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Faeq Abdullah AL-Temimei
Faculty of Science, Department of Physics,
University of Kufa, Iraq
Faeqa.jasim@uokufa.edu.iq

SECTION 10. Astronomy and space research.

ASTRONOMICAL REFRACTION MODEL BY LEAST SQUARE FITTING ABOVE KUFA ASTRONOMICAL OBSERVATORY

Abstract: *In the domains of Astronomy and Geodesy, and according to the different hypotheses dependent on distribution of atmospheric condition , many versions of the computing formulae of Atmospheric refraction have been published, and a lot of versions of Atmospheric refraction tables have also come out to get important and more accurate information. In this study a new mathematical model has been proposed to calculate the Astronomic refraction by using least square fitting method and other mathematical approximations with utilizing data that was collected by the weather link station accompanying Kufa Astronomical Observatory, during the interval 1\5\2011 to 30\4\2012. In this model Astronomic refraction in arc sec, when: P in hPa, T in* ^{0}*C , and the altitude in deg; also is applicable for altitude range 10 - 300, zenith angle from 600 to 900, in comparison with the previous Astronomic refraction models. Thus, the Astronomic refraction value, in arc sec, is presented at weather conditions T0=100C, P0=1005 hpa, and zenith angles 600,650,700,750,800,850 equals to 100.985, 122.504, 154.770, 208.515, 315.891, 637.192 respectively. The results obtained from the study have been compared with other researchers which was in perfect agreement.*

Key words: *Astronomical Refraction, Least Square Fitting, Kufa Astronomical Observatory.*
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1. Introduction

Observational astronomy is the part of Astronomy that deals with research on objects in the sky through scientific observations, where the main source of information about celestial bodies and other objects is the measure of electromagnetic signals. These electromagnetic signals propagate through the atmospheric layers, where they found several phenomena effecting on the light coming from celestial bodies as Astronomic refraction (Kunz et al., 2005; Green, 1985). The real part of the Atmospheric index of refraction is a function of pressure, temperature and density; so, the Atmospheric refraction is the deviation of light from a straight line as it passes through the atmosphere due to the variation of the air refractive index with altitude (Michael E. Thomas and Richard I. Joseph, 1996). Many interesting low-altitude refractive effects are existed because of the atmosphere variations in density and water vapor of partial pressure as a function of position as well as the effect of refraction in the atmosphere shifts the observed position of a star towards the observer's zenith. Therefore, it is a specific effect to a given location.

Recently, the Atmospheric refraction has been a significant subject of studies about its impact on the observations in which there are several hypotheses determine the Astronomic refraction at various wavelength or its theoretical models (Garfinkel, 1967). The simplest precise assumption model of the Atmosphere and the refraction assumes that spherical symmetry of the Earth depends on local weather conditions (Richard J. Mathar, 2004, MAO Wei, Yang Lei and Tie Qiong-xian, 2008).

The effect of the Atmosphere on the propagation of electromagnetic wave signals has been researched by many researchers and some models where the theoretical researches on Astronomical refraction began with the work of B. Tycho, a Danish astronomer, in the end of the 16th century(MAO Wei, Yang Lei and Tie Qiong-xian,2008). (Peter D. Noerdlinger,1999) solved the problem of the refraction angle for a spherical atmosphere by a simple, analytic solution, depending only on the surface index of refraction (μ), the problem of the apparent horizontal displacement of the point viewed is also solved analytically, but approximately, because the result depends weakly on an assumed vertical structure of the atmosphere, (Russell D. Sampson, Edward P. Lozowski, and et al., 2003) studied the variability in the Astronomical refraction of the Rising and Setting Sun in Edmonton, Alberta. They found that the seasonal variation of the observed sunset refraction in that study qualitatively matches the predictions of Sugawa, with a maximum occurring in the colder months, (Changbo Wang, Zhangye Wang and Qunsheng Peng,2008) used a new sky light model for atmospheric scattering and refraction where it calculates the refractional track of light through the atmosphere according to the refraction index by adapting a path tracing algorithm considering refraction; the intensity distribution of sky light was calculated, (Krzysztof G. Hełminiak, 2009) studied the impact of the atmospheric differential chromatic refraction on the measurements and precision of relative astrometry. His study has shown that the Atmospheric refraction must be taken into account in Astrometric studies, (S. Cavazzani, S. Ortolani, C.Barbieri, 2011) counted the delay of the arrival times of visible photons on the focal plane of a telescope and its fluctuations as a function of local atmospheric conditions: temperature, pressure, chemical composition and telescope diameter. They also described a theoretical mathematical model for calculating the radius through the study of delay time fluctuations, also (Alejandro Jenkins 2012) described a method to calculate the position of the Sun in the sky as a function of time and the observer's geographic coordinates; his method discussed the Astronomic refraction effect on the position of the sun in the sky.

2. Astronomic Refraction

Astronomical refraction addresses ray-bending effects for objects outside the earth's atmosphere in relation to an observer within the atmosphere (Michael E. Thomas and Richard I. Joseph, 1996). When light was refracted by the atmosphere, the direction of an object differs from the true direction by an amount depending on the atmospheric conditions along the line of sight. This refraction varies with atmosphere density, pressure and temperature ,if the object is not too far from the zenith, the atmosphere between the object and the observer can be approximated by a stack of parallel planar layers fig. (2-8), each of which has a certain index of refraction (μi) outside the atmosphere(H. Karttunen, P. Kroger, H. Oja, 2006). From Snell's Law of refraction, the index of refraction depends on the density of the air, which further depends on the pressure and temperature. When the altitude angles are over 15^0, we can use an approximate formula:

$$AR = \frac{P}{(T+273.15)}\,0.00452\tan(90-a) \qquad (1)$$

An approximate formula for the Astronomic refraction (for altitude angles from15^0 to 30^0) is given by:

$$AR = \frac{P}{(T+273.15)}\,\frac{0.1594+0.0196a+0.00002a^2}{1+0.505a+0.0845a^2} \qquad (2)$$

Where: (a) is altitude angle in degrees, (T) is Temperature in Celsius degree, (P)pressure in millibars and (AR) is Astronomic refraction.

3. A New Model of Astronomic Refraction

In the present work, a new model has been proposed by adopting least square fitting of Astronomical refraction in terms of atmospheric parameters measured by the weather link station of Kufa observatory which is located (35 m) altitude above sea level with geodetic coordinates (longitude 320.0 and latitude 44.50).

For least square fitting to function in this state, it is used (n) as the degree of the polynomial and (N) is the number of the data.it can be assumed the functional relationship (F.Curtis and O.Patrick, 1989; Angus M. Brown, 2001; K. Madsen, H.B. Nielsen, O. Tingleff, 2004; Mehmet Sari, 2012):

$$y = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n \qquad (3)$$

With errors defined by :

$$e_i = Y_i - y_i = Y_i - c_0 - c_1\,x_i - c_2x_i^2 - \cdots - c_nx_i^n \qquad (4)$$

Where: Y_i represents the observed or experimental value corresponding to x_i ,with x_i free error.

The sum of squares is minimized.

$$S = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N}(Y_i - c_0 - c_1\,x_i - c_2x_i^2 - \cdots - c_nx_i^n)^2 \qquad (5)$$

The optimum values of refraction can be obtained by setting, all the partial derivatives $({}^{\partial S}/_{\partial c_0}\,, {}^{\partial S}/_{\partial c_1}, \ldots, {}^{\partial S}/_{\partial c_n})$ to zero: writing the equations for these given (n+1) equations:

$$\frac{\partial S}{\partial c_0} = 0 = \sum_{i=1}^{N} 2(Y_i - c_0 - c_1\,x_i - c_2x_i^2 - \cdots - c_nx_i^n)\,(-1) \qquad (6)$$

$$\frac{\partial S}{\partial c_1} = 0 = \sum_{i=1}^{N} 2(Y_i - c_0 - c_1\,x_i - c_2x_i^2 - \cdots - c_nx_i^n)\,(-x_i) \qquad (7)$$

$$\frac{\partial S}{\partial c_n} = 0 = \sum_{i=1}^{N} 2(Y_i - c_0 - c_1\,x_i - c_2x_i^2 - \cdots - c_nx_i^n)\,(-{x_i}^n) \qquad (8)$$

Dividing each by (-2) and rearranging gives the (n+1) normal equations to be solved simultaneously:

$$\begin{gathered} c_0 N + c_1 \sum x_i + c_2 \sum {x_i}^2 + \cdots + c_n \sum {x_i}^n = \sum Y_i, \\ c_0 \sum x_i + c_1 \sum {x_i}^2 + c_2 \sum {x_i}^3 \ldots + c_n \sum {x_i}^{n+1} = \sum x_i Y_i, \\ c_0 \sum {x_i}^2 + c_1 \sum {x_i}^3 + c_2 \sum {x_i}^4 \ldots + c_n \sum {x_i}^{n+2} = \sum {x_i}^2 Y_i, \\ c_0 \sum {x_i}^n + c_1 \sum {x_i}^{n+1} + c_2 \sum {x_i}^{n+2} \ldots + c_n \sum {x_i}^{2n} = \sum {x_i}^n Y_i, \end{gathered} \tag{9}$$

Putting these equations in matrix form shows an interesting way in the coefficient matrix:

$$\begin{bmatrix} N & \sum x_i & \sum {x_i}^2 & \sum {x_i}^3 & \ldots & \sum {x_i}^{n2} \\ \sum x_i & \sum {x_i}^2 & \sum {x_i}^3 & \sum {x_i}^4 & \ldots & \sum {x_i}^{n+1} \\ \sum {x_i}^2 & \sum {x_i}^3 & \sum {x_i}^4 & \sum {x_i}^5 & \ldots & \sum {x_i}^{n+2} \\ \sum {x_i}^n & \sum {x_i}^{n+1} & \sum {x_i}^{n+2} & \sum {x_i}^{n+3} & \ldots & \sum {x_i}^{2n} \end{bmatrix} c = \begin{bmatrix} \sum Y_i \\ \sum x_i Y_i \\ \sum {x_i}^2 Y_i \\ \sum {x_i}^n Y_i \end{bmatrix} \tag{10}$$

All the summations in equations (9) and (10) run from 1to N. and the matrix of equation (10) is called the normal matrix for least –squares fitting (F.Curtis and O.Patrick, 1989).

By using this method for equation (1) of Astronomic refraction model and by using data from Weather link station of the Kufa Astronomic Observatory, see Appendix (C), The following expression is obtained :

$$AR = \frac{P}{T+273.15}\left(-1.8706 + \frac{901.3392}{a} - \frac{21.9383}{a^2} + \frac{12.6801}{a^3}\right) \tag{11}$$

This equation represents a new model for Astronomic refraction, which is quite different from previous models by the following:

1- Astronomic refraction in (arc sec), where: P in (hPa), T in (^{0}C), and the altitude a in (deg).

2- Equation (11) is applicable for altitude range 1^0 - 30^0 (zenith angle from 60^0 to 90^0) in comparison with the previous Astronomic refraction models(H. Karttunen ,P. Kroger ,H. Oja, 2006).

4. Data Analysis

The following tables and figures are the statistical analysis for the atmospheric parameters " temperature (in ^{0}C) and pressure (in hpa)" are measured by the Weather link station at Kufa Astronomical Observatory "altitude 35 above sea level, longitude 32^0.0 and latitude 44.5^0 " during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012 .

Table (1) represented the statistical analysis for the temperature (in ^{0}C) measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012 from two o'clock to five o'clock in the morning. The average of minimum and maximum temperatures have been taken in this period. It can be seen from these results that maximum value of temperature at August 2011(40.833 ^{0}C) and the minimum value of temperature at January 2012 (3.558 ^{0}C). Also it can be seen from the results in table (1-1) the highest amount of variance in temperature at March 2012(17.500 ^{0}C^2) and the less amount at July 2011(3.351 ^{0}C^2). And from this statistical table the highest amount of mean in temperature at July 2011(36.897 ^{0}C) and the less amount at December 2011(4.278 ^{0}C).

Table 1

The statistical analysis for the temperature measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012.

Month	**Mean (in ^{0}C)**	**Variance (in^{0}C^2)**	**Max. value (in ^{0}C)**	**Min. value (in ^{0}C)**
May 2011	29.421	11.118	36.558	20.800
June 2011	34.861	4.157	37.863	30.663
July 2011	36.897	3.351	40.238	32.783
Aug. 2011	36.197	3.465	40.833	33.346
Sept. 2011	31.813	13.984	40.400	26.700
Octo. 2011	24.155	9.160	28.421	18.379

Nove. 2011	14.388	10.059	20.333	7.521
Dece. 2011	10.048	4.278	14.929	7.233
Janu. 2012	11.212	7.068	15.358	3.558
Febr. 2012	12.923	6.956	17.592	8.288
March 2012	17.345	17.500	23.688	10.067
April 2012	27.218	8.814	32.692	21.221

The following figure (1) describes the standard deviation of temperature measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012. It can be seen from these results that the highest amount of standard deviation of temperature at March 2012(4.183 ^{0}C) and the less amount standard deviation at July 2011(1.83 ^{0}C).

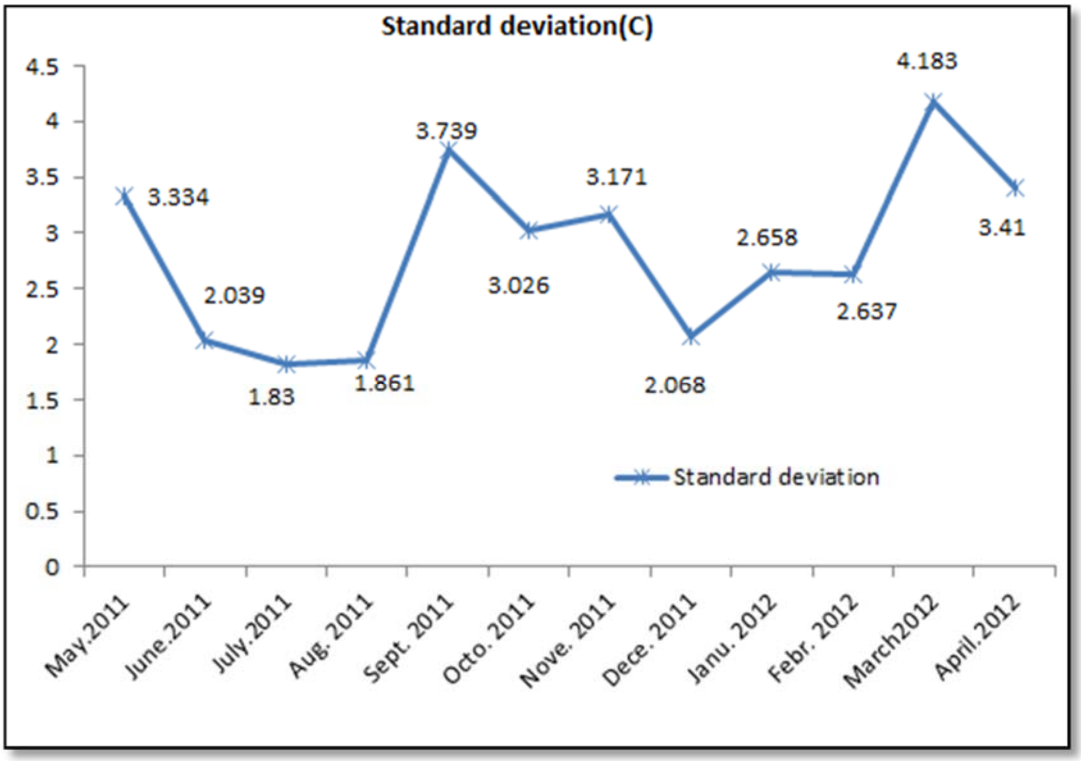

Figure 1 - The standard deviation of temperature at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012.

Table (2) represented the statistical analysis for the pressure (in hpa) measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012 from two o'clock to five o'clock in the morning where the average of pressure in this period was evaluated. It can be seen from these results that the maximum value of pressure at February 2012 (1029.60 hpa) and the minimum value of pressure at August 2011 (994.4 hpa) . Also it can be seen from the results in table (3-2) that the highest amount of variance in pressure at February 2012 (45.533 (hpa)^{2}) and the less amount at September 2011 (5.063 (hpa)^{2}). And it can be seen from this statistical table the highest amount of mean in pressure at December 2011(1021.4 hpa) and the less amount at July 2011(998 hpa).

Table 2

The statistical analysis for the pressure measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012

Month	Mean (hpa)	Variance $(hpa)^2$	Max. value (hpa)	Min. value (hpa)
May 2011	1007.3	6.147	1012.67	1001.01
June 2011	1000.7	12.452	1008.40	995.33
July 2011	998.0	5.679	1003.07	994.93
Aug. 2011	998.1	5.955	1001.87	994.4
Sept. 2011	1003.1	5.063	1010.8	999.73
Octo. 2011	1011.6	5.210	1016.0	1005.73
Nove. 2011	1016.6	8.889	1022.13	1010.27
Dece. 2011	1021.4	10.936	1026.4	1013.07
Janu. 2012	1018.7	8.644	1025.33	1012.67
Febr. 2012	1017.6	45.533	1029.60	1006.27
March 2012	1017.4	40.856	1029.35	1005.60
April 2012	1010.9	10.507	1017.33	1005.07

The following figure (2) describes the standard deviation of pressure measured by the Weather link station at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012. It can be seen from these results that the highest amount of standard deviation of pressure at February 2012(6.747 hpa) and the less amount standard deviation at September 2011(2.25 hpa).

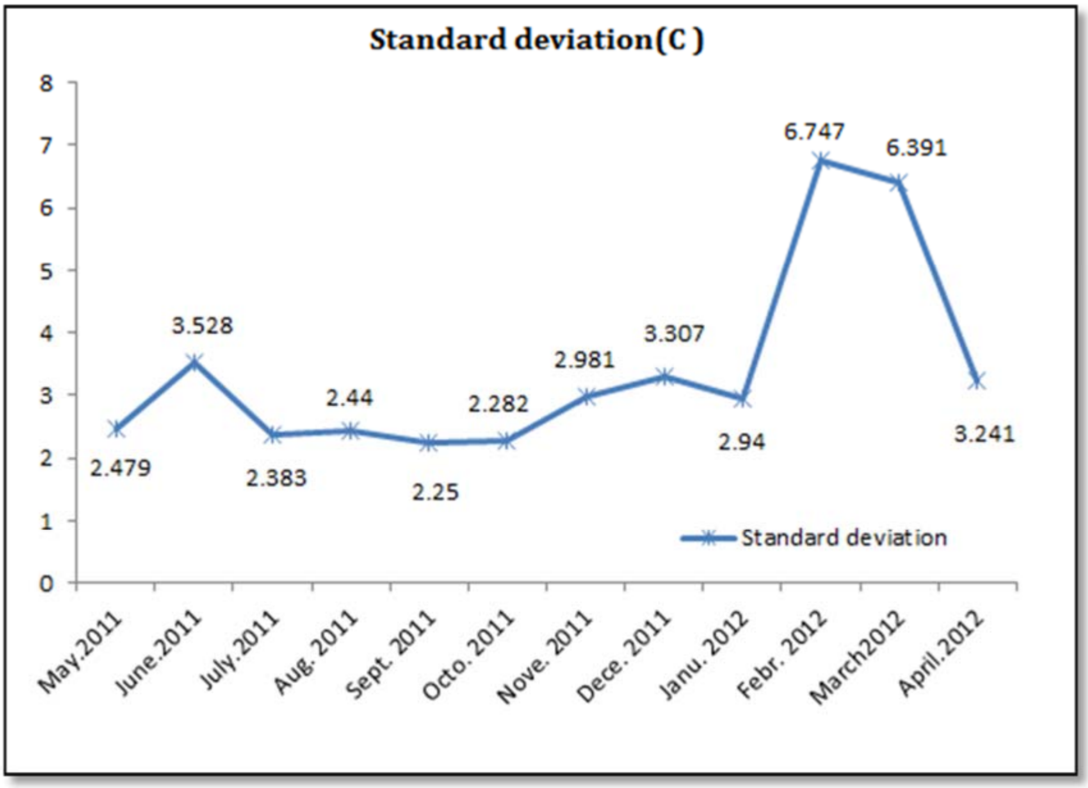

Figure 2 - The standard deviation of pressure at Kufa Astronomical Observatory during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012.

5- Results and Discussion

In the Astronomic refraction model, the Astronomic refraction has been estimated by using two equations(1), and (2) for altitude angles from (0^0-30^0) (H. Karttunen ,P. Kroger ,H. Oja, 2006). While in the new model, the Astronomic refraction has been accounted for altitude angles from (0^0 -30^0) by using the final equation (11) that was obtained from this model directly with units (in arc sec) . Without the need to transfer or change the units of temperature (in^{0}C), altitude angle(in deg) and pressure (in hpa), these changes are included within the derivation of this new model.

The table (3) refers to the value of the Astronomical refraction by using the new model for the interval (1 \ 5 \ 2011 to 30 \ 4 \ 2012) from two o'clock to five o'clock in the morning. The average of minimum and maximum temperatures have been taken as well as the average of pressure in this period. It can be seen from this table the value of Astronomic refraction depending on temperature and pressure with altitude angle and the correlation factor of (AR, T) and (AR, P) at each altitude angle.

Table 3

The Astronomic refraction during the interval 1 \ 5 \ 2011 to 30 \ 4 \ 2012 according to the new model

Date	P (hpa)	T (^{0}C)	AR (arc sec) a=5^0 z=85^0	AR (arc sec) a=10^0 z=80^0	AR (arc sec) a=15^0 z=75^0	AR (arc sec) a=20^0 z=70^0	AR (arc sec) a=25^0 z=65^0	AR (arc sec) a=30^0 z=60^0
15/5/2011	1007.73 ±1.848	25.508 ±2.748	599.344	297.128	196.130	145.577	115.227	94.986
15/6/2011	1006.93 ±3.747	30.663 ±1.843	588.697	291.849	192.645	142.991	113.180	93.299
15/7/2011	998.00 ±2.431	39.842 ±1.311	566.362	280.777	185.337	137.566	108.886	89.759
15/8/2011	1001.87 ±1.000	34.338 ±1.043	578.747	286.917	189.390	140.574	111.267	91.722
15/9/2011	1002.53 ±2.716	29.70 ±3.926	587.982	291.495	192.412	142.817	113.043	93.185
15/10/2011	1014.40 ±2.376	26.396 ±3.045	601.518	298.206	196.841	146.105	115.645	95.331
15/11/2011	1015.47 ±3.160	13.767 ±3.193	628.660	311.661	205.723	152.697	120.863	99.632
15/12/2011	1023.33 ±3.822	10.108 ±1.711	641.712	320.132	209.994	155.868	123.372	101.701
15/01/2012	1020.67 ±2.679	10.279 ±3.137	639.660	317.115	209.323	155.369	122.978	101.376
15/02/2012	1018.67 ±6.808	16.958 ±3.096	623.706	309.205	204.102	151.494	119.911	98.847
15/03/2012	1010.27 ±6.488	22.879 ±4.009	606.193	300.523	198.371	147.240	116.544	96.072
15/04/2012	1008.00 ±1.826	24.025 ±2.811	602.490	298.687	197.159	146.341	115.832	95.485

The following fig. (3) illustrates the changes of the Astronomic refraction above Kufa Astronomical Observatory with altitude angles from (0^0-30^0) during weather conditions through above mentioned period. It can be seen from this figure that there is a gradual exponential decrease in the Atmospheric refraction with an increase in the altitude angles.

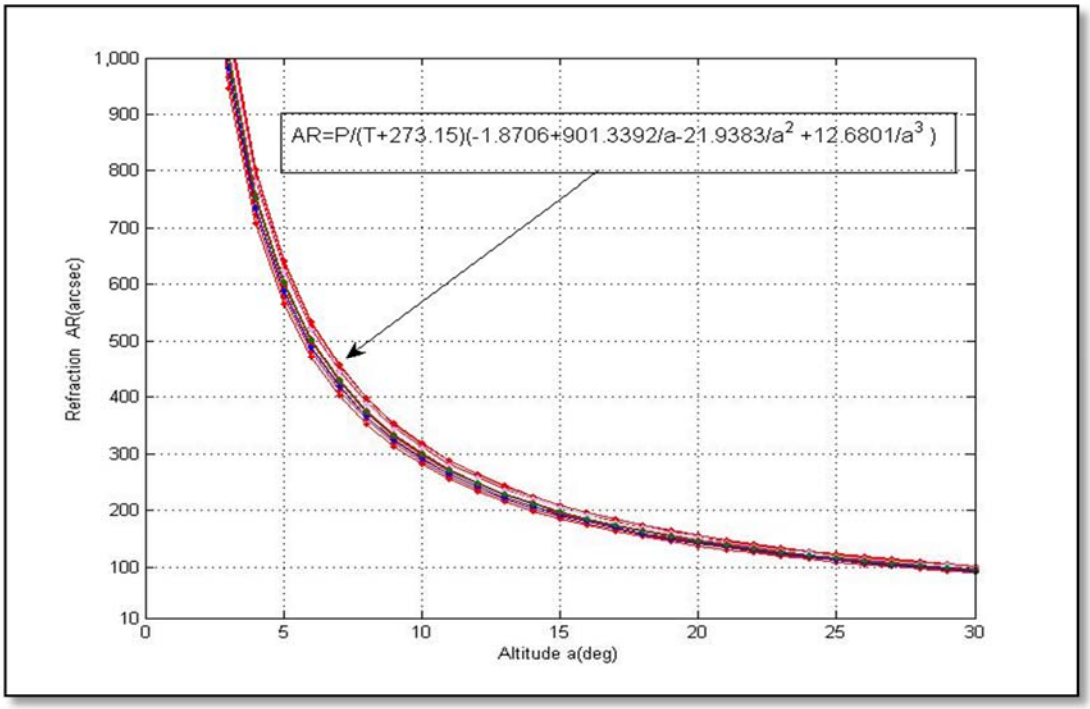

Figure 3 - Astronomic refraction at different altitude angles according to the new model.

Table(4) indicates the extent of the correspondence of the results that are obtained according to the new model and Astronomic refraction model under the same weather conditions, that the result calculated in this present model is very close to that of Astronomic refraction model (H. Karttunen, P. Kroger , H. Oja, 2006).

Table 4

Comparing Astronomic refraction by using the new model with Astronomic refraction model.

Z Deg	New model P=1015.47hpaT=13.76(^{0}C)	Astronomic refraction model P=1015.47hpa T=13.76 ^{0}C	The difference	New model P=1007.73 hpa T=25.50 ^{0}C	Astronomic refraction model P=1007.73hpa T=25.50 ^{0}C	The difference
60	99.632	101.007	-1.375	94.986	96.297	-1.311
65	120.863	121.220	-0.357	115.227	115.567	-0.34
70	152.697	151.590	1.107	145.577	144.521	1.056
75	205.723	205.249	0.474	196.130	195.678	0.452
80	311.661	311.900	-0.239	297.128	297.355	-0.227
85	628.660	628.611	0.049	599.344	599.297	0.047

Here is a comparison between the results with other works under various conditions as they are shown in the following table (5). It can be seen from table (5) that the result calculated in this present model is very close to that in model (Peter D. Noerdlinger ,1999), model (Laweence H. Auer and E. Myles Standish, 2000) and model (Minodora Lipcanu, 2005) .

Table 5

Comparison of Astronomic refraction in the new model with other models

Z(deg)	New model for data at 15/12/2011	Ref. [1999]	Ref. [2000]	Ref. [2005]
60	101.701	99.057	103.99	98.47
65	123.372	122.789	……	……
70	155.868	157.184	……	155.38
75	209.994	213.248	221.34	……
80	320.132	322.96	329.46	312.33
85	641.712	642.152	588.87	577.89

The results observed in the table above are compatible with the compared references, with some differences in the values which belong to the difference in weather conditions and the number of variables used in addition to the type of computation method that was used.

6-Conclusions The tables of refraction used in Astronomy and Navigation are generally based on constitution of the atmosphere. This was unavoidable at a time when the meteorological data available conditions have become ripe for a new approach to problem. Many models derived for this phenomena where the effect of Astronomic refraction in the atmosphere shifts the observed position of a star towards the observer's zenith; therefore, this effect can be considered in specifying the real position to the celestial body. Due to the refraction theory, the ray path through the atmosphere is curved. We can calculate the refraction index in every altitude, and then gain the ray path by the Snell's law. The study of Astronomic refraction for the first layer (troposphere) is very significant as this layer is more effective on propagating light from celestial body. Also The path banding correction of refraction should be taken into account specially when zenith distance is large. Through the results of the study found the Astronomic refraction is inversely proportional to temperature and altitude, but directly proportional to pressure. The outcomes of this study showed that better results are obtained than the previous models and the new model. The future work such a study can be carried out on the measurement of extinction coefficient also measurement of aerosols of the atmosphere using lidar system.

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