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ASSESSING GAUGE RELIABILITY AND REPRODUCIBILITY USING THE CORRELATION BETWEEN TWO MEASUREMENT SYSTEMS

Abstract: *In modern production processes, a large amount of testing and measurement is performed to support decision making and to ensure quality. In order to achieve this, the measurement data needs to be reliable, and the capability of measurement systems needs to be verified. Gauge reliability and reproducibility (GRR) is used for quantifying and analysing the variation in results caused by the measurement system. The purpose of this study is to introduce a simple method to assess GRR performance when there are two parallel measurement systems. The study shows that with certain assumptions, the Pearson correlation between measurement results of two measurement systems can be expressed using the GRR indices of these systems. This implies that the GRR performance of these measurement systems could be analysed based on this correlation. In certain situations this could save significant time compared to regularly performed GRR studies.*

Keywords: *Gauge R&R, correlation, measurement, quality, measurement system analysis*

1. Introduction

Expectations for the performance of production systems are constantly increasing, and these systems are becoming more advanced. This requires management to make decisions based on proper quantitative analysis of data. In the manufacturing process, control of variation with an increasingly high degree of precision demands an improved degree of measurement effectiveness (Hoffa and Laux 2007). For this, it is crucial that the measurement data is reliable; therefore the capability of the measurement systems needs to be monitored.

Measurement Systems Analysis (MSA) is a collection of statistical methods for analysing measurement system capability (AIAG 2002; Smith *et al.*, 2007). Previous research has shown that rising costs of measurement are a cause for concern in the industry (Neely *et al.*, 1994).

This study explores gauge reliability and reproducibility (GRR), one of the tools used in MSA. It is a methodology to quantify and analyse the variation in results caused by the measurement system (Larsen 2002; Larsen 2003; Mader *et al.*, 1999). Repeatability can be determined by measuring a part several times, effectively quantifying the variability

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in a measurement system resulting from the gauge itself (AIAG 2002; Smith *et al.*, 2007; Pan 2006). Reproducibility is determined from the variability created by several operators measuring a part several times each, effectively quantifying the variation in a measurement system resulting from the operators of the gauge and environmental factors, such as time (AIAG 2002; Pan 2006; Burdick *et al.*, 2003; Tsai 1989).

In all industry areas there is a constant need to speed up production processes (Louka and Besseris 2010). Regularly performing these studies can be time consuming, and achieving their goal in a more resource-efficient way would help in reducing Cost of Quality without effecting product quality (Modrak 2007). This is particularly true in a situation where we have parallel measurement systems and a large number of measurements taken within each system, for example, a production system with two similar production lines with similar measurement system setups. In situations like this, it would be convenient to have a way to follow GRR performance without having to conduct full-scale GRR studies on both measurement systems.

The purpose of this study is to introduce a simple method to assess GRR performance when there are two parallel measurement systems and to discuss its implications. The method is based on the correlation of the measurement results between the two systems.

2. Definitions and assumptions

2.1. Definitions

The definitions presented below refer to a situation where we have two measurement systems X and Y measuring parts from the same production process. Both measurement systems and the production process are assumed to be normally distributed. The parts have actual values that the measurement system tries to capture, and observed values, which are the actual values combined with measurement errors caused by the measurement system. The actual value is also referred to as the true value (Burdick *et al.*, 2003).

σ_{ax} is the variation of the *actual* values of parts measured with measurement system X, as shown in equation (1):

$$\sigma_{ax} = \sqrt{\frac{\sum_{i=1}^{n}(x_{ai} - \bar{x}_a)^2}{n}} \qquad (1)$$

where x_{ai} is the actual individual values of the measured parts. The same relationship applies to:

σ_{ay}, the variation of the *actual* values of parts measured with measurement system Y, where y_{ai} is the actual individual values of the measured parts.

$\sigma_{\varepsilon x}$, the variation in *measurement errors* of measurement system X, where ε_{xi} is the individual measurement errors of each part.

$\sigma_{\varepsilon y}$, the variation of *measurement errors* of measurement system Y, where ε_{yi} is the individual measurement errors of each part.

σ_{ox}, the variation of *observed* values in measurement system X, where x_{oi} is the individual observed values of each part. σ_{ox} is referred to as *total variation* in GRR studies.

σ_{oy}, the variation of *observed* values in measurement system Y, where y_{oi} is the observed values of each part. σ_{oy} is referred to as *total variation* in GRR studies.

GRR_x stands for the gauge repeatability and reproducibility index of measurement system X. It is defined with equation (2) and is referred to as the percentage of total variation. GRR_y is defined in the same manner.

$$GRR_x = \frac{\sigma_{\varepsilon x}}{\sigma_{ox}} \qquad (2)$$

The Pearson product–moment correlation coefficient is suitable for modelling linear correlation relationships. The Pearson correlation coefficient R_a for the actual part values x_{ai} and y_{ai} is shown in equation (3). The correlation R_o between the observed values x_{oi} and y_{oi} is defined in the same way.

$$R_a = \frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{\sqrt{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)^2\sum_{i=1}^{n}(y_{ai}-\bar{y}_a)^2}} \quad (3)$$

2.2. Assumptions

The following assumptions about the measurement systems X and Y and the population of measured parts have been made in this study:

1) Parts from the same population are used in the correlation and GRR studies, and this population is normally distributed.
2) Measurement errors $[\varepsilon_{x1},\varepsilon_{x2},\varepsilon_{x3}\ldots\varepsilon_{xi}]$, $[\varepsilon_{y1},\varepsilon_{y2},\varepsilon_{y3}\ldots\varepsilon_{yi}]$ are independent of $[x_{a1},x_{a2},x_{a3}\ldots x_{ai}]$, $[y_{a1},y_{a2},y_{a3}\ldots y_{ai}]$, hence $[\varepsilon_{x1},\varepsilon_{x2},\varepsilon_{x3}\ldots\varepsilon_{xi}]$, $[\varepsilon_{y1},\varepsilon_{y2},\varepsilon_{y3}\ldots\varepsilon_{yi}]$ are independent of $[x_{a1}-\bar{x}_a, x_{a2}-\bar{x}_a, x_{a3}-\bar{x}_a \ldots x_{ai}-\bar{x}_a]$ and $[y_{a1}-\bar{y}_a, y_{a2}-\bar{y}_a, y_{a3}-\bar{y}_a \ldots y_{ai}-\bar{y}_a]$
3) Measurement errors $[\varepsilon_{x1},\varepsilon_{x2},\varepsilon_{x3}\ldots\varepsilon_{xi}]$, $[\varepsilon_{y1},\varepsilon_{y2},\varepsilon_{y3}\ldots\varepsilon_{yi}]$ are normally distributed.
4) The relationship of observed values x_{oi}, actual values x_{ai} and measurement errors ε_{xi} is presented in equation (4). The same relationship exists between y_{oi}, y_{ai} and ε_{yi}
5) $x_{oi} = x_{ai} + \varepsilon_{xi},$ (4)
6) The means of measurement errors $[\varepsilon_{x1},\varepsilon_{x2},\varepsilon_{x3}\ldots\varepsilon_{xi}]$ and $[\varepsilon_{y1},\varepsilon_{y2},\varepsilon_{y3}\ldots\varepsilon_{yi}]$ are equal to 0 i.e., there is no bias. It should be noted that this assumption is made to clarify the calculations. Bias does not actually have an effect on correlation.
7) The mean of the observed values $[x_{o1},x_{o2},x_{o3}\ldots x_{oi}]$ are equal to equations (5) due to assumption 5. The same applies to $[y_{o1},y_{o2},y_{o3}\ldots y_{oi}]$.
8) $\bar{x}_o = \bar{x}_a + \bar{\varepsilon}_x = \bar{x}_a$, (5)
9) The observed value variation σ_{ox}, actual value variation σ_{ax} and measurement error variation $\sigma_{\varepsilon x}$ are related as shown in equation (6), using equation (2). The same relationship exists between σ_{oy}, σ_{ay} and $\sigma_{\varepsilon y}$.

$$\sigma_{ox}^2 = \sigma_{ax}^2 + \sigma_{\varepsilon x}^2 = \sigma_{ax}^2 + GRR_x^2 * \sigma_{ox}^2 = \frac{\sigma_{ax}^2}{1-GRR_x^2}, \quad (6)$$

3. Innovations

In this section, the correlation of observed values R_o presented in equation (7) is expressed as a function of GRR_x and GRR_y, using the definitions and assumptions given above.

$$R_o = \frac{\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)(y_{oi}-\bar{y}_o)}{\sqrt{\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)^2\sum_{i=1}^{n}(y_{oi}-\bar{y}_o)^2}} \quad (7)$$

The numerator can be expressed as in equation (8) by using equations (4, 5):

$$\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)(y_{oi}-\bar{y}_o)=\sum_{i=1}^{n}(x_{oi}-\bar{x}_a)(y_{oi}-\bar{y}_a)=\sum_{i=1}^{n}(x_{ai}+\varepsilon_{xi}-\bar{x}_a)(y_{ai}+\varepsilon_{yi}-\bar{y}_a)=$$
$$\sum_{i=1}^{n}(x_{ai}-\bar{x}_a+\varepsilon_{xi})(y_{ai}-\bar{y}_a+\varepsilon_{yi})=\sum_{i=1}^{n}[(x_{ai}-\bar{x}_a)+\varepsilon_{xi}][(y_{ai}-\bar{y}_a)+\varepsilon_{yi}]= \quad (8)$$
$$\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)+\sum_{i=1}^{n}[\varepsilon_{xi}(y_{ai}-\bar{y}_a)]+\sum_{i=1}^{n}[(x_{ai}-\bar{x}_a)\varepsilon_{yi}]+\sum_{i=1}^{n}(\varepsilon_{xi}\varepsilon_{yi})$$

Based on the assumptions, ε_{xi} and $y_{ai}-\bar{y}_a$, ε_{yi} and $x_{ai}-\bar{x}_a$, ε_{yi} and ε_{xi} are all independent and their means equal 0. This means that equation (8) can be expressed as in equation (9):

$$\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)(y_{oi}-\bar{y}_o)=\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)+\sum_{i=1}^{n}[\varepsilon_{xi}(y_{ai}-\bar{y}_a)]+\sum_{i=1}^{n}[(x_{ai}-\bar{x}_a)\varepsilon_{yi}]+\sum_{i=1}^{n}(\varepsilon_{xi}\varepsilon_{yi})= . \quad (9)$$
$$\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)+0+0+0=\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)$$

We can express the denominator as in equation (10) by using equations (1, 6):

$$\sqrt{\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)^2\sum_{i=1}^{n}(y_{oi}-\bar{y}_o)^2}=\sqrt{n\sigma_{ox}^2 n\sigma_{oy}^2}=n\sqrt{\frac{\sigma_{ax}^2}{1-GRR_x^2}\frac{\sigma_{ay}^2}{1-GRR_y^2}}=\frac{n\sigma_{ax}\sigma_{ay}}{\sqrt{(1-GRR_x^2)(1-GRR_y^2)}} \quad (10)$$

By combining equations (9, 10), R_o can be expressed as in equation (11):

$$R_o=\frac{\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)(y_{oi}-\bar{y}_o)}{\sqrt{\sum_{i=1}^{n}(x_{oi}-\bar{x}_o)^2\sum_{i=1}^{n}(y_{oi}-\bar{y}_o)^2}}=\sqrt{(1-GRR_x^2)(1-GRR_y^2)}\frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{n\sigma_{ax}\sigma_{ay}} \quad (11)$$

The actual correlation coefficient R_a (3) can be expressed as in equation (12) below using equation (1):

$$R_a=\frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{\sqrt{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)^2\sum_{i=1}^{n}(y_{ai}-\bar{y}_a)^2}}=\frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{\sqrt{n\sigma_{ax}^2 n\sigma_{ay}^2}}=\frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{n\sigma_{ax}\sigma_{ay}} \quad (12)$$

By using equation (12), we can express equation (11) as in equation (13):

$$R_o=\sqrt{(1-GRR_x^2)(1-GRR_y^2)}\frac{\sum_{i=1}^{n}(x_{ai}-\bar{x}_a)(y_{ai}-\bar{y}_a)}{n\sigma_{ax}\sigma_{ay}}=R_a\sqrt{(1-GRR_x^2)(1-GRR_y^2)} \quad (13)$$

This can also be expressed as in equation (30):

$$R_o^2 = (1 - GRR_x{}^2)(1 - GRR_y{}^2)R_a^2 \quad (14)$$

4. Discussion

Equation 14 shows how the GRR indices of two measurement systems can be connected with the given assumptions by using the Pearson correlation coefficient. This could potentially be useful in a situation where there are parallel measurement systems and performing GRR studies on a regular basis would require considerable resources.

A practical application could be based on monitoring correlation periodically instead of GRR studies. This could be done by, for example, measuring R_o using a set of master parts and comparing the results against a threshold level. If the same parts are used to calculate GRR_x, GRR_y, and R_o, R_a is theoretically 1. Table 1 presents sample values of R_o^2 using Equation 30 and different combinations of GRR_x and GRR_y.

Table 1. Relationship of GRR_x, GRR_y, and R_o^2 when $R_a = 1$

GRR_x	10%	20%	20%	30%	40%	50%	60%
GRR_y	10%	10%	20%	30%	40%	50%	60%
$R_o^2 = (1 - GRR_x^2)(1 - GRR_y^2)$	0.98	0.95	0.92	0.83	0.71	0.56	0.41

In practice the observed correlation is also affected by other sources of variation besides the variation accounted for in a GRR study. Therefore $R_a \leq 1$ and

$$R_o^2 \leq (1 - GRR_x{}^2)(1 - GRR_y{}^2) \quad (15)$$

Theoretically, worst values for GRR_x and GRR_y related to different R_o^2 levels can be calculated using Equation 15. By setting $GRR_y = 0$, we can calculate worst possible values for GRR_x and the same can be applied to GRR_y. Sample values are presented in Table 2.

Table 2. Relationship of R_o2 and worst GRR_x or GRR_y values

R_o^2	0.99	0.98	0.95	0.9	0.85	0.8	0.75
Worst GRR_x or GRR_y	10%	14%	22%	32%	39%	45%	50%

Using R_o^2 information, we can draw inferences about the worst possible GRR levels of measurement systems X and Y. In other words, if the measurement results correlate highly, both GRR_x and GRR_y must be on a good level. This means we can set a control level for R_o^2 and use the measurement results for assessing GRR performance.

A practical application of the results involves a situation with two measurement systems, an established system and a new one. Each system has 1000 fixtures, and each fixture has 10 cavities. Each cavity needs to be qualified for GRR and bias. The target for the new machine's GRR has been set at $\leq 20\%$ for each cavity. Performing individual GRR studies for each cavity would require considerable resources. Instead, the following inspection plan can be used to assess GRR performance against the target value:

a) Measure a 10-piece sample on the established system. From previous studies it is already known that this system's GRR percentage is 10% for all cavities.

b) Measure the same sample with the new

machine and perform a correlation study with the results. If $R_o^2 > 0.95$, the new machine's cavity in question is qualified based on Table 1.

This method was also piloted in to practice in a case production environment.The pilot results were encouraging; the rates of false passes and false rejections have stayed on predicted levels. This application was based on saving a set of master parts and periodically measuring them. If the production process is in control, it could be possible to use random samples from the production instead of a set of master parts. This would reduce the need for performing GR&R studies, as correlation between the measurement systems could be monitored on a continuous basis.

5. Conclusion

Modern production firms perform large amounts of testing and measurement to support decision making and to ensure quality. This means that making measurement and its supporting processes more efficient could be financially significant. GRR studies quantify and analyse the variation in the results caused by the measurement system. Regularly performing these studies can be time consuming, especially in a situation where we have parallel measurement systems. The purpose of this study is to introduce a simple method to assess GRR performance when there are parallel measurement systems without conducting full-scale GRR studies.

In this study it was shown that with certain assumptions the Pearson correlation between measurement results of certain parts in two measurement systems can be expressed using the GRR indices of these systems, as in equation (14).

These results imply that conclusions about the GRR performance of these two measurement systems can be drawn based on the correlation of measurement results. Thus, the correlation could be used to assess GRR performance of these systems. In practical use this could mean setting a threshold level for correlation and regularly measuring correlation against this level. In certain situations this could save significant resources compared to regularly performed GRR studies.

The limitations of the study relate to the given assumptions. The results apply in a situation where two measurement systems are used to measure parts from the same normally distributed population. The measurement results were assumed to be independent and normally distributed. It was also assumed that there is no bias or linearity, although bias does not affect correlation. Significant linearity could have an effect on correlation.

Potential areas for future research include expanding the results outside the assumptions of this study, such as the correlation of measurement results with distributions other than normal. Another topic could be how to integrate MSA into continuous process flow.

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