Fuzzy Models Applied to Complex Social Systems: Modeling Poverty using Distributed Agencies

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ABSTRACT

This work has as its main objective to apply the distributed agency methodology, which consists of several techniques used in computer science, to solve complex social problems. This approach allows the possibility of analyzing data from a macro level to a micro level where the intermediate part is highlighted, all in an organized integration of a social simulation displayed on geographic information with natural language quantifiers. We present a new model based on Distributed agencies Model and we use an interval type-2 fuzzy logic system is designed to evaluate the poverty levels.

KEYWORDS

Complex Social System, Data Mining, Fuzzy Logic, Distributed Agencies, Poverty.

1 INTRODUCTION

In the beginning, poverty was defined as a static and unidimensional concept, studied primarily in a static fashion with a clear economic nuance, attending to what is considered acceptable lower income levels in society. But in recent decades, it has passed to a concept of dynamic and multidimensional nature. Recent research shows that not only the income levels determine poverty but also absence of resources the and opportunities such as education, housing

and other factors. Thus we see the existence of multidimensional poverty studies with various aspects and multidimensional analysis[1].

As in other developing countries, the primary cause of the existing inequality in Mexico is poverty, which is reflected in the results of economic development.

The purpose of this paper is not to discuss the various determinants factors for the conceptualization of poverty, but instead propose a methodology with different computational techniques that aid complex social simulations and provide an option for the analysis of social problems.

Social systems contain many components which depend on many relationships; this makes it difficult to construct models closer to reality[2]. To analyze these systems with a dynamic and multidimensional perspective, we will consider data mining theory, fuzzy logic [3] and distributed agencies.

Fuzzy logic is based on the way the brain handles imprecise information, while neural networks are based on the physical structure of the brain. Even though the fundamental inspiration for these methods is different, there are a number of parallels between them [citation needed]. Fuzzy logic systems (FLS) and artificial neural networks (ANN) are estimators for free models and dynamic systems; they share the ability of improving results in systems that work with uncertainty, imprecision and noise by adding some level of intelligence [3]. Both have been successfully applied in a variety of control systems and other devices, improving their performance. It has been shown that fuzzy systems and neural networks have the capacity to model complex, non-linear processes to an arbitrary degree of precision [4]

There are several problems in analyzing a complex system, like the interactions between individuals who, because of these relationships, may be forced to change their environments. A new mathematical theory to study this evolution is complex systems, an emerging field in the dynamics of adaptation. This system could explain the long-term effects of interactions between the evolutionary processes [5]. A complex system is a conceptualization of the complexity and the related systems. Complex Adaptive Systems (CAS) established by physicists and economists since two decades ago [6], classification systems, ecosystems and simulation systems, helped lead the creation of evolutionary computation and artificial life. The framework is derived from many natural and artificial complex systems that have collaborated such diverse fields in such as anthropology. psychology. genetics evolution, ecology, and business management theory. The methodologies using simulation CAS on models in the field of biological computing have contributed in this field; CAS is the result of the artificial systems [7-9].



Figure 1. Multiple levels represented.

The objective of our study is to use the methodology and corresponding computational platform that incorporates available mathematical and computational theories that have not been appropriately considered in models of complex social phenomena. Even though applications of Multi-Agent Systems (MAS) have been developed for the social sciences, it has been widely considered in other areas such as Artificial Intelligence (AI) [10, 11]. The state-of-the-art computational in capabilities has been incorporated in multiple areas [7-8], particularly as it refers to distributed systems and distributed agencies [9].

2 METHODOLOGY

The modeling of a realistic social system cannot be achieved by resorting to only one particular type of architecture or methodology. The growing methodology of Distributed Agency (DA) represents a promising research avenue with promising generalized attributes, leading to potentially groundbreaking applications engineering and in the social in sciences—areas in which it minimizes the natural distances between physical and sociological nonlinear systems. In

this work we thus lay the foundations for a DA description of socioeconomic realities, in a process that weaves different available computational techniques in the context of DA to represent social and individual behavior in contextualized fashion а accommodating agents with limited rationality and complex interactions. We believe that this research avenue will improve our understanding of social complexity, as it moves the discussion in the field towards the capability of describing the vast array of linear and nonlinear realities, interlocking levels and currently non-consilient theories in existence.

The methodology we are aiming to create represents a novel approach to simulation architectures, creating a language that links the social sciences to programmable terminology and that can thus be broadly applied. The methodology of DA represents a general theory of collective behavior and structure formation [12], which intends to redefine agency and reflect it in multiple layers of information and interaction, as opposed to the traditional approach in which agency is only reflected in individual, atomized and isolated agents, as shown in Figure 1.

We consider a disentangled agent that is formed by multiple and relatively independent components. Part of the resulting agent's task is to present alternatives, or 'fields of action' to its components. Correspondingly, the composed agent is itself constrained by a field of action that the superstructure to which it belongs presents. We therefore drop customary assumptions made in traditional social disciplines and MAS about what is considered a decisionmaking unit. To arrive at this, we redefine what a unit of decision is by unscrambling behavioral influences to the point of not being able to clearly delineate what the individual is, who is part of a group and who is not, or where a realm of influence ends; the boundary between an individual self and its social coordinates is dissolved.

The proposed intermediate agent can be thought of as a person, a family, a social class, a political party, a country at war, a species as a whole, or a simple member of a species trying to survive. The archetype of the agents we attempt to describe can be summarized as a group of colluded oligopolists, such as the oil-producing countries of OPEC. As a whole, they share the common interest of jointly behaving like a monopoly and restricting their production, but they cannot avoid having an incentive to deviate and produce above their quota.



Figure 2. Multiple levels of identity of poverty.

Reductionist linear science has concentrated on the study of entities that are clearly delineated, where one could separate what belongs to an agent's nature against the backdrop of what does not. The relevant agent is taken to be exogenous, and therefore disconnected from the system to which it belongs. At their core, these traditional disciplines are based on a selfish and unitary agent, or atom of description. Implicitly or explicitly, these paradigms claim that all aggregate complexity can be traced back to the lower level of the system: the strategies and actions of the selfish agent. In other words, these represent research agendas that purposely deemphasize the existence of any level other than that of the individual.

On the other hand, the idea of emergence reflects the fact that different and irreducible levels of interaction will naturally arise in complex systems such as the ones studied by social disciplines, and thus the agent, as we define it in this work, is a combination of levels of interaction. It is through this lens that we would like to consider humans, who will independent partly be creatures. possessors of free will, but who are also partly created by an array of upper levels that 'suggest' agreeable utility functions. This conception stems directly from the concept of complexity, in which wholes are more than the sum of their parts. If we believe in that proposition, then we should expect to find a world full of emergent phenomena, with distinctive levels of interaction that have agency of their own. The proposed language redefines agents in two ways. First, there are no obvious atomic agents, for all actors represent the emerging force resulting from the organization of relatively independent subsets. Second, agents are not created in a vacuum, but are rather the result of what an upper level spawns[13].

This model allows us to evaluate a preference for a candidate considering deferent kinds of identity variables, and different uncertainty degrees as to how these identities interact and determine intent. Agents' interactions could create a dynamic behavior if each agent

perceives and changes its uncertainty values depending on the perception of the behavior of other agents. The agents adapt their preferences by varying degrees of uncertainty in any of the membership functions of input and output variables.

In this work the model is composed by the combination from two theories: for the uncertain and emergente modeling we use fuzzy logic originally introduced by Zaded [3, 14] and ditributed agencies [12],as shown in Figure 2.

4 IMPLEMENTATION

Real world simulations such as poverty must include some means of validation [15]. In econometrics, studies performed on populations and economy have abundant data, the main problem is finding data sets that are adapted to the desired architecture [16].

With the steady increase in availability of information, from existing projects such as databases needed for social simulation, it becomes essential to use data mining. In our case, for the vast amount of data, we obtain the necessary quantitative information on the most important subject of the social and system from government economic databases. Data mining extracts implicit information such as social patterns in order to discover knowledge[17]. The use of these techniques has been wide spread in this field in recent years. Most efforts dedicated research are to effective developing and efficient algorithms that can extract knowledge from data[18].

For the particular case of the City of Tijuana, in the state of Baja California Mexico we use the geographical, demographic and economic indicators provided by the databases of the National Institute of Statistics and Geography (INEGI).

The information for this city is fragmented in 363 "AGEBs". "The AGEB delimitates urban areas, a whole locality of 2,500 inhabitants or more, or a municipal seat, regardless of the number of people, in groups that typically range from 25 to 50 blocks. Rural AGEB's frame an area whose land use is predominantly agricultural and these are distributed communities of less than 2.500 inhabitants that for operational purposes, are denominated rural locations[19]. For each AGEB we determine the extent of poverty, taking into account 10 variables of income and employment rate, 23 variables on education, and 15 variables on the resources available in a home, such as TV, telephone, refrigerator and more. This gives a 48 by 363 matrix containing the information necessary to analyze.

The databases for the model were compiled in a geographic information system that helped in the creation, classification and formatting of the data layers required. This eliminates the issue of having different thematic maps of information; each map, quantifies the spatial structure to display and interpret the different areas and spatial patterns of Tijuana.



Figure 3. Tijuana divided into AGEBS using NetLogo.

We use Fuzzy Logic to modeling poverty and a NetLogo simulation to model population interaction.Using the NetLogo platform[20], we are able to simulate social phenomena, model complex systems and give instructions to hundreds or thousands of independent agents all operating holistically. It also allows us to filter information from geographical information system with spatial and statistical data. This makes it possible to explore the relationship and behavior of agents and the patterns that emerge from the interactions within a geographical space.

Each AGEB contains quantitative information about employment, education, income, articles and infrastructure that a home has.



Figure 4. Levels of agents represented on the social system.

Since simulating each person under such circumstances is unattainable, as it is the case of modeling the social development of a city like Tijuana, the population total and all those relationships that accompany it, such as migration and birth-rates, must be analyzed with some sort of macro or "top-down" model. On the other hand, all low-level interactions at a micro level, such as partner selection or the decision to form a family with a given number of children, must be captured with a "bottom-up" model.

To form the general structure of the simulation, we use dynamic systems to model the top-down aspect of the architecture. The dynamic system allows us to represent all aspects and relations of the structure of the sustainable system and its evolution in time. Also, the dynamic system allows for the creation of mathematical equations that describe the model at a macro-level, recreating stylized facts and the most important variables to be described. On the other side of the spectrum, at the micro level, a Distributed Agency methodology will allow us to represent contextualized agents, interacting with each other, their environment, and other levels of agency.

system Using neuro-fuzzy to automatically generating the necessary rules, this phase of data mining with a fuzzy system becomes complicated as there is no clear way to determine which variables should be taken into account [21]. The Nelder-Mead (NM) search method is used; even though being more efficient on other methods such as genetic algorithms, as shown in other studies[22], since the NM method seems to produce more accurate models with fewer rules. This optimization algorithm is widely used and is a numerical method that minimizes an objective function in a multidimensional space, finding the approximate local optimal solution to a problem with N variables [23].

Using this grouping algorithm we obtain the rules, which are assigned to each agent which represents an AGEB. The agent receives inputs from its geographical environment and also must choose an action in an autonomous and flexible way to fulfill its function [24].

The other hand the concept of a linguistic variable is very important in

fuzzy logic and approximate reasoning, as it has a key role in many applications, specifically in expert systems and fuzzy logic controllers. In essence, a linguistic variable represents a concept in the form of a word or sentence in a natural or artificial language. For example, poverty can be a linguistic variable that can have specific values such as: null, moderate, and extreme. The concept of a linguistic variable was introduced by Zadeh [3] in order to handle approximations of complex phenomena or to describe concepts that cannot be handled satisfactorily by quantitative terms. A formal definition for a linguistic variable will be presented shortly bur first, the concept of a fuzzy variable will be discussed.



Considering the use of agent-based models, is the fact that they are very similar to artificial societies, following same techniques. the Their main differences focus on system simulations, and the research program designs[15]. Thus considering all the properties of the agents [25] is suitable for the purpose of our research. Each agent has a distinct function depending on the rule that it is geographic assigned and the environment where it will decide the tasks dynamically, so there is no global control system.

The purpose is to provide agents with the least rules possible and observe the operation of the system by the interactions: The system itself generates intelligent behavior that was not necessarily planned or defined within the agents themselves; thus achieving an emergent behavior.

Systems with a low degree of complexity or little uncertainty can be described precisely with mathematical models. On the other hand, in systems with a greater degree of complexity but with a significant amount of available data, the uncertainty can be reduced by using model-independent methods such as ANNs, relying on their capabilities for pattern recognition and learning. For the more complex systems, where little data is available, fuzzy reasoning offers a robust alternative to understanding the behavior of a system by performing an approximate interpolation between the inputs and the observed outputs. These types of systems can, so far, only be modeled using fuzzy logic based intelligent systems [definition required], which constitutes a fundamental tool for modeling non-linear complex systems.

Fuzzy logic and fuzzy sets form the basis for fuzzy systems; these concepts have been developed in order to emulate the way imprecise information is manipulated in the brain. While in a traditional type-1 fuzzy set one models imprecision and uncertainty is left implicit, a type-2 fuzzy set models both imprecision and uncertainty. Originally proposed by Zadeh in 1975 [26], type-2 fuzzy sets are essentially fuzzy sets of fuzzy sets in which the membership value is a type-1 fuzzy set.

The rules obtained from the clustering algorithm can tell us which agents have more income; which are at a higher or lower educational level; and what resources are available. Taking the compound poverty index which measures poverty [27] as reference on the three basic dimensions (health, education and an acceptable quality of life) we calculate this factor with.

$$HPI = \left[\frac{1}{3}(P_1^{\alpha} + P_2^{\alpha} + P_3^{\alpha})\right]^{\frac{1}{\alpha}}$$
(1)

P1: Population that has no access to medical services

P2: Adult illiteracy rate

P3: Population with no access to basic services such as running water, pluming or electricity.

HPI: The Human Poverty Index

The Human Poverty Index is used to track poverty, development, and economic population level. These variables correspond to the 363 AGEBs that the municipality of Tijuana contains.



Figure 6. Poverty distribution is compared with the selected variables

Studies within the social sciences (and in particular in economics) are normally performed on large data platforms, in situations in which there is too much rather than too few data points. The most problematic aspect the modeling processes is to define data sets that match the desired architecture [16]. Continuous expansion of available information for social simulation makes the use of data mining unavoidable. In our case study, for example, we have access to many different sources of quantitative and qualitative data describing both economic and sociological aspects of reality. Many of these data sets are readily available from governmental sources.

Data mining provides us with the process for extracting implicit information, such as social patterns, that reveal ingrained knowledge [17]. The use of these techniques has had significant progress in the last decade, with many research agendas and efforts devoted to the development of efficient algorithms to extract relevant information out of data sets [18].



Figure 7.Rule Evolution of Poverty (type-1)



Figure 8. Rule Evolution of Poverty (type2)

Using this grouping algorithm we obtain the rules. Agent-based technology has been considered appropriate for the social development of distributed systems [28]. Distributed agents are a promising strategy that can correct an undesirable centralized architecture[29].

Distributed Agents do not define independent agents. The idea behind the distributed agency modeling language stems from a worldview that is ubiquitous in appearance, in which we find groups that are irreducible to their parts, and exist in different dimensions where different rules apply [30].



Figure 9. Bottom-up model and top-down model at population.

Therefore, the next step is for the agencies to try and solve problems distributed among a group of agents, finding the solution as the result of cooperative interaction. Communication facilitates the processes of cooperation; the degree of cooperation between agents can range from complete cooperation to a hostile [31].

An example of cooperation might be agents of a given area sharing certain resources for the mutual good. A hostile situation may occur by agents blocking the objectives of others fear of sharing resources and bya lack of resources in For cooperation the area. and coordination mechanisms to succeed in a system of agents there must be an additional mechanism: negotiation, by which, members of a system can reach an agreement when each agent defends its own interests, leading to a situation that benefits all, taking into account all points of view [31].

So then, there are three levels of poverty that are being analyzed which are the following:

In the meso level, the type of poverty is called cyclical: it refers to generalized poverty, but temporary that affects a specific population. It produces food scarcity caused by the deficient planning in agriculture or by natural causes, giving way to the cycle of famine that periodically devastates the community.

At the macro level, the type of poverty is called collective: it creates the insufficiency of resources needed to satisfy the basic necessities of life; it can affect an entire population or large sectors of a population immersed in a prosperous society. This type of poverty result of economic is the underdevelopment, exacerbated by the inability to adequately satisfy the needs of a population whose resources are insufficient. The consequences of these conditions are a low life expectancy, illness cause by malnutrition, and a high mortality rate. The proposed solutions to ease the effects of this type of poverty have produced less than desirable results. This has given rise to the vicious cycle of poverty theory that attempts to explain the underdevelopment of the most disadvantaged countries.

At the micro level, poverty is of the individual type. It can be permanent and its distribution area is limited to a single individual or a single family, incapable of providing for the most basic necessities. This situation is caused by either physical or mental impediments that if not present, would allow the individual to attend his needs adequately.



Figure 10. Results Poverty distribution with RMSE=1.5568e-004 (type-1).



Figure 11. Results Poverty distribution with RMSE=0.0032.(type-2)

Poverty analysis is studied at a macro level. This analysis generates an upper level model which both influences and is influenced by the lower levels. One should also mind the middle hierarchies since most analysis in general only consider the extremes. This can be achieved by not losing sight of the links that exist between the components amongst the different levels.

The INEGI database information was normalized for the corresponding 363 AGEBs, establishing an interval from 0 to 1, where values closer to 1 indicate greater poverty. This is shown in figures 6 and 9.

The use of statistics in the study of poverty has greater precision when the number of variables is increased. It is valid when analyzed from only one level, but most social problems, as in the case of poverty, encompass a complex system.

It is possible to observe and analyze the interactions between the multiple levels amongst the different dimensions. To visualize all of this in a global or macro level, and at the same time to observe specific scenarios and the interpersonal relationships between agents, a multi-level complex system analysis is necessary.

Thus, the poverty of a population can be measured as the various multi-levels of population since lower to higher level.

Upper Level: Several studies relating to poverty in different Mexican federal entities have been conducted, placing the cities as the upper most level agent type. In these studies the city of Tijuana appeared as one of the municipalities lowest poverty with the levels. Nonetheless, if we apply a model that considers all the levels that the social system spans, from the highest to the lowest, it can be observed that deficiencies do exist and it shows where they are located.



Figure 12. Results of poverty using six membership functions.

Middle Level: The middle agents indicate the poverty level between AGEBs. At this level, poverty has as much to do with external as with internal migration. where the geographical rearrangements of poverty and the migration effect have direct repercussions. As can be observed in figure 10, the highest concentration of poverty can be found at the city limits.

These zones are at a disadvantage since they are rarely aided by federal, state or municipal social aid. A high value in the variable P3 indicates that the situation is disconcerting. As to basic services such as water, basic pluming, electricity, and paved roads it is seen as out of reach for the inhabitants of these areas.

Lover Level: For every 10 people in Tijuana, at least 4 have some kind of deficiency, be it illiteracy, lack of social security, and/or have no access to basic services such as water, pluming, and electricity; almost half a million people are in this situation. Another interesting fact worthy of notice is that a person with a high income (lower level agent) can be found within a poor AGEB (middle level agent), and is directly affected in their quality of life by the lack of pluming, paved roads, or the smell of sewage.

5 CONCLUTIONS

The resulting model represents a powerful alternative for complementing, substituting or augmenting traditional approaches in the social sciences. The study of interdisciplinary connections, of consilience, and of modeling several levels of reality jointly remains an area of research with vast fields of unexplored territory. growing The disciplines of Computational Social Science and Social Simulation should be trail blazers in this effort

The methodology we are using is developed in a holistic manner, originally focusing on the description and interconnection of different levels of reality, whether these refer to either different dimensions or different time granularities. The applications of the approach are ultimately very general, but they are particularly useful for interdisciplinary analysis, where different disciplines overlap or interact in their description of natural or social phenomena. This general language links together the developments in computational science with those in the social sciences, as they pertain to the nascent paradigm of complexity

We created a system that is composed of agents where each agent represents an AGEB whose adaptation is the result of interactions complex in nonlinear dynamics, emergent phenomena which arise in the system, and to compare reality with the artificial system and observe the properties, processes and relationships by using different computational methods.

The poverty model using different techniques can be a powerful tool for any planning process. The use of distributed agencies allows us to unearth new theoretical models and furthers our understanding the relationships found within distinct levels of reality; and helped us to see very concrete cases. The estimated percentages for the population living below the poverty line are base on quantitative surveys without considering qualitative measurement, a taking estimated results considering the number of people and income. For example, conventional measurements compare data from the World Bank using income of less than 2 dollars a day as a reference for poverty. In this case 2 dollars and 10 cents is above the range and therefore considered, disregarding not the proximity to poverty entirely. By using fuzzy logic it is possible to obtain better results. In future work we will be comparing data from different countries such as Mexico and South Korea whose economies are very similar in size and the effects of income distribution.

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