

Simple Design Method of MIMO Multiuser Downlink System Based on Block-Diagonalization

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ABSTRACT

Block diagonalization (BD) technique is widely used in multiple input multiple output (MIMO) multiuser downlink systems because of its simplicity and high ability for eliminating the interuser interferences. In this study, very simple weight design algorithms based on BD are proposed, in which the order of design steps is swapped, namely, receiver weights are calculated first, and then transmission weights are derived using zero forcing (ZF) approach. It is shown through computer simulations that the proposed method achieves better performance than the conventional BD under certain conditions. Additionally, the condition on the degrees of freedom is released, hence it can be utilized for the transmitters with small number of antennas to which BD cannot be applied.

KEYWORDS

MIMO, multiuser, downlink, eigenanalysis, array signal processing.

1 INTRODUCTION

A decade has past after the active research era of multiple input multiple output (MIMO) communication systems [1], [2] started. The main focus of this technology which utilizes antenna arrays both in the transmitter and receiver sides is now moving to the practical application aspects in a variety of areas [3]. There are many application-oriented studies of MIMO: the lively discussions are still made and new schemes are developed for the design of multiuser system, since it has many applications like local area network (LAN) [4], cognitive radio [5], and recently base station cooperation in coordinated multi-point (CoMP) cellular systems [6]. The multiuser system is classified into two categories: the first is the uplink scheme, and roughly speaking, it is considered in the context of conventional array signal processing [7] if the use of spatial processing is assumed, and the next one is downlink scheme, which is little a bit out of the line of well know receiver beamforming, and

many works have been actively proposed and discussed.

A variety of design approaches have been developed for MIMO multiuser downlink system, which are summarized in [8]. A nonlinear technique which is called dirty paper coding (DPC) [9] is a well known method, but the complicated power control is required for the implementation, hence the mainly used are linear processing approaches simply realized by the multiplication of transmit and receiver weights. For the (near-)optimal weight design, some iterative optimization algorithms have been proposed in [10], [11]. Some of them simultaneously try to resolve another important subject, the resource allocations, but they need high implementation cost and mathematical complexity, which are not easily acceptable for most communication engineers who are familiar with conventional MIMO system. "How it is easy-to-use" has an importance in many applications (for example, a simple codebook-based weight selection scheme is adopted in long term evolution (LTE) system). From such a viewpoint, most widely used approach is block diagonalization (BD) [8]. This method is based on the extension of the concept of conventional singular value decomposition (SVD) for single user system, and simultaneously achieves simplicity and good performance. For BD, some modified versions have been proposed, for example, reference [12] presents imperfect channel knowledge based design

method, and in [13], the antenna selection method is simultaneously used in the receiver side (reference [14] considers multi-stream selection instead). An imperfect BD is described in reference [15], which needs complicated procedure compared to the original BD, but on the other hand, brings a good performance improvement.

This study presents, attempting to achieve a better performance than conventional BD under certain conditions, simple MIMO downlink algorithms, which are based on BD method and zero forcing (ZF) scheme. The fundamental concept of the proposed approach is as follows: in the conventional BD, the procedure of ZF to the nontarget users is first carried out, then the weights to derive a larger gain as possible toward the target user are designed. But this operation might result in the small gain of the target link because the operation to steer zero toward all the antennas of undesired users consumes the degrees of freedom. Our method first considers deriving a strong connection between the transmitter and the receiver sides, and two steps of BD are swapped. By this operation, we should remark that not only the performance improvement is expected if this idea is reasonable, but also the restriction on the degrees of freedom is released since the elimination of one stream of one user requires only one degree (we can regard each of them as a single-antenna user terminal). Consequently, this method can be easily implemented in a small size ar-

ray. Through computer simulations, we investigate the characteristics of our novel methods, and verify the advantage and some their natures.

The organization of the rest of this paper is as follows: in section 2, the model of multiuser MIMO system considered in this study is briefly described, then in section 3, design methods of the system are proposed based on the concept of BD. After computer simulations to verify the effectiveness and characteristics of the proposed method in section 4, conclusions and future works are given in section 5.

2 MULTIUSER DOWNLINK MIMO SYSTEM MODEL

In this section, we shortly describe the system model of multiuser MIMO considered in this study based on a popular scenario of MIMO downlink transmission.

Figure 1 draws a picture of multiuser MIMO system considered in this study: it consists of a transmitter and M receivers each of which corresponds to the terminals of user $m = 0, \dots, M - 1$. The transmitter and the receiver of the m -th user are respectively equipped with N_t and $N_{r,m}$ antenna elements. Matrix $H_m \in \mathbb{C}^{N_{r,m} \times N_t}$ denotes the channel between the transmitter and the m -th receiver, where $(n_{r,m}, n_t)$ -th entry shows the channel response between the n_t -th element of the transmitter and the $n_{r,m}$ -th element of the receiver. By stacking H_m for all users, we can define the total channel $H = [H_0^T, \dots, H_{M-1}^T]^T \in$

$\mathbb{C}^{N_r \times N_t}$ ($N_r = \sum N_{r,m}$ is the sum of the number of all the receiver antennas).

Here we assume multistream transmission where L_m data streams are transmitted to the m -th user. The transmission weight $\mathbf{w}_{t,m,\ell} \in \mathbb{C}^{N_t}$ is used to send the ℓ -th data stream $\{s_{m,\ell}(t)\}$ (t is the time index) of the m -th user: before the transmission, the signals for all the streams of all the users are summed up, then radiated, and arrive at the receiver of m -th user through channel H_m . The signal captured at the m -th receiver is expressed by

$$\mathbf{y}_m(t) = H_m \sum_n \sum_{\ell=0}^{L_n-1} \sqrt{P_{n,\ell}} \times \mathbf{w}_{t,n,\ell} s_{n,\ell}(t) + \mathbf{n}_m(t). \quad (1)$$

In this equation, the transmission power allocated to $s_{m,\ell}(t)$ is denoted by $P_{m,\ell}$ (we establish the unit total power, namely, $P_{s,m} = E[|s_{m,\ell}(t)|^2] = 1$), and notation $\mathbf{n}_m(t)$ shows the additive white Gaussian noise (AWGN) generated at the m -th receiver which has the power of $P_{N,m}$, that is, $E[|\mathbf{n}_m(t)|^2] = P_{N,m} I_{N_{r,m}}$, where I_M expresses the M -dimensional identity matrix. The output signal $\{\hat{s}_{m,\ell}(t)\}$ of the ℓ -th stream of the m -th user is the replica of the transmitted signal $\{s_{m,\ell}(t)\}$. It is derived by multiplying weight vector $\mathbf{w}_{r,m,\ell}$ to the m -th user received signal $\mathbf{y}_m(t)$.

The development of the design method of a set of weight vectors $\{\mathbf{w}_{t,m,\ell}, \mathbf{w}_{r,m,\ell}\}$ for the m -th receiver is our aim in this study, and its proce-

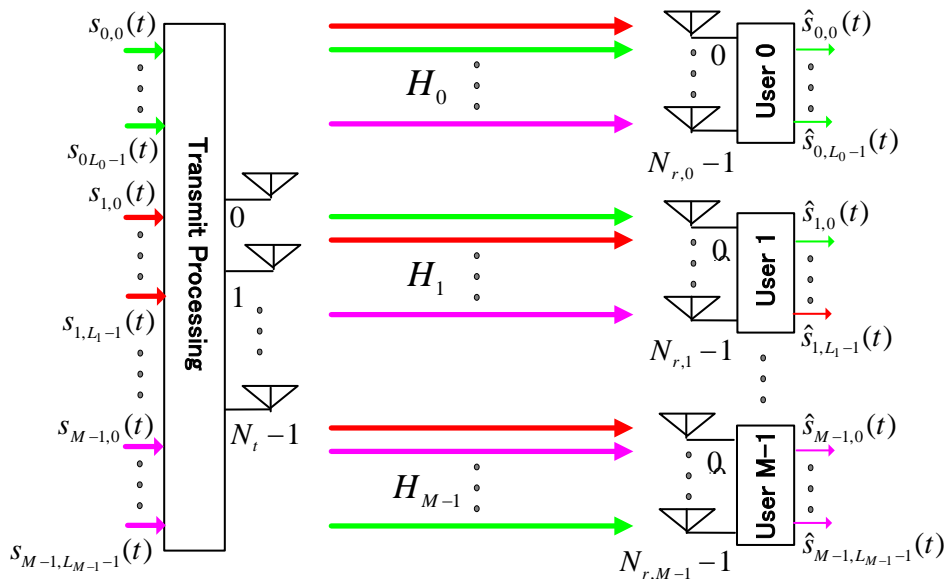


Figure 1. Multiuser MIMO downlink system model.

dures for the derivation of output signals with high precision are presented in the next section.

3 DESIGN ALGORITHMS OF MULTIUSER MIMO DOWNLINK SYSTEM

In this section, a design method of weight vectors for multiuser MIMO downlink system defined in section 2 is developed based on BD scheme. In addition, another type of BD based algorithm method not using ZF but adopting maximum signal to interference plus noise ratio (MSINR) criterion in the transmitter is also considered searching for the further possibility of the performance improvement.

3.1 Weight Design Problem

General aspects of weight design problem are overviewed in this subsection before describing the actual design procedure.

The output signal of the m -th receiver consists of the desired signal corrupted by the interferences components originated from all the streams of all the users except that of the target stream of the target user plus the noise signal. Hence, SINR denoted by $\Gamma_{m,\ell}$ for the output signal of the ℓ -th stream of the m -th user becomes an adequate evaluation criterion, and it is defined by the next equation:

$$\Gamma_{m,\ell} = \frac{|\mathbf{w}_{r,m,\ell}^H \mathbf{H}_m \mathbf{w}_{t,m,\ell}|^2 P_{m,\ell}}{\mathbf{w}_{r,m,\ell}^H \mathbf{R}_{i,t,m,\ell} \mathbf{w}_{r,m,\ell} + \|\mathbf{w}_{r,m,\ell}\|^2 P_{N,m}} \quad (2)$$

$$R_{i,t,m,\ell} = \sum_{(n,k) \in \mathcal{I}_{m,\ell}} H_n \mathbf{w}_{t,n,k} \mathbf{w}_{t,n,k}^H H_n^H P_{n,k}$$

where $\mathcal{I}_{m,\ell} = \{(n,k) \neq (m,\ell)\}$ for the target user m and the stream ℓ . The simultaneous maximization of the set $\{\Gamma_{m,\ell}\}$ using a certain adequate criterion, for example, sum capacity which is approximately represented by $C = \sum_{m,\ell} \log(1 + \Gamma_{m,\ell})$ is

first enumerated as a straight way of the weight design, but we need a much complicated optimization. Some algorithms with improved design efficiency have been proposed, but they still need heavy burden for the implementation and high mathematical complexity, and they are not engineer-familiar. That is the reason we consider simple methods based on the idea of BD in the following section.

3.2 ZF Based Design Method

A BD based method utilizing ZF in the transmitter is proposed in this section.

Conventional BD first attempts ZF to all the antennas of non-target users using the transmitter weights. In other words, the transmission weight vector is chosen so that it belongs to the kernel of $\tilde{H}_m = [H_0^T, \dots, H_{m-1}^T, H_{m+1}^T, \dots, H_{M-1}^T]^T$. The problem of this method is that it consumes the degrees of freedom $\sum_{n \neq m} N_{r,n}$. This fact means a large number of antennas are required even

if a small number of streams are used for each user, and in addition, it might result in small transmission gain between the transmitter and the target receiver.

Therefore, here the design of the receiver weight vectors is considered first. Finding the optimal vector before the determination of the transmission weights is difficult, but instead, we use the SVD based design [16] as one reasonable choice, which is the optimal approach in the case of conventional single user MIMO. Namely, the left singular value vector of H_m corresponding to the ℓ -th largest singular value $\sqrt{\lambda_{m,\ell}}$ is used as weight vector $\mathbf{w}_{r,m,\ell}$, where the ℓ -th largest eigenvalue of the covariance matrix of H_m is expressed by $\lambda_{m,\ell}$.

Next, total MIMO channel H is converted to $H_r \in \mathbb{C}^{L \times N_t}$, where L denotes the total number of streams which is given by $L = \sum L_m$. Matrix H_r is defined as $H_r = [H_{r,0}^T, \dots, H_{r,M-1}^T]^T$, where $H_{r,m} = W_{r,m}^H H_m$, and the ℓ -th column of $W_{r,m} \in \mathbb{C}^{N_{r,m} \times L_m}$ is $\mathbf{w}_{r,m,\ell}$. We can derive the transmitter weight vectors by applying ZF approach to this matrix. Let us define

$$\tilde{H}_{r,m,\ell} = [H_{r,0}^T, \dots, H_{r,m-1}^T, \underline{H}_{r,m,\ell}^T, H_{r,m+1}^T, \dots, H_{r,M-1}^T]$$

where $\underline{H}_{r,m,\ell} = \tilde{W}_{r,m,\ell}^H H_m$ and

$$\tilde{W}_{r,m,\ell} = [\mathbf{w}_{r,m,0}, \dots, \mathbf{w}_{r,m,\ell-1}, \mathbf{w}_{r,m,\ell+1}, \dots, \mathbf{w}_{r,m,L_m-1}].$$

The vector $\mathbf{w}_{t,m,\ell}$ is given by $V_{m,\ell} \mathbf{v}_{m,\ell}$, where $\mathbf{v}_{m,\ell}$ is the right singular value

vector of $H_m V_{m,\ell}$ corresponding to the largest singular value. The columns of matrix $V_{m,\ell} \in \mathbb{C}^{N_t \times (N_t - L + 1)}$ consist of $(N_t - L + 1)$ different right singular value vectors of $\tilde{H}_{r,m,\ell}$ corresponding to the null singular value.

The interference cancellation from undesired users is ignored in the first step, since the suppression of them could be considered in the second step using ZF technique.

We can anticipate that this approach has two advantages given below if the satisfaction of the condition on N_t is assumed:

(1) We can expect the performance improvement which comes from the establishment of strong connection between the transmitter and the target stream of the target receiver under the assumption that this idea works toward a positive direction. In the next section, we will verify the detailed nature of this approach through computer simulations.

(2) The number of degrees of freedom consumed for the user m could be reduced to L_m . This number is less than or equal to the case of conventional BD always steering zeros toward all the $\sum_{n \neq m} N_{r,m}$ antennas in the receiver side. Therefore, even in the case the number of the transmit antennas is restricted to a small number, we can transmit minimum one stream if $N_t \geq M$ is satisfied.

Together with the advantages above, the proposed approach is still very

simple both in the computational and theoretical aspects.

3.3 MSINR Based Design

This subsection attempts further improvement from the ZF based approach in 3.2, namely, another approach adopting MSINR criterion in the transmitter side is shown. The concept of this approach is, however, developed based on the above method and expressed using the descriptions in 3.2.

The idea of ZF eliminating interferences to all the antennas of non-target users is simple and easily understandable. This is because BD approach is widely used, but here, going back to the origin of interference and noise mitigation, another approach using MSINR design instead of ZF in the transmitter side is considered. Being maximized here is the criterion shown below

$$\Gamma_{t,m} = \frac{|\mathbf{w}_{r,m,\ell}^H H_m \mathbf{w}_{t,m,\ell}|^2 P_{S,m}}{\mathbf{w}_{t,m,\ell}^H R_{i,r,m} \mathbf{w}_{t,m,\ell} + \|\mathbf{w}_{r,m,\ell}\|^2 P_{N,m}} \quad (3)$$

$$R_{i,r,m} = \sum_{(n,k) \in \mathcal{I}_{m,\ell}} H_n^H \mathbf{w}_{r,n,k} \mathbf{w}_{r,n,k}^H H_n P_{n,k}$$

The transmit weights cannot control the receiver noise, but under the situation where it can be ignored (or experimentally assuming it does not exist), this method becomes equivalent

to ZF. Strictly speaking, it is desirable that the design procedure contains the simultaneous calculation of the transmit powers of all users. But here, for the simplicity, they are kept to one during weight design process, and then the water filling [17] described in the next section is utilized for the power allocation. After calculating equation (3) keeping the unit norm condition, the optimal weight condition is derived as an eigenproblem

$$R_S \mathbf{w}_{r,m,\ell} = \Gamma_{t,m} \left(R_{i,r,m,\ell} + \sum_n \|\mathbf{w}_{r,n,\ell}\|^2 P_{N,n} I_{N_t} \right) \mathbf{w}_{t,m,\ell} \quad (4)$$

$$R_S = H_m^H \mathbf{w}_{r,m,\ell} \mathbf{w}_{r,m,\ell}^H H_m$$

and the transmit weights are derived by solving this problem.

4 SIMULATIONS

This section carries out computer simulations to evaluate the design methods of multiuser MIMO downlink system proposed in section 3, together with their effectiveness and features.

4.1 Simulation Conditions

This subsection shows default simulation conditions.

The evaluation measure for the proposed methods is SINR which is defined by equation (2) in section 3.1. Because the transmit and receive weight vectors are under the condition

of the unit norm and $E[|s_{m,\ell}(t)|^2] = 1$ for all the signals, the transmit power allocation is calculated utilizing water filling theorem [17] so that the total power of each user keeps unity, that is, $P_m = \sum_{\ell} \{P_{m,\ell}\} = 1$. The signal to noise ratio (SNR) of the m -th user is given by $\text{SNR} = \frac{P_m}{P_{N,m}}$, namely, the ratio of total signal power and the noise of user m . The number of user is assumed to be $M = 3$, and the antennas number is $N_t = 12$ and $N_{r,m} = 4$ for all users, which means the condition on the degrees of freedom is satisfied even if the conventional BD is adopted. The default number of streams is $L_m = 2$, but if it is impossible because of the channel condition, the maximum number of streams less than L_m is chosen instead. Binary phase shift keying (BPSK) modulation is considered here, but the results could be applied to other schemes.

We assume independent and identically distributed (i.i.d.) Rayleigh fading channel with unit variance, the mean SINR is calculated using 500 sample channels.

The default simulation conditions are summarized in Table 1.

4.2 Results and Discussions

Simulation results for the evaluation and discussions related to them are given in this subsection.

The empirical distribution functions of the output SINR of multiuser MIMO downlink methods for $N_t = 12$ and $N_{r,m} = 4$ are depicted in Fig. 2

Table 1. Simulation Conditions

Number of Users	$M = 3$
Number of Transmit Antennas	$N_t = 12$
Number of Receive Antennas	$N_{r,m} = 4$
Number of Streams per User	$L_m = 2$
Energy Constraint	$P_m = 1$
SNR	SNR = 20dB
Channel Statistics	i.i.d. Rayleigh Fading with unit variance

in case of enough antennas number for the maximum $L_m = 4$ stream transmission. Subplots (a) to (d) correspond the cases of stream number $L_m = 1$ to $L_m = 4$, where L_m is set to a constant not concerning m . Figure 2 shows only the curves of proposed method using ZF in the second step, since the difference between the curves of ZF from those of MSINR based approach is indistinguishable small under this situation. What can be observed from Fig. 2 is that the proposed method outperforms the conventional BD with respect to the output SINR characteristics when small number of streams $L_m = 1, 2$ is adopted. But the conventional BD is superior to the proposed approach in case the number of streams is increased. The reason is that, while the conventional BD consumes the same degrees of freedom not concerning the number of streams, the proposed method consumes smaller number of them when the streams are less than the maximum possible number (rank of the channel matrix). Hence the transmit weights for the target user can be selected from the space with a larger dimension. But their difference becomes smaller as the increment of stream

number. This is because as the stream number approaches to its maximum, the optimality of the weight under the ZF condition of the conventional BD gradually invokes its advantage. We can observe the influence of the order change of transmit and receive weight design in this nature.

Figure 3 draws the curves of SNR given in section 4.1 versus output SINR. From this figure, we can observe that the output SINR improves almost in proportional to SNR. Figure 3 depicts the case the degrees of freedom is not sufficient to use the conventional BD, hence its curves are not plotted there. The evaluation in the low SNR region shows that the MSINR based approach outperforms ZF based one. But the difference between them is decreased as SNR becomes higher, and almost same around SNR = 30dB, which is a natural result coming from the performance improvement of ZF method under the high SNR condition [8].

The relation between number of users and the output SINR is given in Fig. 4 for $N_t = 12$, $N_{r,m} = 2$, and $L_m = 1$ or $L_m = 2$. We can observe from this figure that the difference of two methods is enlarged as the num-

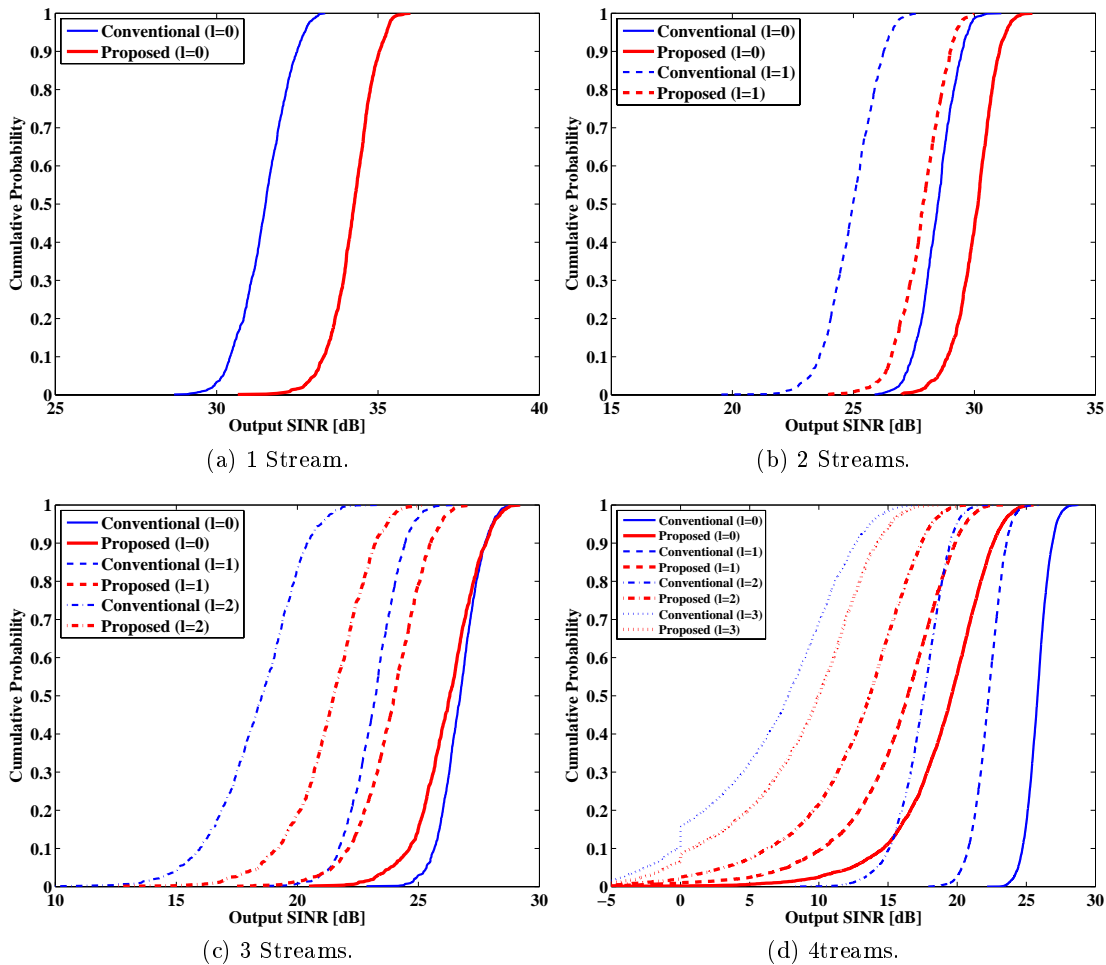


Figure 2. Distribution functions of output SINR for $N_t = 12$, $N_{r,m} = 4$, and $\text{SNR} = 20\text{dB}$.

ber of users is increased not depending on the proposed method outperforms ($L_m = 1$) or not ($L_m = 2$) the conventional one. Those results tell us that the selection of the algorithm is particularly important in a system assuming a relatively large number of users.

Figure 5 depicts the curves of the relation between the receive antennas number and the output SINR for $M = 3$, $N_t = 14$ and $L_m = 1, 2$. What

should be paid attention is that the number of streams same as that of the receive antennas is not always possible since the limitation of the degrees of freedom exist (for example, the maximum number of stream is $L_m = 2$ even under $N_{r,m} = 6$). This figure shows that the performance gap of the conventional and the proposed methods is enlarged as the number of receive antennas increases.

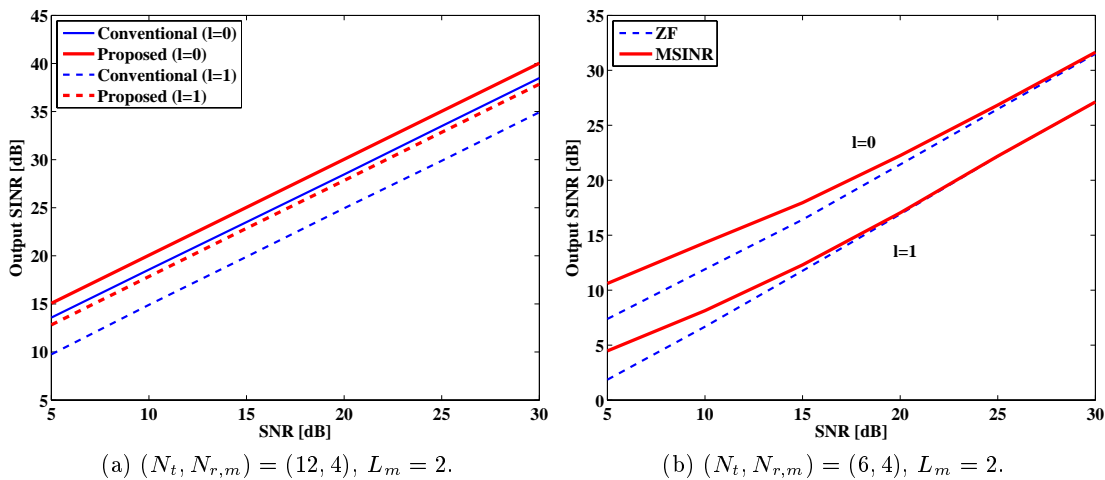


Figure 3. SNR versus output SINR characteristics.

Next, we consider the case different numbers of streams are adopted among different users ($m = 0 \sim 3$). Figure 6 depicts examples of SINRs for $L_0 = 3$, $L_1 = 2$, and $L_2 = 1$ in cases of SNR = 10dB and SNR = 30dB. The SINR of user 1 adopting the largest stream number $L_0 = 3$ is the lowest in the stream $\ell = 0$, but a higher SINR is achieved as the increment of the stream number. On the other hand, this order is reversed ($\text{SINR}_0 > \text{SINR}_1$) for SNR = 10dB in the stream $\ell = 1$, and almost equal for SNR = 30dB. A simple rule is not found observing those result, but through investigation of other cases (e.g., $N_r = 12$ though the result is not shown here), it is verified that when the number of N_r is sufficiently large so that the conventional BD is possible, SINR of the ℓ -th path becomes higher in descent order of the stream number (generally, $L_m \geq L_n$ results in $\Gamma_{m,\ell} \leq \Gamma_{n,\ell}$).

Based on those observations, we can summarize following two results:

- (1) By adopting the proposed method, a multistream transmission using small number of antennas to which the conventional BD could not be applied becomes possible. This is one advantage if we assume a home-use access point which has a restriction on its physical size and cannot be equipped with a large array.
- (2) Also under the situation the degrees of freedom is sufficient for the conventional BD, a better performance is derived by the proposed method if the stream number is small compared with the maximum possible value. MIMO system often does not use all the streams to avoid solving a complicated scheduling problem, and simple rule of adaptive modulation choosing the maximum one or two streams is preferred

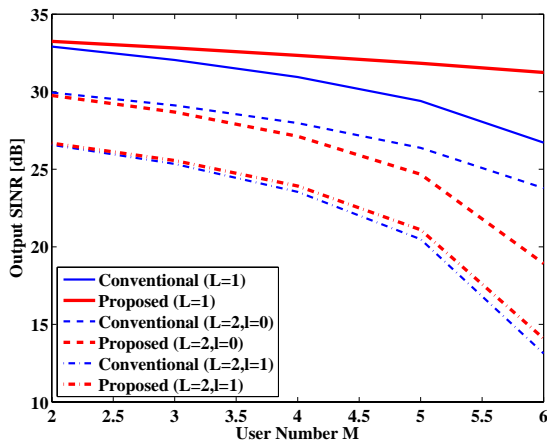


Fig. 4. Number of users versus output SINR for $(N_t, N_{r,m}) = (12, 2)$ and SNR = 20dB.

in some applications. So our recommendation is to switch between the conventional and the proposed methods depending on the situation (number of users and streams) in which the multiuser system is actually used.

Other than the above-mentioned features, we can consider investigations changing some parameters. If some of user are under worse condition, for example, under fewer receive antenna number and/or smaller channel variance, the SINR characteristics might be degraded. But it is a phenomenon common to most multiuser transmission schemes, hence not considered here.

5 CONCLUSIONS

This paper have presented very simple weight design methods for multiuser MIMO downlink transmission based

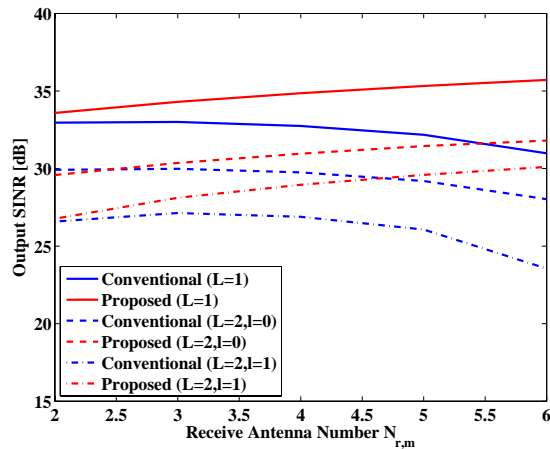


Fig. 5. Number of receive antennas versus output SINR for $M = 3$, $N_t = 14$ and SNR = 20dB.

on BD. In those approaches, the order of design steps is swapped from that of conventional BD scheme, which means the receiver weights are first designed, and then transmission weights are calculated based on ZF technique for the establishment of a large gain through the channel between the transmitter and the receiver of the target user. By so doing, the degrees of freedom required in the transmitter side could be also reduced. Computer simulations show that the performance of the proposed methods is better than that of conventional BD under certain conditions, and in that case, the SVD based receiver weight design is reasonable.

The choice of the receiver weight vectors here is simple and reasonable, but we believe that is not the best because of the existence of the interferences. Therefore, as a future work, we consider the improvement of the proposed approach based on more rea-

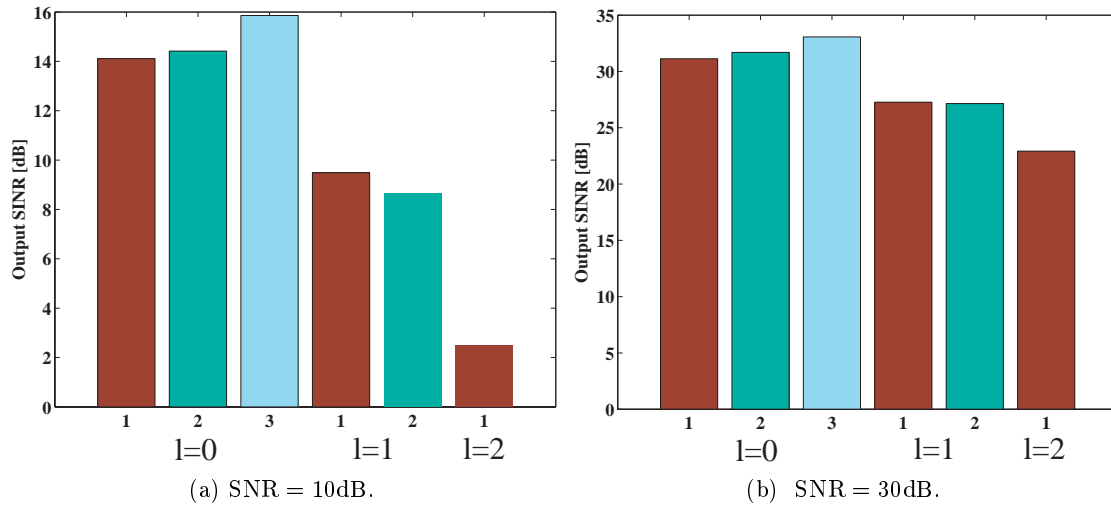


Figure 6. Output SINR in case of $L_0 = 3$, $L_1 = 2$, and $L_2 = 1$. Numbers in horizontal axis denote user index m .

sonable choice of the receiver weights which achieves a better performance. Application of the proposed method to CoMP transmission and extension to broadband case [18] are also important themes of study.

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