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Building Fuzzy Goal Programming with Fuzzy Random Linear Programming for Multi-level Multi-Objective Problem

Nureize Arbaiy[1], and Junzo Watada[2]
[1,2]Graduate School of Information, Production and System, Waseda University, 2-7 Hibikino, Wakamatsu, Kitakyushu, 808-0135 Japan
[1]Faculty of Computer Science and Information Technology, University Tun Hussein Onn Malaysia, 86400 Johor, Malaysia.
(Email: [1]nureize@uthm.edu.my, [2]junzow@osb.att.ne.jp})

ABSTRACT

This paper proposes a fuzzy goal programming that is developed by fuzzy random regression approach, to solve a multi-level multi-objective problem. Fuzzy random regression enables us to deal with fuzzy random circumstances and approximate the coefficients for the developed model. A numerical example of the production planning problem illustrates the proposed solution approach. The proposed method is important where the fuzzy random data is dealt in the mathematical model to solve the multi-level multi-objective decision making problem which can attain a satisfactory solution.

KEYWORDS

Fuzzy goal program, fuzzy random linear programming, multi-level multi-objective problem.

1 INTRODUCTION

The mathematical model is important to translate real-world problem to find the solution. However, translating real-world problem into a mathematical model becomes more complicated when uncertainties are contained in the system. Decision maker faced with environments in which both fuzziness and randomness are included causes the developed mathematical model should carefully treat these uncertainties.

Since the model coefficients are usually decided by decision makers, it makes these decisions crucial and influential to the result of the model [1]. A regression analysis will be possibly used to estimate the coefficients of the model [1], [2], [3]. However, classical regression models consider crisp variables and values, and produces crisp kinds of model. That is, the obtained statistical model does not consider randomness and vagueness included in the data or in a system [3]. In view of the nature of the real world, the information available to a decision maker is often imprecise due to inaccurate attribute measurements and inconsistent priority judgments. It makes the treatment of such circumstances is necessary.

The real situations of making a decision in an organization involve a diversity of evaluation such as evaluating alternatives and attaining several goals at the same time. In many practical decision making activities, decision making structure has been changing from a single decision maker with a single criterion to multiple decision makers with multi-criteria and even to multi-level situations. A resource planning problem in an organization usually consists of several objectives and requires a compromise among several committing individuals or units.

Typically, these groups of decision making are arranged in an administrative hierarchical structure to supervise the independent and perhaps conflicting objectives. In this type of multi-level organization, decision planning should concern issues of central administration and coordination of decision making among lower-level activities to achieve the overall organization target. Each decision maker is responsible for one decision making unit of the hierarchical decision-making levels and controls a decision to optimize the objectives at each level. Although the execution of decision moves sequentially from an upper level to a lower level, the reaction, behavior and decision of a lower-level decision maker should affect the optimization of the decision at an upper level decision maker ([4], [5], [6], [7]). Because of conflicting objectives over different levels, the dissatisfaction with the decision results is often observed among the decision makers. In such cases, a proper distribution of decision authority must be established among the decision levels for most multi-level decision situations.

A mathematical multi-level multi-objective programming has often served as a basis for structuring the underlying goals and a hierarchical decision making situation of such organizations ([8], [9], [10], [11], [12]). Subsequently, a multi-objective linear programming problem aims to optimize various conflicting linear objective functions simultaneously under given linear constraints to find compromise solutions ([13], [14]).

Let $\mathbf{c}_i = (c_{i1},\dots,c_{ik}), i = 1,\dots,p$ denote a vector of coefficients of the i^{th} objective function $f_i(\mathbf{x})$. Then, the multi-objective linear programming problem is written as:

$$\begin{aligned} & opt \quad (f_1(\mathbf{x}),\dots,f_p(\mathbf{x})) \\ & \text{s.t.} \quad \mathbf{Ax} \le \mathbf{b}, \mathbf{x} \ge \mathbf{0}, \end{aligned} \tag{1}$$

where opt indicates optimization operation (minimization or maximization), $\mathbf{x}$ is an $n-$ vector with components $x_1,\dots,x_n$, $\mathbf{Ax} \le \mathbf{b}$ denotes system constraints written in vector notation and $f_i(\mathbf{x}) = \mathbf{c}_i\mathbf{x}$ are the objectives function. Nevertheless, the standard mathematical programming of multi-objective problem (1) cannot accommodate problems in a multi-level decision making structure as it is assumed that each of all objectives comes from a single decision maker at a single level. Therefore, a multi-level multi-objective programming problem solution is a necessity.

In a multi-level decision-making context, each decision maker represents a decision-making unit at a different level. All decision makers should cooperate with others in making the decision. For necessity in the sequential multi-level decision making structure, a decision maker at the highest level determines the plan and distributes this information to all decision makers in the subordinate levels. To ensure all decisions are made in cooperatively and decision authorities are distributed properly in the organization, the satisfaction of decision makers at the lower level must be considered. From this standpoint, it is desirable to develop a fuzzy programming method that facilitates multiple objectives in multi-level and fuzzy decision-making situations.

In this paper, we introduce an additive model of a Fuzzy Goal Programming (FGP) [15] to realize the multi-level multi-objective decision making. The FGP approach is used to achieve the highest degree of achievement for each goal by maximizing fuzzy achievement functions. The algorithm uses the concept of satisfaction to multi-objective optimization at every level until a preferred solution is attained. The problem model was also developed by means of fuzzy random regression ([1], [16]) approach, to overcome the difficulties in determining the model coefficients and in treating the hybrid uncertainties that exist in the data used to construct the model coefficients. From that we emphasize that the proposed method has significant advantages in solving multi-objective problem in the multi-level organizational situation in which fuzzy random information coexisting.

The remainder of this paper is divided into six sections. Section II provides preliminary knowledge for a multi-level multi-objective problem and fuzzy random regression model. Section III explains the main components of FGP model. Section IV describes the FGP solution algorithm for solving multi-level multi-objective problems. An illustrative example is presented in Section V, and finally, discussions and conclusions are given in Section VI.

2 MULTI-LEVEL MULTI OBJECTIVE DECISION MAKING

In any organization with a hierarchical decision structure, the sequential and preemptive nature of the decision process increases complexities in making organization decision. In the multi-level programming, sequential decision making process used to start at the highest level. A decision maker at one level controls or coordinates the decision makers on the subordinate levels. Moreover, it is assumed that a decision maker at each level has a certain degree of autonomy, where the decision maker has an authority to decide the best option among the alternatives in their decision making unit. Planning in such an environment has been recognized as an important decision making process ([17], [18], [19]).

A multi-level multi-objective programming problem is characterized when a multiple decision makers optimize several objectives in the multi-level structured organization ([8], [9]). In a multi-level programming, chosen decision variables x_{ik}^* are controlled by the decision maker for each level and are distributed down to the following level so that the decision-making process at the present level can include the decision from the upper level simultaneously. As each decision making level deals with several conflicting objectives, the situation creates multi-level programming problems in a set of nested optimizations over a single feasible region. In such a situation, the coordination of decision authority demonstrates that the decision variables of one level affect the decisions of the other levels. Hence, it explains that the important feature of the multi-level programming problem is essentially related to the coordination of the decisive powers among all levels and that decisions at the lower levels are influenced from the upper levels.

There are many plans and/or decision making situations that can be properly represented by a multi-level programming model. All of them appear whenever a hierarchical structure exists in the decision making process. Let us consider an organization that has a multi-level programming problem with multi-objective function $F_i(\mathbf{x})$ for $i = 1,\cdots,p$, defined over a jointly dependent strategy set S. Let the vector of decision variables $\mathbf{x} = (x_1,\ldots,x_p)$ takes values in R^n. Assume that decisions are made sequentially beginning with DM_1, which controls a vector $\mathbf{x}_1 \in X_1$, down through DM_p, which controls a vector $\mathbf{x}_p \in X_p$, where X_i is a nonempty subset of $\Re^{n_i}, i = 1,\cdots,p$ and $n_i = n_1 + \cdots + n_p$.

The decision maker DM_i at the i^{th} level has authority over the decision variable $\mathbf{x}_i$. The multi-level multi-objective linear programming problem is a nested optimization problem ([4], [20], [21]), and has the following structure:

Find $\mathbf{x}$ so as to

$$\min_{x_1 \in X_1} F_1(\mathbf{x}) = \min_{x_1}\{f_{11}(\mathbf{x}),\cdots, f_{1m_1}(\mathbf{x})\},$$

where x_1 solves

$$\min_{x_2 \in X_2} F_2(\mathbf{x}) = \min_{x_2}\{f_{21}(\mathbf{x}),\cdots, f_{2m_2}(\mathbf{x})\},$$

$\vdots$

where $x_2,\ldots,x_p$ solves

$$\min_{x_p \in X_p} F_p(\mathbf{x}) = \min_{x_p}\{f_{p1}(\mathbf{x}),\cdots, f_{pm_p}(\mathbf{x})\}$$

$$\text{s.t.:} \quad \mathbf{x} \in X,$$
$$S = \{\mathbf{x} \in \Re^n : \mathbf{Ax}(\le,\ge,=)\mathbf{b}\}, \qquad (2)$$
$$\mathbf{x} \ge 0,$$

where $f_{ij}(\mathbf{x}) = c_1^{ij}\mathbf{x}_1 + \cdots + c_p^{ij}\mathbf{x}_p$, $i = 1,\cdots,p$, $j=1,\cdots,m_i$, are linear objective functions. Let us indicate c_k^{ij} as constants, $\mathbf{A}_i$ as coefficient matrices of size $m \times n_i$ and n_i as the number of involved decision makers.

The execution of decision-making units moves from higher to lower levels. Each decision-making unit optimizes its objective function independent of other units but is affected by the actions of another level. The lower-level decision maker independently optimizes the unit's plan of action according to the goals and limitations determined in the unit, disregarding the goals of the higher-level decision maker. Thus, the problem with decision authority coordination in this multi-level structure is to identify the best compromising solution at each decision-making level to attain overall organization targets.

3 FUZZY RANDOM REGRESSION APPROACH FOR BUILDING MULTI-OBJECTIVE MATHEMATICAL MODEL

Typical multi-objective problem is a decision problem to optimize a set of objectives. The mathematical model is then used to represent and solve the problem. Though, the model coefficients play a pivotal role in the mathematical modeling and the value of model coefficient should be determined in prior to construct the mathematical model. The coefficients of the mathematical model are commonly decided by a decision maker with their knowledge and expertise. Nonetheless, sometimes it is not easy to determine the coefficients, as relevant data are occasionally not given or difficult to obtain. This task may cause difficulties, and thus it makes the decisions of model coefficient is crucial and influential to the model's result. The occurrence of errors in the determination of the coefficients might ruin the model formulation [22]. Therefore, a number of studies have suggested various methods to minimize these potential errors and to address the problem ([2], [23], [24], [25], [26], [27]). The regression analysis will possibly work to estimate the coefficients of the model ([1], [16]).

A regression method analyzes statistical data to estimate the model coefficients in developing effective models. The conventional mathematical programming problem uses numerical deterministic values to these coefficients. In contrary, it is more realistic to take the estimated value of the coefficients as imprecise values rather than precise ones. In practical systems, probabilistic or/and vague situations include uncertain information such as predictions of future profits and incomplete historical data. Therefore, the mathematical programming models should be able to handle the above problems. That is, the above situations should be explicitly considered in the decision making process. For that reason, the fuzzy random regression model is introduced to solve such a problem with the existence of the randomness and fuzziness in historical data used for the approximation [3]. The property of fuzzy random regression model is used to allow for the co-existence of fuzziness and randomness in the data.

In this paper, one sigma confidence interval is used to express the confidence interval that expresses the expectation and variance of a fuzzy random variable as follows:

$$I[e_X, \sigma_X] \triangleq \left[E(X) - \sqrt{var(X)},\ \ E(X) + \sqrt{var(X)}\right] \quad (3)$$

The fuzzy random regression model with one sigma confidence intervals [13] is described as follows:

$$\begin{aligned} &\min_{A} \quad J(c) = \sum_{j=1}^{m}\left(c_j^r - c_j^l\right) \\ &c_j^r \geq c_j^l, \\ &Y_i = \sum_{j=1}^{m} c_j\, I[e_{X_{ij}}, \sigma_{X_{ij}}] \supset_{\tilde{h}} I[e_{Y_i}, \sigma_{Y_i}] \\ &\text{for} \quad i = 1, \cdots, p \quad j = 1, \cdots, m. \end{aligned} \quad (4)$$

where $\supset_h$ denotes the fuzzy inclusion at level h.

Thus, the fuzzy random regression model with confidence intervals is given in the following expression:

$$Y_i = \sum_{j=1}^{m} c_j I\left[e_{X_{ij}} + \sigma_{X_{ij}}\right], \quad i = 1, \ldots, p. \quad (5)$$

Developing a mathematical programming model requires an appropriate model setting to avoid solutions from being mislead. Thus, the fuzzy random regression approach has been introduced in the construction of a multi-level multi-objective model.

4 FUZZY GOAL PROGRAMMING FOR MULTI-LEVEL MULTI-OBJECTIVE PROBLEM

Let us consider the multi-objective problem (1). In the fuzzy multi-objective problem, the objective functions are denoted as $F_i(x) \tilde{\geqq} g_i$ where $\tilde{\geqq}$ represents fuzzy inequality and g_i is the goal target for the objective function.

Let V represent the fuzzy achievement function consisting of membership functions μ_i for fuzzy objectives. In the FGP approach, the weighted additive model [15] is formulated by aggregating the membership functions with an additive operator as follows:

$$\begin{aligned} \max \quad & V(\mu) = \sum_{i=1}^{m} \omega_i \mu_i \\ \text{subject to} \quad & \mu_i = \frac{\mathbf{A}_i \mathbf{X}_i - L_i}{g_i - L_i}, \\ & \mathbf{Ax} \le \mathbf{b}, \\ & x \le 0, \\ & \mu_i \in [0,1]; i = 1, \dots, p. \end{aligned} \tag{6}$$

In this section, we explain the important components required to build the additive model of FGP consisting of the objective function, achievement function, goal and tolerance, and membership function.

4.1 Objective Function

The term 'objective' is the terminology used in goal programming approach and referred as a criterion with additional information about the direction (maximize or minimize) in which the decision maker prefers on the criterion scale [28]. In a multi-objective problem, the objective function $F_i(\mathbf{x})$ is created for each objective to solve. The objective function is represented in the form of

$$F_i(\mathbf{x}) = c_1 \mathbf{x}_{i1} + \cdots + c_m \mathbf{x}_{im}, \quad i = 1, \dots, p, \; j = 1, \dots, m. \tag{7}$$

In this proposed model, the coefficient value of c_{ij} is decided by the fuzzy random regression approach. The coefficient value derived from fuzzy random regression model (4) however results in an interval denoted by the bracketed numbers $[c_j^l, c_j^r]$. Considering the midpoint value of $\xi_{ij} = \frac{(c^l + c^r)}{2}$, then the fuzzy random based objective functions (7) for FGP are rewritten as follows:

$$F_i(\mathbf{x}) = \xi_1 \mathbf{x}_{i1} + \cdots + \xi_m \mathbf{x}_{im}, \quad i = 1, \dots, p, \; j = 1, \dots, m \tag{8}$$

where ξ_{ij} is the fuzzy random based coefficient.

Hence, the coefficients ξ_{ij} of each objective function are identified by the regression model and these objective functions are further used in the setting of the multi-objective model.

4.2 Fuzzy Achievement Function

The fuzzy achievement function V is the total achievement of all the objectives. All the membership functions of the fuzzy objectives are multiplied by a weight ω that reflects their relative importance and are added together to form the achievement function.

The first level achievement function is expressed as follows:

$$\max \; V(\mu_1) = \sum_{j=1}^{m_1} \omega_{1j} \mu_{1j}. \tag{9}$$

For the subsequent lower level, the achievement function $V(\mu_p)$ is written as

$$\max\ V(\mu_p) = \sum_{j=1}^{m_1}\omega_{1j}\mu_{1j} + \cdots + \sum_{j=1}^{m_p}\omega_{pj}\mu_{pj} + \sum_{k=1}^{n_{(p-1)}}\left[\omega_{(p-1)k}\mu(x)^L_{(p-1)k} + \omega_{(p-1)k}\mu(x)^R_{(p-1)k}\right] \quad (10)$$

$k = 1,\ldots,n_i$

where the weight of decision variables and controlled decision vector x_{ij} is elicited with Equations (14.1) and (14.2), respectively. The weighting scheme is explained in the sub-section 4.5.

4.3 Goal and Tolerance

A goal in goal programming is known as a numerical target value that decision makers desire to achieve [28]. Usually, decision makers assign values to the goal and the tolerance based on their experience and knowledge. The mathematical model can also be used to determine the goal and the tolerance values by computing the individual optimal solutions to obtain the satisfaction degree [29]. In this study, the goal and tolerance are assumed provided by the experts.

4.4 Membership Functions

The fuzzy objectives in a multi-objective problem are characterized by their associated membership functions, based on fuzzy set theory [30]. The membership functions are used to formulate the corresponding objective functions.

The linear membership functions μ_i for the i^{th} fuzzy objective $F_i(x) \gtrsim g_i$ can be formulated as follows [29]:

$$\mu_{F_i}(x) = \begin{cases} 1 & \text{if } L_{ij} \le f_{ij}(x) \\ \dfrac{f_{ij}(x) - L_{ij}}{g_{ij} - L_{ij}} & \text{if } g_{ij} \le f_{ij}(x) \le L_{ij} \\ 0 & \text{if } f_{ij}(x) \le g_{ij} \end{cases} \quad (11)$$

The membership function for $F_i(x) \lesssim g_i$ is as follows:

$$\mu_{F_i}(x) = \begin{cases} 1 & \text{if } f_{ij}(x) \le g_{ij} \\ \dfrac{L_{ij} - f_{ij}(x)}{L_{ij} - g_{ij}} & \text{if } g_{ij} \le f_{ij}(x) \le L_{ij} \\ 0 & \text{if } L_{ij} \le f_{ij}(x) \end{cases} \quad (12)$$

where $i = 1,\ldots,p,\quad j = 1,\ldots,m_i$ and L_{ij} is the tolerance limit for fuzzy objectives. The membership function of each fuzzy objective was built to find the optimal solutions of the i^{th} level of the multi objective linear programming problem $x^{i*} = (x_1^{i*},\ldots,x_p^{i*}),\ i = 1,\ldots,p-1$.

In a multi-level decision-making situation, the decision at each subordinate level influences the upper level's decision as well as the upper-level decision makers control the subordinate level's decision. The decision denoted as x_{ik} at the present level is sent down to the next lower level. To take care of the vagueness of this decision x_{ik} , let $t_k^{i_L}$ and $t_k^{i_R}$ for $i = 1,\ldots,p-1;\quad k = 1,\ldots,n_i$ be the maximum negative and positive of tolerance values, respectively, for the decision vectors x_{ik} with values specified by the i^{th} level decision maker.

The triangular fuzzy numbers of the decision vectors x_{ik} are stated as $(x_{ik}^* - t_k^{i_L}, x_{ik}^*, x_{ik}^* + t_k^{i_R})$. Thus, as in Baky [4], the linear membership functions for each of the n_i components of the decision vector $x_i^{i*} = (x_1^{i*},\ldots,x_p^{i*})$ controlled by the decision makers of the upper $p-1$ levels can be formulated as follows:

$$\mu_{x_{ik}}(\mathbf{x}) = \begin{cases} \dfrac{x_{ik} - \left(x_{ik}^{*} - t_k^{i_L}\right)}{t_k^{i_L}} & \text{if } x_{ik}^{*} - t_k^{i_L} \le x_{ik} \le x_{ik}^{*} \\ \dfrac{\left(x_{ik}^{*} + t_k^{i_R}\right) - x_{ik}}{t_k^{i_R}} & \text{if } x_{ik}^{*} \le x_{ik} \le x_{ik}^{*} + t_k^{i_R} \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

where $i = 1,\ldots, p-1; \quad k = 1,\ldots,n_i$.

4.5 Relative Importance

Let the numerical coefficients ω_{ij}^{+} , ω_{ik}^{R} and ω_{ik}^{L} denote the relative importance of achieving the aspired levels. The relative importance of the fuzzy goal is then determined using the weighting scheme [21].

$$\omega_{ij}^{+} = \frac{1}{u_{ij} - g_{ij}}, \quad (14.1)$$

$$\omega_{ik}^{L} = \frac{1}{t_k^{i_L}}, \; \omega_{ik}^{R} = \frac{1}{t_k^{i_R}}. \quad (14.2)$$

ω_{ij}^{+} is the weight for the objective functions, and ω_{ik}^{R} and ω_{ik}^{L} represent the weights for the membership functions of the decision vectors. u_{ij} and g_{ij} are the goal and tolerance, respectively.

5 THE SOLUTION ALGORITHM

A solution to the multi-level multi-objective programming problem is obtained as follows on the basis of the main components of additive FGP. For two continuous levels in the decision making tree, the decision-making process is carried out in two sequential stages. The higher level decision maker determines the top plan of action and followed by the lower level decision maker that executes the plan which is decided by the higher level decision maker.

The additive FGP model for multi-level decision making is written as follows:

$$\begin{aligned} \max\ V(\mu_p) &= \sum_{j=1}^{m_1} \omega_{1j}\mu_{1j} + \cdots + \sum_{j=1}^{m_p} \omega_{pj}\mu_{pj} \\ &+ \sum_{k=1}^{n_{(p-1)}} \left[\omega_{(p-1)k}\,\mu(x)_{(p-1)k}^{L} + \omega_{(p-1)k}\,\mu(x)_{(p-1)k}^{R}\right] \\ \text{s.t.}\quad & \mu_{\mathrm{F_i}(\mathbf{x})} = \frac{\left(c_1^{ij}x_1 + \cdots + c_p^{ij}x_p\right) - \mu_{ij}}{g_{ij} - \mu_{ij}}, \\ & i = 1,\ldots,p,\ j = 1,\ldots,m_i, \\ & \mu_{x_{ik}}^{L}(\mathbf{x}) = \frac{x_{ik} - \left(x_{ik}^{*} - t_k^{i_L}\right)}{t_k^{i_L}}, \\ & i = 1,\ldots,p-1, k = 1,\ldots,n_i, \\ & \mu_{x_{ik}}^{R}(\mathbf{x}) = \frac{\left(x_{ik}^{*} - t_k^{i_R}\right) - x_{ik}}{t_k^{i_R}}, \\ & i = 1,\ldots,p-1, k = 1,\ldots,n_i, \\ & A_i x_i (\le,\ge,=) b, \quad \mathbf{x} \ge 0, \\ & \mu_i \in [0,1], \quad i = 1,\ldots,m. \end{aligned} \quad (15)$$

The multi-level additive FGP is separately solved for the i^{th} level multi-objective program with $i = 1,\ldots, p-1$.

Phase I: Building fuzzy random objective functions

1) Describe the problem. Determine the parameters, objectives and decision level for the problem.
2) Solve each objective described in the model to find the estimate's coefficients. For each of the goal constraints $f_{ij}(x)$,
   - prepare the fuzzy random input-output data. Readers are directed to [3], [16] for the explanation of preparing the fuzzy random-input output data.
   - calculate the one sigma confidence interval in terms of Equation (3).
   - apply FRR model by means of Equation (4).

3) Build fuzzy random based objective function as Equation (5) including the confidence interval value.
4) Formulate the MLMO linear program model (11) to illustrate the model for the respective problem. Substitute the fuzzy random-based objective function as Equation (8) that is derived from Step 3) in the MLMO model.

Phase II: Solving MLMO problems

For each objective established at each level in the MLMO Model (2), determine the target goal g_{ij} and tolerance μ_{ij} , respectively from the expert.

A: First-Level Additive-FGP

1) Assume $p = l$. Set $l = 1$.
2) Create the fuzzy achievement function V_{μ_1} consisting of μ_{ij} for $i = 1,\ldots, p \quad j = 1,\ldots,m_i$.
3) Create the fuzzy objective membership function $\mu_{F_i(x)}$ with the target goal g_{ij} and tolerance μ_{ij} for the fuzzy goals in the first level controlled by DM_1 .
4) Formulates first level additive FGP model based on Model (15) to obtain a satisfactory solution.
5) Send the obtained solutions of the objective function $x^{1*} = \left(x_1^{1*},\ldots,x_p^{1*}\right)$ decisions to the next lower level.

B: Second-Level towards p^{th} -Level Additive-FGP.

1) Set $l = l + 1$. Determine the obtained solutions $x^{l-1*} = \left(x_1^{l-1*},\ldots,x_p^{l-1*}\right)$ decisions from the higher level l–1 .
2) Let the decision maker decide the lower bound and upper bound of the tolerance values $t_k^{i_L}$ and $t_k^{i_R}$ on the decision vector $x^{l-1*} = \left(x_1^{1*},\ldots,x_p^{1*}\right)$.
3) Develop the fuzzy achievement function V_{μ_p} consisting of $\mu_{f_{ij}}$ for $i = 1,\ldots, l, \quad j = 1,\ldots, m_l$ and $\mu_{x_{ik}}$ for $i = 1,\ldots, l-1; \ k = 1,\ldots, n_{l-1}$.
4) Create the fuzzy objectives membership function $\mu_{f_i(x)}$ with the target goal g_{ij} and tolerance μ_{ij} for the fuzzy goals for the present level controlled by the DM_p .
5) Develop the decision vector membership function $\mu_{x_{ik}(x)}, i = 1,\ldots, p-1, k = 1,\ldots, K_i$ based on Equation (12).
6) Formulate the additive-FGP Model (15) of the p -level problem to obtain a satisfactory solution to the present $p-$level FGP problem.
7) Solve the model to obtain $x^{l*} = \left(x_1^{l*},\ldots,x_p^{l*}\right)$.
8) Repeat step 1 until $l < p$, to generate a satisfactory solution $x^{l*} = \left(x_1^{l*},\ldots,x_p^{l*}\right)$ for the MLMO linear programming problem.

6 NUMERICAL EXAMPLES

One export-oriented country is concentrating on producing three important products x_1 , x_2 and x_3 which are manufactured by a company C_d , $d = 1,\cdots,D$ with the given capabilities. This company has distributed branch B_d, $d = 1,\cdots,D$, in city level for producing the products. This situations result in 3 levels decision making and each level is responsible to accomplish the objectives that are decided in the prior.

The initial step in this phase is the data preparation to determine the decision variable's coefficient through a Fuzzy Random Regression Model (4) and further develop the objective function for multi-level multi-objective problem (15). The previously collected data set is then pre-processed ([3], [16]). The probabilities are assigned as the proportion of product being produced in i^{th} plant to the total production numbers.

Table 1 summarizes the information needed to construct the multi-level multi-objective model (2). Let us assume f_{ij} represents the DM_i objective(s) in each level. Based on the information in Table 1, the multi-level multi-objective problem can be summarized as follows:

Table 1. The coefficients for objective functions and goal's target

Decision Making Level	Goal		Fuzzy Random-Based Coefficient			Target	Tolerance
			ξ_1	ξ_2	ξ_3		
Government level, DM_1 (first-level)	f_{11}	Maximize the export revenue	2.078	0.260	0.170	4.58	0.5
	f_{12}	Maximize the national level profit	1.010	1.700	0.476	5.50	0.5
	f_{13}	Minimize the capital,	0.438	0.796	0.512	5.00	0.5
State level, DM_2 (second-level)	f_{21}	Maximize the production volume	1.126	0.100	0.097	3.90	0.5
	f_{22}	Maximize the profit for state level	0.856	1.473	0.443	4.50	0.5
	f_{23}	Minimize the cost of production	0.380	0.737	0.277	4.80	0.5
City level, DM_3 (third-level)	f_{31}	Maximize the production volume	0.921	0.050	0.526	3.00	0.5
	f_{32}	Minimize the cost of production	0.380	0.737	0.216	4.00	0.5

Find x so as to satisfy:

[Level 1]

$$\min_{x_1} = \begin{pmatrix} 2.078x_1 + 0.260x_2 + 0.170x_3 \geq g_{11}, \\ 1.010x_1 + 1.700x_2 + 0.476x_3 \geq g_{12}, \\ 0.438x_1 + 0.796\ x_2 + 0.512x_3 \leq g_{13}. \end{pmatrix}$$

[Level 2]

where x_2 and x_3 solves, (16)

$$\min_{x_2} = \begin{pmatrix} 1.126x_1 + 0.100x_2 + 0.097x_3 \geq g_{21}, \\ 0.856x_1 + 1.473x_2 + 0.443x_3 \geq g_{22}, \\ 0.380x_1 + 0.737x_2 + 0.277x_3 \leq g_{23}. \end{pmatrix}$$

[Level 3]

where x_3 solves,

$$\min = \begin{pmatrix} 0.921x_1 + 0.050x_2 + 0.526x_3 \geq g_{31}, \\ 0.380x_1 + 0.737x_2 + 0.216x_3 \leq g_{32}. \end{pmatrix}$$

Subject to constraints:

$$\begin{aligned} &\text{raw} && 3.815x_1 + 0.910x_2 + 0.220x_3 <= 87.75; \\ &\text{labor} && 0.650x_1 + 0.900x_2 + 0.125x_3 <= 4.425; \\ &\text{mills} && 17.50x_1 + 2.160x_2 + 4.775x_3 <= 95.20; \\ &\text{capital} && 1.350x_1 + 0.980x_2 + 0.890x_3 <= 20.15; \end{aligned} \quad (17)$$

Note that all the coefficients for the objective functions in the problem model (16) are derived from Fuzzy Random Regression Model (4).

The first step to solve MLMO problem starts with computation of the individual optimal solutions to determine the goal g_{ij} and its tolerance u_{ij} of each objective function. Table 2 tabulates the individual optimal solutions of all objectives functions for the three levels of the MLMO problem.

Based on the procedure stated in Phase II, the equivalent linear program (19) for the first-level decision making as follows:

$$\begin{aligned} &max = 0.245u_1 + 0.200u_2 + (-0.220)u_3; \\ &u_{f_{11}} = \frac{((2.078x_1 + 0.260x_2 + 0.170x_3) - 0.5)}{4.58 - 0.5}; \\ &u_{f_{12}} = \frac{((01.010x_1 + 1.700x_2 + 0.476x_3) - 0.5)}{5.50 - 0.5}; \\ &u_{f_{13}} = \frac{(0.5 - (0.438x_1 + 0.796x_2 + 0.512x_3))}{5.50 - 0.5}; \\ &\text{system constraints (17);} \\ &x_i \geq 0, \mu_{f_{ij}} \in [0,1]. \end{aligned} \quad (18)$$

LINGO© computer software is used to run the equivalent ordinary linear programming model. The optimal solution of the first level is $x^{1*} = (x_1^{1*}, x_2^{1*}, x_3^{1*}) = (1.94, 2.08, 0.00)$. Let DM_1 decide $x_1^1 = 1.94$ from the first level solution and the negative and positive tolerance are decided to be $t_1^{1L} = 0.5$ and $t_1^{1R} = 0.5$ respectively.

The second level solution proceeds as follows:

$$\begin{aligned} &max = 0.245u_1 + 0.200u_2 + (-0.220)u_3 \\ &+ 0.294u_4 + 0.250u_5 + (-0.232)u_6 \\ &+ 2u_{x_1^L} + 2u_{x_1^R}; \\ &u_{f_{11}} = \frac{((2.078x_1 + 0.260x_2 + 0.170x_3) - 0.5)}{4.58 - 0.5}; \\ &u_{f_{12}} = \frac{((01.010x_1 + 1.700x_2 + 0.476x_3) - 0.5)}{5.50 - 0.5}; \\ &u_{f_{13}} = \frac{(0.5 - (0.438x_1 + 0.796x_2 + 0.512x_3))}{5.50 - 0.5}; \\ &u_{f_{21}} = \frac{((1.126x_1 + 0.1006x_2 + 0.097x_3) - 0.5)}{3.90 - 0.5}; \\ &u_{f_{22}} = \frac{((0.856x_1 + 1.473x_2 + 0.443x_3) - 0.5)}{4.50 - 0.5}; \\ &u_{f_{23}} = \frac{(0.5\text{-}(0.380x_1 + 0.737x_2 + 0.277x_3))}{0.5 - 4.80}; \\ &ux_1^L = 2x_1\text{-}2.44; \\ &ux_1^R = 4.88\text{-}2x_1; \\ &\text{system constraints } (17); \\ &x_i \geq 0, u_{f_{ij}} \in [0,1], ux_1^{R,L} \leq 1. \end{aligned} \quad (19)$$

The optimal solution of the second level is observed as $x^{2*} = (x_1^{2*}, x_2^{2*}, x_3^{2*}) = (1.94, 1.92, 0.00)$. Let DM_2 decide $x_2^2 = 1.92$, $t_1^{2L} = 0.75$ and $t_1^{2R} = 0.25$.

The solution for the third level is shown in the following equivalent linear programming problem.

$$
\begin{aligned}
&max = 0.245u_1 + 0.200u_2 + (-0.220)u_3 \\
&+ 0.294u_4 + 0.250u_5 + (-0.232)u_6 \\
&+ 2u_{x_1^L} + 2u_{x_1^R} + 1.33u_{x_2^L} + 4u_{x_2^R}; \\
&u_1 = \frac{((2.078x_1 + 0.260x_2 + 0.170x_3) - 0.5)}{4.58 - 0.5}; \\
&u_2 = \frac{((01.010x_1 + 1.700x_2 + 0.476x_3) - 0.5)}{5.50 - 0.5}; \\
&u_3 = \frac{(0.5 - (0.438x_1 + 0.796x_2 + 0.512x_3))}{5.50 - 0.5}; \\
&u_4 = \frac{((1.126x_1 + 0.1006x_2 + 0.097x_3) - 0.5)}{3.90 - 0.5}; \\
&u_5 = \frac{((0.856x_1 + 1.473x_2 + 0.443x_3) - 0.5)}{4.50 - 0.5}; \\
&u_6 = \frac{(0.5\text{-}(0.380x_1 + 0.737x_2 + 0.277x_3))}{0.5 - 4.80}; \\
&u_7 = \frac{((0.921x_1 + 0.050x_2 + 0.526x_3) - 0.5)}{3.00 - 0.5}; \\
&u_8 = \frac{0.5\text{-}(0.380x_1 + 0.737x_2 + 0.216x_3)}{0.5 - 4.00}; \qquad (20) \\
&ux_1^R = 2x_1\text{-}2.88; \\
&ux_1^R = 4.88\text{-}2x_1; \\
&ux_2^R = 1.333x_2\text{-}1.56; \\
&ux_2^R = 8.68\text{-}4x_2; \\
&\text{systemconstraints } (17); \\
&x_i \geq 0, uf_{ij} \in [0,1], \\
&ux_i^{L,R} \leq 1.
\end{aligned}
$$

The satisfactory solution of the MLMO problem is $x^{3*} = \left(x_1^{3*}, x_2^{3*}, x_3^{3*}\right) = (1.94, 1.92, 0.02)$. As $l = p = 3$ the procedure brings to an end and the satisfactory solution is obtained.

Based on the procedure stated in Section 4, three equivalent linear programs are constructed in sequence. Table 1 tabulates the goal and tolerance that are pre-determined by the experts for all objectives functions of the three levels of the multi-level multi-objective problem. Computer software LINGO© is used to solve the equivalent ordinary linear programming model.

6 DISCUSSIONS AND CONCLUSIONS

In this section, the results of the industrial problem causes are explained. Section 6.1 explains the results of building the objective function from the fuzzy random regression approach and Section 6.2 explains the solution results of hierarchical structure decision making with multi-objective problem based on the fuzzy goal programming method.

6.1 The Evaluation from FRRM Results

The first phase of the solution presented in this paper is to use fuzzy random regression to estimate the coefficients and build the fuzzy random based objective functions. The FRR regression models based on (4) were applied to the datasets of the manufacturing production. The coefficients are obtained as shown in Table 2, where each coefficient is represented in interval-valued form. This result illustrates the coefficients for each attribute and shows the range of their evaluations. The coefficients ξ_i are the fuzzy random coefficients for the decision variables x_i.

There are five criteria in this situation example which are export revenue, production volume, profit, capital and cost of production. Criterion represents a single measure by which the goodness of any solution to a decision problem can be measured [28]. The result illustrates that for the export revenue criterion, product 1, x_1 has a significant contribution as compared to the other two products of, x_2 and x_3. For the profit returns, each level shows that the product 2 x_2 contributes an important weight to the total evaluation followed by product 1 x_1 and product 3 x_3. This evaluation is similar to the capital and cost of production criteria.

Table 2. Fuzzy Random based Coefficient for the objective functions

Decision Making Level	Goal	Fuzzy Random-Based Coefficient		
		ξ_{x_1}	ξ_{x_2}	ξ_{x_3}
1	Export revenue	[2.031, 2.125]	[0.200, 0.320]	[0.120, 0.220]
	Profit	[0.935, 1.086]	[1.206, 2.194]	[0.420, 0.532]
	Capital	[0.365, 0.512]	[0.735, 0.857]	[0.412, 0.613]
2	Production	[1.098, 1.154]	[0.000, 0.200]	[0.092, 0.103]
	Profit	[0.856, 0.856]	[1.006, 1.940]	[0.354, 0.352]
	Cost	[0.305, 0.455]	[0.680, 0.794]	[0.203, 0.532]
3	Production	[0.872, 0.971]	[0.000, 0.100]	[0.082, 0.970]
	Cost	[0.305, 0.455]	[0.680, 0.794]	[0.200, 0.232]

From the coefficients values derive by the fuzzy random regression approach, we can see that the flexibility which reflects the fuzzy judgments in this evaluation are represented by the interval valued form. It is also supports the range of values around expected value that is likely to contain the estimation target.

The proposed method uses one-sigma confidence intervals when developing the objective functions. In the proposed model, the confidence intervals were constructed based on the expected value and the variance, instead of the actual values of the statistical data. The fuzzy random regression model includes the mean interval values of all samples in the model and is used to handle the presence of hybrid uncertainty contains in the statistical data. Using the confidence intervals, we could provide a more complete description of the information in the data about the estimation target. Our model also supports the range of values around expected value that is likely to contain the estimation target.

Table 3. The optimal solutions and the tolerance

Decision Making Level	Solutions $x = \{x_1, x_2, x_3\}$	Controlled decision variables	Controlled decision variables tolerance	
			$t_k^{i_L}$	$t_k^{i_R}$
1	$x^{1*} = \{1.94, 2.08, 0.00\}$	x_1^{1*}	0.5	0.5
2	$x^{2*} = \{1.94, 1.92, 0.00\}$	x_2^{2*}	0.75	0.25
3	$x^{3*} = \{1.94, 1.92, 0.02\}$	-	-	-

6.2 Multi-level Multi-objective Fuzzy Goal Programming

The optimal solution for each level with the controlled decision variables for the problem is obtained as in Table 2. The experiment's results show that the administrative government level (first level), objectives f_{11} and f_{21} attained nearly full satisfaction achievement which were 98% and 94%, respectively. However, the objective of minimizing the capital only partly achieved about 42% at this level. The objective to maximize the profit at the government state level has fully achieved, whereas the other objectives gained 55% and 38%. In the city level, the objectives satisfied about 55% and 47% achievements. The results show that decision makers can perform decision analysis under consideration of the solution derived from the mathematical approach. The decision maker can re-examine the solution and change the decision and repeat the process. Since the proposed method is based on the satisfaction approach, the decision makers may involve themselves in evaluating the results to attain better satisfying solution.

In this study, we demonstrated the use of the additive model of an FGP approach to solve multi-level multi-objective programming problems, where the initial problem model was developed in terms of a fuzzy random regression approach to treat the uncertainties in the data and to overcome the difficulties in determining the coefficients values. In summary, the proposed procedures have properly used the additive method in the FGP evaluation to solve multi-level multi-objective problems. The procedure also enables the decision maker of a respective decision-making unit to decide the decision value by means of the mathematical based on their satisfaction. Although it is an iterative process, it is practical for the decision maker to re-evaluate the results to attain the satisfaction of the overall system target. In addition, the decision maker's preferences toward the goals are considered in the computation of the decision process by introducing the relative importance evaluation of the additive FGP model.

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