

Student questionnaire responses described how mathematics and physics might be learned and the relationship between learning mathematics and physics. Table 3 presents students' conceptions of mathematics and physics learning and the relationship between learning mathematics and learning physics.

Table 3. Conceptions of mathematics and physics learning and the relationship between learning mathematics and learning physics.

Conceptions of learning mathematics	Conceptions of learning physics	Conceptions of the relationship between learning mathematics and learning physics
Lower-level conceptions	Lower-level conceptions	Lower-level conceptions
• Solving problems (N=69)	• Solving problems (N=12)	• Practicing more problems (N=64)
• Learning proceeds from easier to more difficult (N=48)		• Using rules (N=26)
• Using proofs (N=5)		• Using calculations (N=22)

Conceptions of learning mathematics	Conceptions of learning physics	Conceptions of the relationship between learning mathematics and learning physics
Higher-level conceptions • Using visual materials (N=33) • Providing examples from daily life (N=22) • Learning in depth (N=8)	Higher-level conceptions • Engaging in hands-on or laboratory activities (N=74) • Providing examples from daily life (N=41) • Learning in depth (N=30) • Using visual materials (N=28)	Higher-level conceptions • Providing examples from daily life (N=44) • Using visual materials (N=23)

As Table 3 indicates, students exhibited more lower-level conceptions of mathematics learning, but more, higher-level conceptions of physics learning. Moreover, the number of students who exhibited lower-level conceptions of mathematics and physics learning was higher than the number of students who exhibited higher-level conceptions of mathematics and physics learning.

Lower-level Conceptions of Learning Mathematics

The most frequently occurring conception of learning mathematics involved solving problems. In this view, students could improve mathematics learning by engaging in more problem-solving. The problems would increase their familiarity with certain types of questions and enable them to successfully solve similar problems. In addition, some students thought that learning that progressed from easier to more difficult topics was critical to learn mathematics and motivate students to learn mathematics. In this view, learners who had not mastered basic mathematical operations find it more difficult to understand complex operations. A few students exhibited the conception that it was important to use proofs in learning mathematics because using proofs developed the logical thinking skills necessary for success in mathematics. The following excerpts from a questionnaire and interview responses illustrate these conceptions:

> O54: Solving many problems. If you solve many problems, the probability of being successful increases.
> I54: Mathematics should be learned by solving problems … [The interviewer asks "why do you think so?"]. Mathematics can be learned by paying attention to the teacher because it is based on quantitative calculations. I think that the courses based on the quantitative calculations cannot be learned without the teacher's instruction. If you practice [solving problems] more, you can learn better. For mathematics, solving a large number of problems helps us understand it better. In fact, if necessary, homework can be given so students solve more problems.
> O85: Learning that progresses from the easy to the difficult. Mathematics always begins with easy operations and then progresses to more difficult ones.
> O114: Mathematics learning should progress from the easy to the difficult. Everybody could successfully complete easy mathematics operations. Because of this, they might be motivated to learn more difficult subjects.
> O45: Using proofs. This develops our thinking and we begin to think more analytically.

Higher-level Conceptions of Learning Mathematics

Some students believed that visual aids were necessary in learning mathematics because this enabled students to learn mathematics concretely. In addition, the use of instructional materials might improve student motivation. Some students also emphasised the importance of discussing everyday examples in mathematics learning because this teaching style helps the students attend the lesson and appreciate the usefulness of mathematics. Another conception involved identifying the connections among mathematical operations. In this view, understanding how to use the information provided to solve a mathematical equation and being aware of what students do while solving a problem is important. Addressing these issues would improve learning. The following excerpts from questionnaire and interview responses illustrate these conceptions:

> O71: Mathematics is abstract. You have to make it concrete by using certain strategies. For example, you can demonstrate certain concepts to students. So, students not only learn better, but also become more willing to learn.
> I71: It can be learned by visualising because mathematics is abstract. Computers can be used for this. [The interviewer asks "Can you give an example?"]. For example, if you want to calculate the volume of a cone, this is difficult to imagine mentally without being able to observe. Because of it [using a computer to present a demonstration], it will be easy.
> O97: Why do we learn mathematics? If it is not learned by relating it to daily life, learning it is a waste of time.
> O123: By giving examples from daily life. This increases students' interest in the lesson because it attracts students' attention.
> O87: To improve the learning of mathematics, we need to analyse the relations among given.

Lower-level Conceptions of Learning Physics

A few students reported that learning physics required solving problems. Students frequently expressed the conception that physics involved memorisation of formulas and that a comprehensive understanding of physics required solving a large number of physics problems so that students would not forget what they had learned. The following excerpts illustrate this conception:

> O49: Solving physics problems. Learning physics requires more practice solving problems so that students do not forget [it].
> I49: Learning should be reinforced by providing lots of examples. [The interviewer asks "What do you mean by providing examples?"]. That is, after instruction in the subject, problems should be solved until students understand [it]. If the number of the problems to be solved is increased, learning can be improved.
> O78: Physics includes many formulas. Many physics problems are solved with the help of [these formulas]. More frequently engaging in physics problem-solving, means better learning.

Higher-level Conceptions of Learning Physics

Most students believed that performing hands-on activities was required to learn physics and that hands-on activities improve learning because students are physically and mentally active while learning, which enables them to remember what they had learned. In addition, this type of learning increases student curiosity and interest in physics. Another conception involved the importance of relating physics to daily life to enable students to understand why they were learning physics. Without knowledge of the role of physics in everyday life, the subject had no meaning for students. In addition, using physics to understand everyday life provided learners with a deeper understanding of physics. Some students also emphasised the importance of identifying connections among subjects or concepts and relating newly learned knowledge to previous learning. In addition, students noted that it was necessary to learn how physics formulas were related to physics content. Some students stated, that because physics was abstract it was necessary to use visual materials to enable learners to imagine what physics actually means. This method would facilitate learning physics and allow students to understand what they learned instead of simply memorising. In addition, they could apply their acquired knowledge to life situations. The following excerpts from a questionnaire and interview responses illustrate these conceptions:

> O88: Learning by doing. Going to the laboratory or performing activities so that students can learn through investigation.
> I88: I think [it] is best learned in the laboratory. There, students investigate a topic. Therefore, they can learn better. [The interviewer asks the interviewee to explain why he thinks so.] Students in the laboratory observe the experiments and try to discover what is going on. Consequently, they do not forget what they have experienced. Learning should be more in depth.
> O67: I think physics is best learned through observations and experiments. These attract students' attention more.
> O53: I think that if someone learns something, he/she has to use it in everyday life. Otherwise, it will be meaningless.

> O34: Learning physics requires more in-depth analyses of a topic. That is, when something is learned only superficially, it is impossible to succeed. For example, you cannot understand how to apply formulas.
> O1: Physics includes many abstract concepts. It is difficult to learn them without using visual aids.

Lower-level Conceptions of the Relationship between Learning Mathematics and Physics

Some students perceived both mathematics and physics learning as primarily involving problem solving and believed that learning required solving many problems and that increased problem-solving practice, improved learning. In addition, some students stated, that it was necessary for learners to know how to apply formulas for physics problems or apply rules to solve mathematics equations to succeed in mathematics and physics. Finally, some students reported that mathematics and physics involved performing calculations and that learning was primarily based on knowing how to use calculations to solve problems. If learners had not mastered the ability to perform calculations, it was difficult to learn physics. The following excerpts from a questionnaire and interview responses illustrate these conceptions:

> O49: Both of them include lots of problems. The more you solve, the better you learn.
> I49: I think that learning these [subjects] is based on problem solving. When the number of problems to solve is increased, student success is greater. It is difficult to learn [these subjects] without practicing problems.
> O37: Memorisation of certain rules is necessary for both. Without this, problems cannot be solved.
> O51: Both of them are based on memorisation. If the rules or formulas are not memorised, you cannot solve any problems.
> O53: Both mathematics and physics include calculations. Without understanding them [calculations], it is difficult to learn.

Higher-level Conceptions of the Relationship between learning Mathematics and Physics

Many students thought that mathematics and physics learning were similar because both could be related to everyday life and that without discussing examples from daily life it was difficult to understand mathematics and physics in depth. Because students viewed mathematics and physics as part of life, they believed that it was necessary to use everyday examples in mathematics and physics learning. Some students also thought that mathematics and physics could be learned better using visual materials because of their abstract nature and that using visual materials could convert abstract knowledge into concrete knowledge. The following excerpts from a questionnaire and interview responses illustrate these conceptions:

> O98: Both of them are closely related to daily life. There are many events in the environment that physics and mathematics explain. If they [examples from daily life] were used, we could learn better.
> O134: Both could be learned by discussing examples from daily life. They are closely related to life.
> I134: Both disciplines are also related to daily life. Therefore, attracting students' attention is easy. For example, teachers can give examples from daily life when teaching.
> O4: Using visual material. These [subjects] are abstract. In my opinion, the best way to make them concrete is [using visual materials].
> O26: Physics and mathematics are abstract. There is a need to make them concrete using materials that can help students visualise concepts.

Discussion

The first important result of this study was the finding, that more students had fragmented conceptions of mathematics, but more students had cohesive conceptions of physics. However, when we consider the nature of these two disciplines, this result is not surprising. Due to the nature of physics, its development requires laboratory activities integrated with the scientific processes of hypothesis-testing and experimentation (Hewitt, 2006). This is not true for mathematics. Consequently, students might exhibit more cohesive conceptions of physics.

Moreover, when students described the relationship between mathematics and physics, the number of the students with fragmented conceptions was higher. This result suggests, that students' fragmented conceptions of mathematics might have contributed to their conceptions of the relationship between mathematics and physics. One of the study findings supports this claim. The most frequently occurring student conception was that physics was dependent on physics ("mathematics as a prerequisite for physics"). Many students with this conception associated success in physics with success in mathematics. This finding is consistent with Basson (2002), who found that lecturers exhibited the conception, that mathematics was the language of physics and was required to explain physical phenomena. The results of the current analysis also revealed that students exhibited the same conception. This conception might have been transferred to students directly or indirectly by earlier instructors, who might frequently mention the mathematics-dependent nature of physics in their lessons or might teach physics by emphasising problem-solving, that focused more on mathematics than on laboratory or hands-on activities.

The conception of "mathematics as a prerequisite for physics" might also create problems. For instance, students with negative attitudes towards mathematics might also acquire negative attitudes towards physics because of this conception. To overcome this problem, instructors who teach physics should reduce the focus on mathematics at the elementary and high school levels and discuss the nature of physics more often in their lessons. Because the current study sample included pre-service elementary science and mathematics teachers, the students with this conception might transfer it to their students when they begin to teach. To address this problem, they should be informed of the actual nature of mathematics and physics and how mathematics and physics knowledge emerged and developed throughout the history of science.

Similarly, the number of the students with lower-level conceptions of the relationship between learning mathematics and physics was higher than the number of students with lower-level conceptions of learning physics. In addition, although more students exhibited higher-level conceptions of learning physics, more students exhibited lower-level conceptions of learning mathematics. For example, relatively few students believed that learning physics required solving problems; in contrast, more students believed that learning mathematics required solving problems. This finding suggests that students' lower-level conceptions of learning mathematics influenced their conception of the relationship between learning mathematics and physics. In particular, student conceptions of the relationship between learning mathematics and physics might reflect the conception that physics was dependent on mathematics (i.e., calculations, equations). However, the conception that physics is based on mathematics might lead students to adopt negative attitudes towards physics. Martinez-Torregrosa et al. (2006) claims that physics courses that are integrated with a great deal of mathematics result in negative student attitudes towards physics. As McDermott (1991, 1993) notes, conceptual understanding is more important than memorising problems in physics, and conceptual understanding should be emphasised in teaching physics. This approach might encourage students to adopt higher-level conceptions of the relationship between mathematics and physics.

In summary, the results of the present study indicate that more students exhibited traditional conceptions of mathematics and mathematics learning, while more students exhibited constructivist conceptions of physics and physics learning. Although the mathematics and physics curricula in Turkey incorporate constructivist views, students were more likely to exhibit traditional conceptions of mathematics and mathematics learning. However, students were more likely to exhibit constructivist conceptions of physics and physics learning. As noted above, the distinct nature of these two disciplines and what they explain (numbers or natural events) might produce these results. Therefore, what students perceive while they are learning might shape their conceptions. When the subjects of high school mathematics and physics are examined, there are more concrete examples in physics compared to mathematics. For example, individuals cannot apply the derivatives and integrals of mathematics in their everyday life or ordinary events and thus cannot internalise mathematics as a part of their lives. However, this is not true of physics. Many individuals confront problems related to electricity, lightening or temperature in their daily lives and solve these problems based on the knowledge of physics (whether they are aware of it or not). That is, everyday events can lead individuals to associate physics with their lives. However, apart from basic mathematical operations such as addition and subtraction, they do not frequently use higher-order mathematics that includes derivatives, integrals, and functions in daily life. For these reasons, individuals can exhibit distinct conceptions of mathematics and physics despite the curricula that emphasise constructivist views. The everyday experiences of students and the nature of mathematics and physics might be more influential than the curricula in shaping student conceptions of mathematics and physics and mathematics and physics learning. Consequently, taking the nature of the disciplines and students' life experiences into account, might make it possible to develop more effective curricula and educational programmes to modify these conceptions.

Conclusions and Implications

The current study contributes to the literature because it explored the relationship between conceptions of mathematics and physics and the relationship between mathematics and physics learning. One important study result was the finding, that most of the students viewed mathematics as an indispensible aspect of physics. They primarily associated their success or lack of success in physics with the extent to which they understood mathematics. In this view, it was difficult to succeed in learning physics without understanding mathematics.

In addition, students seemed to exhibit more constructivist conceptions of physics and physics learning compared to their conceptions of mathematics and mathematics learning. As noted above, the nature of each discipline might be more influential than the high school curricula in shaping student conceptions.

This study also reveals the need for additional research to increase our understanding of the extent to which the nature of the discipline itself affects conceptions of the discipline. Because students' epistemological beliefs about science (i.e., beliefs about the nature of scientific knowledge) influence their conceptions of science learning (Tsai et al., 2011), investigating domain-specific conceptions or conceptions of domain-specific learning for fields such as natural science, physics, and mathematics should enable us to identify domain-specific epistemological beliefs. To address this issue, the methods of the current study might be used to investigate other disciplines, and new classifications of conceptions of scientific disciplines and conceptions of learning might emerge.

Limitations

One limitation of the present study was the self-report nature of the questionnaire data. However, interviews with selected participants confirmed the results of the questionnaire data analysis. Another limitation was that the frequency of occurrence of the codes representing the conceptions did not equal the number of the students participating in the study, because some participants failed to respond to all questionnaire items. However, the number of incomplete responses was very low.

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