ISSN № 9125 0912. ВІСНИК ДНУ. Серія "Моделювання". № 8. 2012. Вип. 4. С. 95–115

Проблеми математичного моделювання
та теорії диференціальних рівнянь

УДК 517.91

DYNAMIC FIELD OF ELASTIC DISPLACEMENTS IN A ROPE WHICH IS REELED UP ON THE DRUM AT LIFTING OF LOADS

V. A. Ostapenko

Department of Differential Equations Dnepropetrovsk National University, pr. Gagarina, 72, 49010, Dnepropetrovsk, Ukraine, E-mail: victor.ostapencko@yandex.ru

The boundary-value problem about construction of the displacement waves and the strain waves arising in ropes of elevating devices, such as lifts, mine lifts and so on is considered. The rope at lifting of loads is reeled up on a drum. In a case when the friction coefficient of a rope about a drum is not too big, occurs frictional sliding a rope on a drum. Therefore the behavior of a rope on a drum is described by the telegraph equation. The behavior of a hanging part of a rope is described by the wave equation. It means, that in different parts of a rope the displacements are solutions of the different equations. That is from this point of view the rope is shared on two zones. Thus owing to reeling of a rope on a drum the border which shares these two zones is a variable. In such model the waves not only reflect from ending points of a rope. There is also their reflection and refraction on moving border of the sharing of zones. Is developed methods for obtaining of exact solutions for the boundary-value problems with mobile borders for both the wave and telegraph equations. They are based on maintenance of a continuity of the displacements in points of reflection of waves. The exact solution of such problem is obtained for the case of sagging a rope prior to the beginning of rise.

Key words: dynamic field in ropes, wave and telegraph equations, variable border, exact solution.

Introduction

Investigation of a dynamic field of stress in ropes of elevating devices, such as lifts and mine lifts, represents the important problem for practical use. First of all, it is connected to necessity of a safety of work of elevating devices, and also with aspiration to reduce weight of a rope. Therefore to studying of a problem of calculation of the dynamic stress arising in ropes, a plenty of works is devoted. Long time was supposed, that elastic displacements in ropes are described by the wave equations. Besides at statement of initial boundary value problems it was supposed, that the length of a rope does not change. Such statement of problems predetermined their solutions with the help of a separation of variables method [1, 2].

Lacks of such approach were quite obvious. Therefore, since the second half twentieth century works in which attempts to take into account change of length of a rope [3-8] were undertaken began to appear. The basic feature of all such works has consisted in intention to use for their solution still a separation of variables method. However the initial boundary value problems describing processes of distribution of waves in ropes of variable length, did not suppose separation of variables. Therefore such initial boundary value problems were transformed to the integro-differential equations [4-6] with variable borders of integration. The approached solutions of the integro-differential equations again with the help of a separation of variables method or its updatings [5, 6] further were under construction. Besides were under construction asymptotic expansion of the solutions of the integro-differential equations on small parameter. The role of such small parameter carried out speed of winding of a rope, in the assumption, that it was essentially less speeds of a sound in a rope [6].

Strictly speaking, the integro-differential equations with variable limits of integration have no simple own functions. Therefore expansion of solutions was carried out on eigenfunctions of the simplified operators. Simplification was, that the integro-differential equation for a rope of variable length provided that speed change of length of a rope is small, was replaced with the equation for a rope of constant length and expansion of the solution was carried out on eigenfunctions of this last equation. Besides, the kernel of the equation was replaced approximately with a kernel of Fredholm's type [5-7]. Asymptotic estimations of such approximation where obtained [6]. However they only show that as small parameter tends to zero (that is if the length of a rope does not vary) solutions for ropes of a variable and constant length in a limit coincide. In such solutions it is still difficult to observe character of distribution of waves along a rope.

The described researches have shown, that without taking into account change of length of a rope in essence it will not be possible to obtain model of the system consisting of a rope and a drum, adequate enough natural. Besides it became clear, that breaks of stress inevitably arising in ropes can be investigated only with the help of construction of the solution as propagating waves. Such research has been carried out in [9,10] in the assumption, that force of friction of a rope about a drum is so great, that slipping the rope on a drum cannot occur. At such assumption the problem has been transformed to the solution of the wave equation in area with variable borders. Due to the developed method of construction of the waves reflected from mobile border, the exact solution of such problem representing set of propagating waves has been obtained.

If the factor of friction of a rope about a drum is not too great slipping a rope on a drum occurs. In this case elastic displacements to that part of a rope which is reeled up on a drum are described by the telegraph equation while displacements to a hanging part of a rope are described by the wave equation. Such initial boundary value problem at the additional assumption that force of friction always is directed aside points of fastening of a rope to a drum, is considered here.

The boundary-value problem about construction of the displacement waves and the strain waves arising in ropes of elevating devices, such as lifts, mine lifts and so on is considered. The rope at lifting of loads is reeled up on a drum. In a case when the friction coefficient of a rope about a drum is not too big, occurs frictional sliding a rope on a drum. Therefore the behavior of a rope on a drum is described by the telegraph equation. The behavior of a hanging part of a rope is described by the wave equation. It means, that in different parts of a rope the displacements are solutions of the different equations. That is from this point of view the rope is shared on two zones. Thus owing to reeling of a rope on a drum the border which shares these two zones is a variable. In such model the waves not only reflect from ending points of a rope. There is also their reflection and refraction on moving border of the sharing of zones.

So the problem consists in obtaining solution to the system different parts of which are described with different equations. Besides, border of dividing these parts is movable. For a long time such problem could not to be solved. Now we suggest the exact solution to this problem.

The author has developed methods for obtaining of exact solutions for the boundary-value problems with mobile borders for both the wave and telegraph equations [11-20]. They are based on maintenance of a continuity of the displacements in points of reflection of waves. On mobile border at construction of the reflected and refracted waves the conditions of a continuity of displacements and strain are used. Application of such methods to a considered problem has allowed obtaining the exact solution of this problem. The solution is submitted as sequence of extending waves. With the help of such representation of the solution it is possible to reveal the most loaded sections of elevating ropes, and also propagation of breaks of pressure which arise, for example, in case of sagging a rope prior to the beginning of rise.

1. Statement of a problem

Let the rope, which is suspended vertically and having initial length L, at $t = 0$ starts to be reeled up on a drum. Prior to the beginning of rise on a drum the part of a rope in length l_0 is reeled up on a drum. The average radius of winding of a rope on a drum is equal to r. As the rope is considered as a flexible string, it can be arranged entirely along the rectilinearly axes ξ. A motionless axis ξ we shall direct vertically downwards on a longitudinal axis of a rope, for a reference mark $\xi = 0$ we shall accept a point of fastening of a rope to a drum. Alongside with a motionless axis ξ we shall enter also a mobile axis x. The axis x is directed the same as also an axis ξ, and at $t = 0$ beginnings of coordinates $O\xi$ and Ox coincide.

The axis x goes together with a rope, making linear moving

$$\nu(t) = r \int_0^t \int_0^s \varepsilon(\tau)\, d\tau\, ds \ . \tag{1.1}$$

Here $\varepsilon(t)$ - angular acceleration of rotation of a drum; also is supposed, that $\nu(0) = 0, \nu'(0) = 0$. On the end of a rope the cargo of mass m is suspended. That

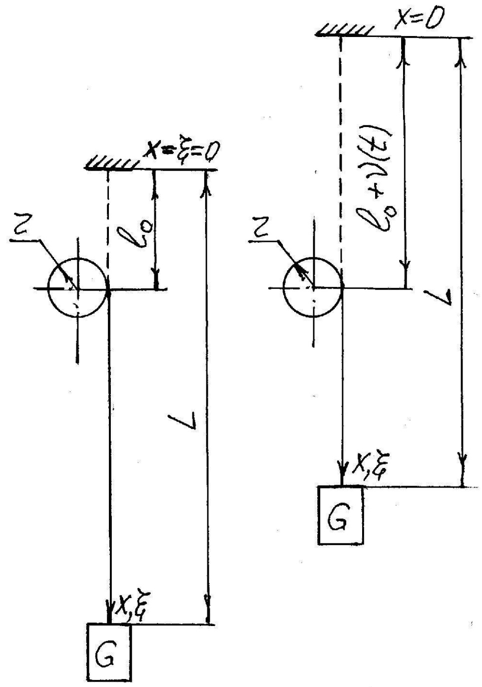

Figure 1. Calculation scheme of load lifting

is weight of cargo $G = mg$, where g - acceleration of a gravity. The rope has the area of cross-section S, the module of elasticity E, density ϱ. Then linear density of a rope $\varrho_l = S\varrho$. It is supposed, that positive elastic displacements are directed along an axis x. Moving $\nu(t)$ of a rope together with an axis x is considered as portable. Hence, elastic displacements to a rope will occur in relative movement (see Figure 1). The factor of friction of a rope about a drum is designated as β. We consider the rope as flexible thread.

For elastic displacements of a rope $u(x,t)$, having a trailer cargo and reeling up on a drum, in relative movement the following boundary-value problem is obtained. In the domain $0 < x < L, t > 0$ to search function $u(x,t)$, satisfying at $0 < x < l_0 + \nu(t)$ to the equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{a^2} \cdot \frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\beta}{r} \frac{\partial u(x,t)}{\partial x} = 0 \ , \tag{1.2}$$

and at $l_0 + \nu(t) < x < L$ - to the equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{a^2} \cdot \frac{\partial^2 u(x,t)}{\partial t^2} = -\frac{g}{a^2} \tag{1.3}$$

Function $u(x,t)$ should satisfy also to some initial conditions

$$u(x,0) = \varphi(x); \quad u_t(x,0) = \psi(x) \ , \tag{1.4}$$

and also to boundary conditions

$$u(0,t) = 0, \tag{1.5}$$

$$u_x(L,t) = \frac{m}{ES}(g + \varepsilon(t)r - u_{tt}(L,t)) \tag{1.6}$$

in mobile system of coordinates.

As above and below the point $x_k = l_0 + \nu(t)$ the solution of a boundary-value problem should satisfy to different conditions, we begin to represent this solution as

$$u\left(x,t\right) = \begin{cases} w\left(x,t\right), & l_0 + \nu(t) < x < l; \\ v\left(x,t\right), & 0 < x < l_0 + \nu(t). \end{cases} \tag{1.7}$$

It is necessary also that the solution of a boundary-value problem $u(x,t)$ satisfied to conditions of a continuity of displacements and deformations in a point of initial contact of a rope with a drum $x_k = l_0 + \nu(t)$. With the account of (1.7) such conditions will become:

$$v(l_0 + \nu(t), t) = w(l_0 + \nu(t), t); \quad v_x(l_0 + \nu(t), t) = w_x(l_0 + \nu(t), t) \tag{1.8}$$

It is necessary to take into account, that if during the initial moment of time the cargo hangs on a rope there in rope already exist initial displacements $u_s(x)$, created by the weight both of a cargo and a rope and determined by such formula

$$u_s\left(x\right) = \begin{cases} w_s\left(x\right), & l_0 < x < L; \\ v_s\left(x,\right), & 0 < x < l_0. \end{cases} \tag{1.9}$$

$$w_s(x) = \frac{G(x - l_0)}{ES} - \frac{\varrho_l(L-x)^2 g}{2ES} + \frac{\varrho_l(L-l_0)^2 g}{2ES} + \\ + \frac{G + \varrho_l(L-l_0)g}{ES}\frac{r}{\beta}\left(1 - e^{-\frac{\beta}{r}l_0}\right) ; \tag{1.10}$$

$$v_s(x) = \frac{G + \varrho_l(L-l_0)g}{ES}\frac{r}{\beta}\left(e^{\frac{\beta}{r}(x-l_0)} - e^{-\frac{\beta}{r}l_0}\right) . \tag{1.11}$$

Thus, elastic displacements should satisfy to initial conditions

$$u(x,0) = u_s(x) \; ; \quad u_t(x,0) = 0 \; . \tag{1.12}$$

Function $w_s(x,t)$ (1.10) is accepted as the stationary solution on an interval $l_0 + \nu(t) < x < L$. Function $v_s(x,t)$ (1.11) is accepted as the initial condition on an interval $0 < x < l_0 + \nu(t)$. Therefore on last interval initial conditions will be those:

$$v(x,0) = v_s(x) \; ; \quad v_t(x,0) = 0 \; , \quad 0 < x < l_0 \; . \tag{1.13}$$

Let's note that outside of interval $(0, l_0)$ function $v_s(x)$ identically equals to zero.

2. Construction of the solution of a problem

On the ends of the rope there arise the reflected waves only. On the dividing border there arise both the reflected and refracted waves.

The author obtained the solution of the telegraph equation (1.2) satisfying to initial conditions (1.13) as

$$v_{n1}(x,t) = \frac{1}{2}e^{\frac{\beta}{2r}at}v_s(x-at) + \frac{1}{2}e^{-\frac{\beta}{2r}at}v_s(x+at) + \\ + \frac{1}{2}\int_{x-at}^{x+at} cat\frac{J_1(z)}{z}e^{\frac{\beta}{2r}(x-\xi)}v_s(\xi)\,d\xi, \quad (2.1)$$

where

$$c = -\frac{\beta^2}{4r^2}\ ; \quad z = \sqrt{c\big[(\xi-x)^2 - a^2t^2\big]}\ . \quad (2.2)$$

Thus, the initial conditions (1.13) generate on an interval $0 < x < l_0 + \nu(t)$ waves (2.1). It is necessary, that these waves satisfied to a boundary condition (1.5) and to conditions (1.8) continuity of displacements and deformations in a point of initial contact of a rope with a drum $x_k = l_0 + \nu(t)$.

However function $v_{n1}(x,t)$ to any of the listed boundary conditions does not satisfy. Really, from (2.1) it is obtained

$$v_{n1}(0,t) = \frac{1}{2}e^{\frac{\beta}{2r}at}v_s(-at) + \frac{1}{2}e^{-\frac{\beta}{2r}at}v_s(at) + \\ + \frac{1}{2}\int_{-at}^{at} cat\frac{J_1(z)}{z}e^{-\frac{\beta}{2r}\xi}v_s(\xi)\,d\xi = \sigma(t), \quad (2.3)$$

where

$$z = \sqrt{c(\xi^2 - a^2t^2)}\ . \quad (2.4)$$

$$v_{n1}(l_0+\nu(t),t) = \frac{1}{2}e^{\frac{\beta}{2r}at}v_s(l_0+\nu(t)-at) + \frac{1}{2}e^{-\frac{\beta}{2r}at}v_s(l_0+\nu(t)+at) + \\ + \frac{1}{2}\int_{l_0+\nu(t)-at}^{l_0+\nu(t)+at} cat\frac{J_0'(z)}{z}e^{\frac{\beta}{2r}(l_0+\nu(t)-\xi)}v_s(\xi)\,d\xi, \quad (2.5)$$

where

$$z = \sqrt{c\big[(\xi-(l_0+\nu(t)))^2 - a^2t^2\big]}\ . \quad (2.6)$$

Hence, to satisfy to a boundary condition (1.5), it is necessary to introduce reflected from the fixed end of a rope $x = 0$ wave $v_{n20}(x,t)$ which should be the solution of the telegraph equation (1.2) and satisfy to a boundary condition

$$v_{n20}(0,t) = -\sigma(t)\ , \quad t > 0\ . \quad (2.7)$$

Such solution is obtained by the author in [18] and looks as

$$v_{n20}(x,t) = \Lambda_0(t-\frac{x}{a})e^{\frac{\beta}{2r}x} - ae^{\frac{\beta}{2r}x}\int_0^{t-\frac{x}{a}}\big[-\frac{\beta}{2r}J_0(z) - c\frac{x}{z}J_1(z)\big]\Lambda_0(\eta)\,d\eta\ . \quad (2.8)$$

Here

$$z = \sqrt{c[x^2 - a^2(t-\eta)^2]}\ . \tag{2.9}$$

For construction of the solution (2.8) in [18] continuation of function $-\sigma(t)$ on all axis t is executed as

$$\Lambda\,(t) = \begin{cases} -\,\sigma\,(t)\,, & t > 0; \\ 0, & t < 0. \end{cases} \tag{2.10}$$

Function Λ_0 is the solution to the integral equation of Volterra's type:

$$\Lambda_0(\tau) + a\int_0^\tau \frac{\beta}{2r} J_0(a(\tau-\eta)\sqrt{-c})\Lambda_0(\eta)\,d\eta = \Lambda(\tau). \tag{2.11}$$

Thus function (2.8) satisfies to zero initial conditions.

It is necessary to note, that function $v_{n20}(x,t)$ will not satisfy to conditions of continuity (1.8). However it is necessary to take into account, that at

$$t < \frac{l-\nu(t)}{a} \tag{2.12}$$

this function owing to (2.10) is equal to zero and consequently at values t, satisfying an inequality (2.12), it will not render influence on satisfaction to conditions (1.8).

At the same time function $v_{n1}(x,t)$ does not satisfy to conditions of a continuity of displacements and deformations at all $t > 0$. Differentiating equality (2.1) on x, we shall obtain

$$\frac{\partial v_{n1}(x,t)}{\partial x} = \frac{1}{2}e^{\frac{\beta}{2r}at}v_s'(x-at) + \frac{1}{2}e^{-\frac{\beta}{2r}at}v_s'(x+at) +$$
$$+\frac{1}{2}cat\Big\{\left.\frac{J_1(z)}{z}\right|_{\xi=x+at} e^{-\frac{\beta}{2r}at}v_s(x+at) - \left.\frac{J_1(z)}{z}\right|_{\xi=x-at} e^{\frac{\beta}{2r}at}v_s(x-at) +$$
$$+\int_{x-at}^{x+at}\Big[\frac{\partial}{\partial x}\Big(-\frac{J_0'(z)}{z}\Big) - \frac{\beta}{2r}\frac{J_0'(z)}{z}\Big]e^{\frac{\beta}{2r}(x-\xi)}v_s(\xi)\,d\xi\Big\}.$$

From (2.2) follows, that

$$z|_{\xi=x+at} = 0\ ; \quad z|_{\xi=x+at} = 0\ .$$

Therefore

$$J_0 = 1\ ; \quad \left.\frac{J_1(z)}{z}\right|_{\xi=x+at} = \left.\frac{J_1(z)}{z}\right|_{z=0} = \frac{1}{2}\ , \quad \left.\frac{J_1(z)}{z}\right|_{\xi=x-at} = \left.\frac{J_1(z)}{z}\right|_{z=0} = \frac{1}{2}\ ,$$

Besides

$$\frac{\partial}{\partial x}\Big(-\frac{J_0'(z)}{z}\Big) = \Big(-\frac{J"_0(z)}{z} + \frac{J_0'(z)}{z^2}\Big)\frac{\partial z}{\partial x} = -\frac{c(\xi-x)}{z}\Big(-\frac{J"_0(z)}{z} + \frac{J_0'(z)}{z^2}\Big)\ .$$

Hence,

$$\frac{\partial v_{n1}(x,t)}{\partial x}=\frac{1}{2}e^{\frac{\beta}{2r}at}v_s'(x-at)+\frac{1}{2}e^{-\frac{\beta}{2r}at}v_s'(x+at)+ \\ +\frac{1}{2}cat\Big\{\frac{1}{2}e^{-\frac{\beta}{2r}at}v_s(x+at)-\frac{1}{2}e^{\frac{\beta}{2r}at}v_s(x-at)- \\ -\int_{x-at}^{x+at}\big[c(\xi-x)(-\frac{J"_0(z)}{z^2}+\frac{J_0'(z)}{z^3})-\frac{\beta}{2r}\frac{J_0'(z)}{z}\big]e^{\frac{\beta}{2r}(x-\xi)}v_s(\xi)\,d\xi\Big\}. \quad (2.13)$$

From (2.13) it is obtained

$$v_{n1,x}(l_0+\nu(t),t)=\frac{1}{2}e^{\frac{\beta}{2r}at}v_s'(l_0+\nu(t)-at)+\frac{1}{2}e^{-\frac{\beta}{2r}at}v_s'(l_0+\nu(t)+at)+ \\ +\frac{1}{2}cat\Big\{\frac{1}{2}e^{-\frac{\beta}{2r}at}v_s(l_0+\nu(t)+at)-\frac{1}{2}e^{\frac{\beta}{2r}at}v_s(l_0+\nu(t)-at)- \\ -\int_{x-at}^{x+at}\big[c(\xi-(l_0+\nu(t)))(-\frac{J"_0(z)}{z^2}+\frac{J_0'(z)}{z^3})- \\ -\frac{\beta}{2r}\frac{J_0'(z)}{z}\big]e^{\frac{\beta}{2r}(l_0+\nu(t)-\xi)}v_s(\xi)\,d\xi\Big\}. \quad (2.14)$$

where

$$z=\sqrt{c\big[(\xi-(l_0+\nu(t)))^2-a^2t^2\big]}\ . \quad (2.15)$$

Functions $v_{n1}(x,t)$ and $w_s(x)$ everyone generates in a point $x_k = l_0+\nu(t)$ the reflected and refracted waves. We shall designate as $v_{n10}(x,t)$ a wave being superposition of a wave, arising as a result of reflection from a point $x_k = l_0+\nu(t)$ waves $v_{n1}(x,t)$, and the refracted wave arising owing to falling in a point $x_k = l_0+\nu(t)$ of a wave $w_s(x)$. We shall designate as well as $w_{n10}(x,t)$ a wave being superposition of a wave, arising as a result of reflection from a point $x_k = l_0+\nu(t)$ waves $v_{n1}(x,t)$, and the refracted wave arising owing to falling in a point $x_k = l_0+\nu(t)$ of a wave $w_s(x)$. We shall designate unknown while values of these functions in a point $x_k = l_0+\nu(t)$ as:

$$v_{n10}(l_0+\nu(t),t)=\mu(t) \quad w_{n10}(l_0+\nu(t),t)=\theta(t)\ . \quad (2.16)$$

Let's note, that functions $\mu(t)$ and $\theta(t)$ are determined only at t > 0. For the further we shall continue these functions on all axis t as

$$M\,(t)=\begin{cases}\mu\,(t)\,, & t>0;\\ 0, & t<0.\end{cases}\ ;\quad \Theta\,(t)=\begin{cases}\theta\,(t)\,, & t>0;\\ 0, & t<0.\end{cases}\ . \quad (2.17)$$

Then on the basis [20] function $v_{n10}(x,t)$ as the wave radiated by function $M(t)$ in a point $x_k = l_0+\nu(t)$, is under construction as

$$v_{n10}(x,t)=2M_0(t+\frac{x}{a})e^{\frac{\beta}{2r}x}- \\ -2ae^{\frac{\beta}{2r}x}\int_0^{t+\frac{x}{a}}\big[\frac{\beta}{2r}J_0(z)+c\frac{x}{z}J_1(z)\big]M_0(\eta)\,d\eta\ , \quad (2.18)$$

where z looks like (2.9), and function $M_0(t)$ is the solution to the following integral equation:

$$2M_0(t+\frac{l_0+\nu(t)}{a})e^{\frac{\beta}{2r}(l_0+\nu(t))}- \\ -2ae^{\frac{\beta}{2r}(l_0+\nu(t))}\int_0^{t+\frac{l_0+\nu(t)}{a}}\big[\frac{\beta}{2r}J_0(z)+c\frac{l_0+\nu(t)}{z}J_1(z)\big]M_0(\eta)\,d\eta=M(t)\ . \tag{2.19}$$

In turn, function $w_{n10}(x,t)$ as a wave radiated by function $\Theta(t)$ in a point $x_k=l_0+\nu(t)$, on the basis [15] is under construction as

$$w_{n10}(x,t)=\chi(x-at)\ . \tag{2.20}$$

Having substituted of the form of the solution (2.20) in continued on all axis t the second equality (2.16) we obtain

$$\chi(l_0+\nu(t)-at)=\Theta(t)\ . \tag{2.21}$$

Let's introduce into (2.21) transformation

$$\zeta=l_0+\nu(t)-at\ . \tag{2.22}$$

It is natural to assume, that reeling a rope on a drum is carried out with subsonic speed. It means, that at all t will be valid inequality

$$|\nu'(t)|<a\ . \tag{2.23}$$

On the basis of an inequality (2.23) function in the right part of equality (2.22) will be strictly monotonously decreasing. Therefore for equality (2.22) there will be an inverse function $t_0(\zeta)$, also strictly monotonously decreasing. As $\nu(0)=0$, from (2.22) follows, that $\zeta(0)=l_0$ and at $t>0$ by virtue of an inequality (2.23) $\zeta<l_0$. Thus, continued on all axis ζ inverse function $t_0(\zeta)$ will possess the following properties:

$$t_0\left(\zeta\right)=\begin{cases}>0, & \zeta<l_0;\\ =0, & \zeta=l_0;\\ <0, & \zeta>l_0.\end{cases} \tag{2.24}$$

Let's note, that from (2.24) follows, that at $\zeta<l_0$ such identity is valid

$$t_0(l_0+\nu(t)-at)=t\ . \tag{2.25}$$

Now from (2.20) and (2.21) follows

$$w_{n10}(x,t)=\Theta(t_0(x-at))\ . \tag{2.26}$$

With the purpose of substitution in conditions of a continuity (1.8) we shall differentiate functions (1.10), (2.18) and (2.26) on x. We shall obtain

$$w_{s,x}(x) = \frac{G}{ES} + \frac{\varrho_l(L-x)g}{ES} \ . \tag{2.27}$$

$$w_{n10,x}(x,t) = \Theta'(t_0(x-at))t_0'(x-at) \ . \tag{2.28}$$

$$\frac{\partial v_{n10}(x,t)}{\partial x} = 2e^{\frac{\beta}{2r}x}\Big\{\frac{\beta}{2r}\Big[M_0(t+\frac{x}{a}) - a\int_0^{t+\frac{x}{a}}\big[\frac{\beta}{2r}J_0(z) + c\frac{x}{z}J_1(z)\big]M_0(\eta)\,d\eta\Big] +$$
$$+\frac{1}{a}M_0'(t+\frac{x}{a}) - \big[\frac{\beta}{2r}J_0(z) + c\frac{x}{z}J_1(z)\big]\Big|_{\eta=t+\frac{x}{a}} M_0(t+\frac{x}{a}) -$$
$$- a\int_0^{t+\frac{x}{a}}\big[\frac{\beta}{2r}J_0'(z)\frac{\partial z}{\partial x} - c\frac{J_0'(z)}{z} + cx\frac{\partial}{\partial x}(-\frac{J_0'(z)}{z})\big]M_0(\eta)\,d\eta\Big\} \ .$$

Let's take into account, that in last equality as follows from (2.9),

$$z|_{\eta=t+\frac{x}{a}} = 0 \ ,$$

Therefore

$$J_0(z)|_{\eta=t+\frac{x}{a}} = J_0(z)|_{z=0} = 0 \ , \ \frac{J_1(z)}{z}\Big|_{\eta=t+\frac{x}{a}} = \frac{J_1(z)}{z}\Big|_{z=0} = \frac{1}{2} \ ,$$

Besides

$$\frac{\partial z}{\partial x} = \frac{cx}{z} \ ; \quad \frac{\partial}{\partial x}\big(-\frac{J_0'(z)}{z}\big) = \big(-\frac{J"_0(z)}{z} + \frac{J_0'(z)}{z^2}\big)\frac{\partial z}{\partial x} = \frac{cx}{z}\big(-\frac{J"_0(z)}{z} + \frac{J_0'(z)}{z^2}\big) \ .$$

Hence,

$$\frac{\partial v_{n10}(x,t)}{\partial x} = 2e^{\frac{\beta}{2r}x}\Big\{-c\frac{x}{2}M_0(t+\frac{x}{a}) + \frac{1}{a}M_0'(t+\frac{x}{a}) -$$
$$- a\int_0^{t+\frac{x}{a}}\big[(\frac{\beta}{2r})^2J_0(z) - c\frac{J_0'(z)}{z} + c^2\frac{x^z}{z}(-\frac{J"_0(z)}{z} + \frac{J_0'(z)}{z^2})\big]M_0(\eta)\,d\eta\Big\} \ . \tag{2.29}$$

Let's note, that at the moments of time $t > 0$, enough close to the moment of time $t = 0$, in a point of contact $x_k = l_0 + \nu(t)$ there are only four waves. Above point $x_k = l_0 + \nu(t)$ are exist waves $v_{n1}(x,t)$, generated by initial conditions, and reflected from a point x_k a wave $v_n10(x,t)$. Below a point x_k the static solution $w_s(x)$ and the refracted wave $w_{n10}(x,t)$ are exist. Hence, for such t conditions of a continuity of displacements and deformations (1.8) will become

$$v_{n1}(l_0+\nu(t),t) + v_{n10}(l_0+\nu(t),t) = w_s(l_0+\nu(t),t) + w_{n10}(l_0+\nu(t),t);$$
$$v_{n1,x}(l_0+\nu(t),t) + v_{n10,x}(l_0+\nu(t),t) =$$
$$= w_{s,x}(l_0+\nu(t),t) + w_{n10,x}(l_0+\nu(t),t). \tag{2.30}$$

Having substituted in conditions (2.30) calculated values of functions v_{n10} and w_{n10}, and also their derivatives from (2.18), (2.26), (2.28) and (2.29), we shall obtain system of two equations for definition of functions $M_0(t)$ and $\Theta(t)$:

$$v_{n1}(l_0+\nu(t),t)+2M_0(t+\frac{l_0+\nu(t)}{a})e^{\frac{\beta}{2r}(l_0+\nu(t))}-$$
$$-2ae^{\frac{\beta}{2r}(l_0+\nu(t))}\int_0^{t+\frac{l_0+\nu(t)}{a}}\big[\frac{\beta}{2r}J_0(z)+c\frac{l_0+\nu(t)}{z}J_1(z)\big]M_0(\eta)\,d\eta=$$
$$=w_s(l_0+\nu(t))+\Theta(t_0(l_0+\nu(t)-at))\ ;$$
$$v_{n1,x}(l_0+\nu(t),t)+2e^{\frac{\beta}{2r}(l_0+\nu(t))}\Big\{-\frac{c(l_0+\nu(t))}{2}M_0(t+\frac{l_0+\nu(t)}{a})+$$
$$+\frac{1}{a}M_0'(t+\frac{l_0+\nu(t)}{a})-a\int_0^{t+\frac{l_0+\nu(t)}{a}}\big[(\frac{\beta}{2r})^2J_0(z)-c\frac{J_0'(z)}{z}+$$
$$+c^2\frac{(l_0+\nu(t))^2}{z}(-\frac{J"_0(z)}{z}+\frac{J_0'(z)}{z^2})\big]M_0(\eta)\,d\eta\Big\}=$$
$$=\frac{G}{ES}+\frac{\varrho_l(L-(l_0+\nu(t)))g}{ES}+\Theta'(t_0(l_0+\nu(t)-at))t_0'(l_0+\nu(t)-at)\ . \quad (2.31)$$

In system of the equations (2.31)

$$z=\sqrt{c[(l_0+\nu(t))^2-a^2(t-\eta)^2]}\ . \quad (2.32)$$

Besides, as at $t>0$ is valid $l_0+\nu(t)-at<0$, on the basis of identity (2.25) in these equations such equalities are valid

$$\Theta(t_0(l_0+\nu(t)-at))=\Theta(t)\ ;\quad \Theta'(t_0(l_0+\nu(t)-at))t_0'(l_0+\nu(t)-at)=\Theta'(t)\ . \quad (2.33)$$

Values of functions $v_{n1}(l_0+\nu(t),t)$ and $v_{n1,x}(l_0+\nu(t),t)$ in the equations (2.31) it is necessary to substitute from (2.5) and (2.14) accordingly.

Having expressed $\Theta(t)$ from the first equation (2.31) and having substituted this value in the second equation (2.31), we shall obtain the integro-differential equation with unknown function $M_0(t)$. After solution of this equation function $\Theta(t)$ will easily be determined from the first equation (2.31).

The boundary condition (1.6) in a point $x=L$ will generate also a wave which we shall designate as $w_{n2}(x,t)$. At the some values of $t>0$, close to value $t=0$, displacements in vicinities of a point $x=L$ will be determined by the sum of functions

$$w_{n2}(x,t)+w_s(x)\ . \quad (2.34)$$

Function $u_s(x)$ (1.9) in a point $x=L$, obviously, satisfies to a boundary condition

$$u_{s,x}(L,t)=\frac{m}{ES}(g-u_{s,tt}(L,t)) \quad (2.35)$$

and at $t=0$ conditions of a continuity of displacements and deformations in a point $x=l_0$:

$$v_s(l_0)=w_s(l_0)\ ;\quad v_{s,x}(l_0)=w_{s,x}(l_0)\ . \quad (2.36)$$

But as function $w_s(x)$ satisfies to a boundary condition (2.35) that the sum of functions (2.34) satisfied to a boundary condition (1.5), it is necessary that function $w_{n2}(x,t)$ satisfied to a boundary condition

$$u_{n,x}(L,t) = \frac{m}{ES}(\varepsilon(t)r - u_{n,tt}(L,t)) \ . \qquad (2.37)$$

Taking into account, that under conditions of statement of a problem $\varepsilon(t) = 0$ at $t < 0$, we shall write down a boundary condition (2.37) as

$$u_{n,x}(L,t) = \frac{m}{ES}(H(t)\varepsilon(t)r - u_{n,tt}(L,t)) \ , \qquad (2.38)$$

where $H(t)$ - function of Heaviside.

If to search for function $w_{n2}(x,t)$ as the solution of the homogeneous wave equation corresponding (1.3), which satisfies to a boundary condition (2.38) as

$$w_{n2}(x,t) = \chi(x + at - L), \qquad (2.39)$$

where function χ is accepted equal to zero at negative values of argument. Substitution of the form of the solution (2.39) in a boundary condition (2.38) results in the equation for function χ:

$$\chi"(\tau) + \frac{ES}{ma^2}\chi'(\tau) = \frac{1}{a^2}\varepsilon(\frac{\tau}{a})rH(\frac{\tau}{a}) \ . \qquad (2.40)$$

The general solution to the equation (2.40) will be function

$$\chi(\tau) = \int_0^{\tau} \left[C + \frac{1}{a^2}\int_0^{\xi} e^{\frac{ES}{ma^2}\zeta}\varepsilon(\frac{\zeta}{a})rH(\frac{\zeta}{a})\, d\zeta\right] e^{-\frac{ES}{ma^2}\xi}\, d\xi + \chi(0) \ . \qquad (2.41)$$

If in the formula (2.41) with the purpose of maintenance of a continuity of function $\chi(\tau)$ in a point $\tau = 0$ to accept $\chi(0) = 0, \quad \chi'(0) = C = 0$, function (2.39) will become

$$w_{n2}(x,t) = \frac{1}{a^2}\int_0^{x+at-L} e^{-\frac{ES}{ma^2}\xi} \int_0^{\xi} e^{\frac{ES}{ma^2}\zeta}\varepsilon(\frac{\zeta}{a})rH(\frac{\zeta}{a})\, d\zeta]\, d\xi \ . \qquad (2.42)$$

Thus, it is established, that function $u(x,t)$, having structure (1.7), in which

$$v(x,t) = v_{n1}(x,t) + v_{n10}(x,t) + v_{n20}(x,t) \qquad (2.43)$$

$$w(x,t) = w_s(x) + w_{n10}(x,t) + w_{n2}(x,t) \ , \qquad (2.44)$$

at initial values of $t > 0$ will satisfy to all conditions of statement of a considered boundary-value problem, that is will be its solution. Thus it is important to note, that all components of functions (2.43), except for $w_s(x)$ and $v_{n10}(x,t)$, satisfy to zero initial conditions. In formulas (2.43) function $v_{n1}(x,t)$ fills in all area $0 < x < l_0 + \nu(t)$, and function $w_s(x)$ - all area $l_0 + \nu(t) < x < L$. Other

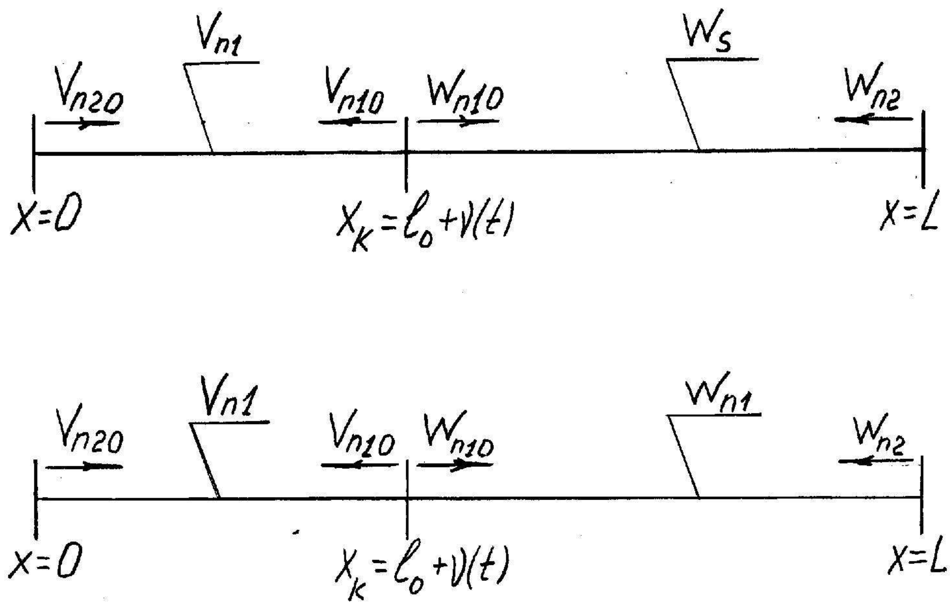

Figure 2. Primary reflected and refracted waves

components of formulas (2.43) are propagating waves. Position and directions of propagations of these primary waves are depicted in Figure 2.

Until any of extending waves will not reach the opposite end of a rope or up to a point x_k, function $u(x,t)$ with components (2.43) will be the solution of a boundary-value problem. However, as soon as one of waves will reach one of the listed critical points, such function will cease to be the solution of a boundary-value problem owing to infringement of a boundary condition in a critical point. Therefore to obtain the solution of a problem at the bigger values of t, it is necessary in points $x = 0$ and $x = L$ to build the reflected waves, and in a point $x = x_k$ - the reflected and refracted waves.

3. Construction of the reflected and refracted waves

At $l_0 < L$ the first up to a critical point $x = 0$ the forward front of a wave v_{n10} will reach. It will take place during an interval of time in length

$$\tau_1 = \frac{l_0}{a} \ . \tag{3.1}$$

At $t > \tau_1$ boundary condition (1.5) will not be satisfied. For satisfaction it is necessary for this condition to build a wave v_{n10b}, as the result of reflection of a wave v_{n10} from the end $x = 0$. Construction of such reflected wave is executed in [20] where it is shown, that

$$v_{n10b}(x,t) = 2e^{\frac{\beta}{2r}x}\big[M_1(t-\frac{x}{a}) + M_0(t-\frac{x}{a})\big] + \\ + 2ae^{\frac{\beta}{2r}x}\int_0^{t-\frac{x}{a}} \big[\frac{\beta}{2r}J_0(z) + c\frac{x}{z}J_1(z)\big][M_1(\eta) + M_0(\eta)]\,d\eta\ . \quad (3.2)$$

Here function M_1 is the solution of the integral equation [20]

$$-M_1(t) - a\int_0^t \frac{\beta}{2r}J_0(z)M_1(\eta)\,d\eta = 2M_0(t)\ . \quad (3.3)$$

in which [20]

$$M_0(t) = -v_{n1}(0,t)\ ; \quad z = a(t-\eta)\sqrt{-c}\ . \quad (3.4)$$

In turn, as the rope is reeled up on a drum with subsonic speed, the forward front of a wave v_{n10b} will reach a point of contact $x_k = l_0 + \nu(t)$ at the moment of time τ_{11} which is the least positive root of the equation

$$at = 2l_0 + \nu(t)\ . \quad (3.5)$$

Therefore at $t > \tau_{11}$ for a wave v_{n10b} boundary conditions (1.8) in a point $x_k = l_0 + \nu(t)$ will cease to satisfy. With the purpose of satisfaction at $t > \tau_{11}$ it is necessary for these boundary conditions to construct reflected v_{n10bb} and refracted w_{n10br} the waves generated by a wave v_{n10b}.

It is made by the technique enough similar to construction of reflected and refracted waves v_{n10} and w_{n10}. With this purpose we shall designate unknown while values of these functions in a point $x_k = l_0 + \nu(t)$:

$$v_{n10bb}(l_0 + \nu(t), t) = \mu_{v1}(t)\ ;\ w_{n10br}(l_0 + \nu(t), t) = \theta_{v1}(t)\ . \quad (3.6)$$

Let's note, that functions $\mu_{v1}(t)$ and $\theta_{v1}(t)$ are determined only at $t > \tau_{11}$. For the further we shall continue these functions on all axis t as

$$M_{v1}(t) = \begin{cases} \mu_{v1}(t), & t > \tau_{11}; \\ 0, & t < \tau_{11}. \end{cases} ; \quad \Theta_{v1}(t) = \begin{cases} \theta_{v1}(t), & t > \tau_{11}; \\ 0, & t < \tau_{11}. \end{cases} . \quad (3.7)$$

Then on the basis [20] function $v_{n10bb}(x,t)$ as the wave radiated by function $M_{v1}(t)$ in a point $x_k = l_0 + \nu(t)$, is under construction as

$$v_{n10bb}0(x,t) = 2M_{v10}(t+\frac{x}{a})e^{\frac{\beta}{2r}x} - 2ae^{\frac{\beta}{2r}x}\int_0^{t+\frac{x}{a}} \big[\frac{\beta}{2r}J_0(z) + c\frac{x}{z}J_1(z)\big]M_{v10}(\eta)\,d\eta\ . \quad (3.8)$$

where z looks like (2.9), and function $Mv10(t)$ is the solution to the following integral equation:

$$2M_{v10}(t+\frac{l_0+\nu(t)}{a})e^{\frac{\beta}{2r}(l_0+\nu(t))} - 2ae^{\frac{\beta}{2r}(l_0+\nu(t))}\int_0^{t+\frac{l_0+\nu(t)}{a}} \big[\frac{\beta}{2r}J_0(z) + \\ + c\frac{l_0+\nu(t)}{z}J_1(z)\big]M_{v10}(\eta)\,d\eta = M_{v1}(t). \quad (3.9)$$

In turn, function $w_{n10br}(x,t)$ as a wave radiated by function $\Theta_{v1}(t)$ in a point $x_k = l_0 + \nu(t)$, on the basis [15] is under construction as

$$w_{n10br}(x,t) = \chi(x - at). \tag{3.10}$$

Then the same as and for function w_{n10} it is obtained

$$w_{n10br}(x,t) = \Theta_{v1}(t_0(x - at)) \ . \tag{3.11}$$

It is necessary to take into account, that the wave (3.11) operates in domain $x > l_0 + \nu(t)$. In such domain the inequality $x - at < l_0 + \nu(t) - at < l_0$ is valid as function (2.22) decreases, and its initial value is equal to l_0. Therefore on the basis of identity (2.25) from the formula (3.11) follows

$$w_{n10br}(l_0 + \nu(t), t) = \Theta_{v1}(t). \tag{3.12}$$

At $t > \tau_{11}$ function $u(xt)$, having structure (1.7), will consist of components

$$\begin{aligned} v(x,t) &= v_{n1}(x,t) + v_{n10}(x,t) + v_{n20}(x,t) + v_{n10b}(x,t) + v_{n10bb}(x,t) \\ w(x,t) &= w_s(x) + w_{n10}(x,t) + w_{n2}(x,t) + w_{n10br}(x,t) \ . \end{aligned} \tag{3.13}$$

In view of that boundary conditions (2.30) are executed, boundary conditions (1.8) will become

$$\begin{aligned} & v_{n10b}(l_0 + \nu(t), t) + v_{n10bb}(l_0 + \nu(t), t) = w_{n10br}(l_0 + \nu(t), t); \\ & v_{n10b,x}(l_0 + \nu(t), t) + v_{n10bb,x}(l_0 + \nu(t), t) = w_{n10br,x}(l_0 + \nu(t), t) \ . \end{aligned} \tag{3.14}$$

Substituting in (3.14) values of functions v_{n10b}, v_{n10bb} and w_{n10br} from (3.2), (3.8) and (3.12), we shall obtain the equations for definition of functions M_{v10} and Θ_{v1}:

$$\begin{aligned} & v_{n10b}(l_0 + \nu(t), t) + 2M_{v10}(t + \frac{l_0 + \nu(t)}{a}) e^{\frac{\beta}{2r}(l_0 + \nu(t))} - \\ & - 2ae^{\frac{\beta}{2r}(l_0+\nu(t))} \int_0^{t + \frac{l_0 + \nu(t)}{a}} [\frac{\beta}{2r} J_0(z) + c\frac{l_0 + \nu(t)}{z} J_1(z)] M_{v10}(\eta)\, d\eta = \Theta_{v1}(t) \ ; \\ & v_{n10b,x}(l_0 + \nu(t), t) + 2e^{\frac{\beta}{2r}(l_0+\nu(t))} \Big\{ - \frac{c(l_0 + \nu(t))}{2} M_{v10}(t + \frac{l_0 + \nu(t)}{a}) + \\ & + \frac{1}{a} M'_{v10}(t + \frac{l_0 + \nu(t)}{a}) - a \int_0^{t + \frac{l_0 + \nu(t)}{a}} [\left(\frac{\beta}{2r}\right)^2 J_0(z) - c\frac{J'_0(z)}{z} + \\ & + c^2 \frac{(l_0 + \nu(t))^2}{z} (- \frac{J"_0(z)}{z} + \frac{J'_0(z)}{z^2})] M_{v10}(\eta)\, d\eta \Big\} = \Theta'_{v1}(t) \ . \end{aligned} \tag{3.15}$$

Here z it is determined by equality (2.32).

Further the forward front of a wave v_{n10bb} at the moment of time $\tau_{12} = \frac{3l_0 + 2\nu(\tau_1)}{a}$ will reach a point $x = 0$, and for satisfaction to boundary condition (1.5) it will be necessary to construct a wave reflected from this point. To make this it will be possible precisely the same as it is made at reflection of a wave v_{n10} from a point $x = 0$.

The forward front of a wave w_{n10br}, in turn, at the moment of time $\tau_{13} = \tau_{11} + \frac{L-(l_0+2\nu(\tau_{11}))}{a}$ will reach a point $x = L$, and for satisfaction to boundary condition (2.37) it will be necessary to construct a wave reflected from this point. To make this it will be possible precisely the same as it will be made below at reflection of a wave w_{n20r} from a point $x = L$. Process of construction of the reflected and refracted waves generated by a wave v_{n10}, it is necessary to continue up to a stop of system during some moment of time $t = T_k$.

Let's consider now process of movement of a wave v_{n20}. At the moment of time τ_2 which is the least positive root of the equation

$$at = l_0 + \nu(t) , \tag{3.16}$$

the forward front of a wave will catch up a point $x_k = l_0+\nu(t)$, and for satisfaction to boundary condition (1.8) it is necessary to build reflected v_{n20b} and refracted w_{n20r} waves. Boundary conditions (1.8) for three considered waves will become

$$\begin{aligned} v_{n20}(l_0 + \nu(t), t) + v_{n20b}(l_0 + \nu(t), t) = w_{n20r}(l_0 + \nu(t), t); \\ v_{n20,x}(l_0 + \nu(t), t) + v_{n20b,x}(l_0 + \nu(t), t) = w_{n20r,x}(l_0 + \nu(t), t). \end{aligned} \tag{3.17}$$

Construction of waves v_{n20b} and w_{n20r} is carried out by the technique enough similar to a technique of construction of reflected and refracted waves v_{n10} and w_{n10}. With this purpose we shall designate unknown while values of functions v_{n20b} and w_{n20r} in a point $x_k = l_0 + \nu(t)$ as

$$v_{n20b}(l_0 + \nu(t), t) = \mu_{v2}(t) \ ; \ w_{n20r}(l_0 + \nu(t), t) = \theta_{v2}(t) \ . \tag{3.18}$$

Again functions $\mu_{v2}(t)$ and $\theta_{v2}(t)$ are determined only at $t > \tau_2$. For the further we shall continue these functions on all axis t as

$$M_{v2}(t) = \begin{cases} \mu_{v2}(t), & t > \tau_2; \\ 0, & t < \tau_2. \end{cases} ; \quad \Theta_{v2}(t) = \begin{cases} \theta_{v2}(t), & t > \tau_2; \\ 0, & t < \tau_2. \end{cases} . \tag{3.19}$$

Then on the basis [20] function $v_{n20b}(x, t)$ as the wave radiated by function $M_{v2}(t)$ in a point $x_k = l_0 + \nu(t)$, is under construction as

$$v_{n20b}(x, t) = 2M_{v20}(t+\frac{x}{a})-2ae^{\frac{\beta}{2r}x} \int_0^{t+\frac{x}{a}} [\frac{\beta}{2r}J_0(z)+c\frac{x}{z}J_1(z)] M_{v20}(\eta)\, d\eta \ . \tag{3.20}$$

where z looks like (2.9), and function $M_{v20}(t)$ is the solution to the following integral equation:

$$\begin{aligned} 2M_{v20}(t + \frac{l_0 + \nu(t)}{a})e^{\frac{\beta}{2r}(l_0+\nu(t))} - 2ae^{\frac{\beta}{2r}(l_0+\nu(t))} \int_0^{t+\frac{l_0+\nu(t)}{a}} [\frac{\beta}{2r}J_0(z)+ \\ + c\frac{l_0 + \nu(t)}{z}J_1(z)] M_{v20}(\eta)\, d\eta = M_{v2}(t). \end{aligned} \tag{3.21}$$

In turn, function $w_{n20r}(x,t)$ as a wave radiated by function $\Theta_{v2}(t)$ in a point $x_k = l_0 + \nu(t)$, it is similar (3.11) turns out as

$$w_{n20r}(x,t) = \Theta_{v2}(t_0(x - at)). \tag{3.22}$$

Substituting in (3.17) values of functions v_{n20}, v_{n20b} and w_{n20r} from (2.8), (3.20) and (3.22), we shall obtain the equations for definition of functions M_{v20} and Θ_{v2}:

$$v_{n20}(l_0 + \nu(t), t) + 2M_{v20}(t + \frac{l_0 + \nu(t)}{a})e^{\frac{\beta}{2r}(l_0+\nu(t))} -$$
$$- 2ae^{\frac{\beta}{2r}(l_0+\nu(t))} \int_0^{t+\frac{l_0+\nu(t)}{a}} [\frac{\beta}{2r} J_0(z) + c\frac{l_0 + \nu(t)}{z} J_1(z)] M_{v20}(\eta)\, d\eta = \Theta_{v2}(t) \ ;$$
$$v_{n20,x}(l_0 + \nu(t), t) + 2e^{\frac{\beta}{2r}(l_0+\nu(t))}\Big\{ - \frac{c(l_0 + \nu(t))}{2} M_{v20}(t + \frac{l_0 + \nu(t)}{a}) +$$
$$+ \frac{1}{a} M'_{v20}(t + \frac{l_0 + \nu(t)}{a}) - a\int_0^{t+\frac{l_0+\nu(t)}{a}} [\left(\frac{\beta}{2r}\right)^2 J_0(z) - c\frac{J'_0(z)}{z} +$$
$$+ c^2 \frac{(l_0 + \nu(t))^2}{z}\big(- \frac{J"_0(z)}{z} + \frac{J'_0(z)}{z^2})] M_{v20}(\eta)\, d\eta\Big\} = \Theta'_{v2}(t) \ . \tag{3.23}$$

Here still z it is determined by equality (2.32).

Further it is necessary at $t > \tau_{21} = \frac{l_0+\nu(\tau_2)}{a}$ to construct a wave v_{n20bb} as result of reflection of a wave v_{n20b} from the end $x = 0$. To make that it is possible precisely the same as it is made at construction of reflection of a wave v_{n10} in this point.

The forward front of a wave w_{n20r} will reach a point $x = L$ at the moment of time $\tau_{22} = \tau_2 + \frac{L-(l_0+\nu(\tau_2))}{a}$. At $t > \tau_{22}$, it is necessary to build the wave w_{n20rb} reflected from this point and generated by a wave w_{n20r}. Construction of the reflected wave w_{n20rb} is carried out by a technique of construction of the reflected wave generated by the falling wave w_{n10} stated above.

The forward front of a wave w_{n10} will reach a point $x = L$ at the moment of time $\tau_3 = \frac{L-l_0}{a}$. Therefore at $t > \tau_3$ it is necessary to build the wave w_{n10b} reflected from this end. At construction of a wave it is necessary to take into account, that at $t < \tau_3$ in a vicinity of a point $x = L$ the solution of a problem is represented as

$$u(x,t) = w_s(x) + w_{n2}(x,t) + w_{n10}(x,t) \ , \tag{3.24}$$

and at $t > \tau_3$ it will be already represented as four distinct from zero of waves:

$$u(x,t) = w_s(x) + w_{n2}(x,t) + w_{n10}(x,t) + w_{n10b}(x,t) \ . \tag{3.25}$$

Hence, in view of that function $w_s(x)$ satisfies to a boundary condition

$$w_{s,x}(L) = \frac{mg}{ES} \ ,$$

and function $w_{n2}(x,t)$ - to a boundary condition (50), at substitution of function (3.25) in a boundary condition (1.6) it is obtained, that the sum of functions $w_{n10}(x,t)$ and $w_{n10b}(x,t)$ should satisfy in a point $x = L$ to a boundary condition

$$w_{n10,x}(L,t) + w_{n10b,x}(L,t) = -\frac{m}{ES}(w_{n10,tt}(L,t) + w_{n10b,tt}(L,t)) \ . \qquad (3.26)$$

The reflected wave $w_{n10b}(x,t)$ is under construction as

$$w_{n10b}(x,t) = \chi_1(x - at). \qquad (3.27)$$

Substitution of the form of the solution (3.27) in a boundary condition (3.26) allows obtaining differential equation for determination of function χ_1:

$$a^2\chi"_1(L+at) + \frac{ES}{m}\chi_1'(L+at) = -\frac{ES}{m}w_{n10,x}(L,t) - w_{n10,tt}(L,t) \ ,$$

which after introduction of transformation of an independent variable

$$\tau = L + at \qquad (3.28)$$

will become

$$\chi"_1(\tau) + \frac{ES}{ma^2}\chi_1'(\tau) = -\frac{ES}{ma^2}w_{n10,x}(L, \frac{\tau - L}{a}) - \frac{1}{a^2}w_{n10,tt}(L, \frac{\tau - L}{a}) \ . \qquad (3.29)$$

The solution of the equation (3.29) is obtained in [19] and for this case looks as

$$w_{n10b}(x,t) = \chi_1(x+at) = -\int_0^{x+at} e^{-\frac{ES}{ma^2}\xi}\Big[\int_0^{\xi} e^{\frac{ES}{ma^2}\zeta}\times$$
$$\times\big[\frac{1}{a^2}w_{n10,tt}(L, \frac{\zeta - L}{a}) + \frac{ES}{ma^2}w_{n10,x}(L, \frac{\zeta - L}{a})\big]\, d\zeta\Big]\, d\xi \ . \qquad (3.30)$$

Moving to a direction negative x, forward front of a wave $w_{n10b}(x,t)$ at the moment of time $t = \tau_{31}$ which is a root of the equation

$$at = a\tau_3 + L - (l_0 + \nu(t)) \ , \qquad (3.31)$$

will reach a point $x_k = l_0 + \nu(t)$. Therefore at $t > \tau_{31}$ there is a necessity of construction reflected $w_{n10bb}(x,t)$ and refracted $v_{n10br}(x,t)$ waves, satisfying boundary conditions

$$v_{n10br}(l_0 + \nu(t), t) = w_{n10b}(l_0 + \nu(t), t) + w_{n10bb}(l_0 + \nu(t), t) \ ;$$
$$v_{n10br,x}(l_0 + \nu(t), t) = w_{n10b,x}(l_0 + \nu(t), t) + w_{n10bb,x}(l_0 + \nu(t), t) \ , \qquad (3.32)$$

Process of construction of waves $w_{n10bb}(x,t)$ and $v_{n10br}(x,t)$ is similar to process of construction stated above reflected $w_{n2b}(x,t)$ and refracted $v_{n2r}(x,t)$ the waves generated by the falling wave $w_{n2}(x,t)$.

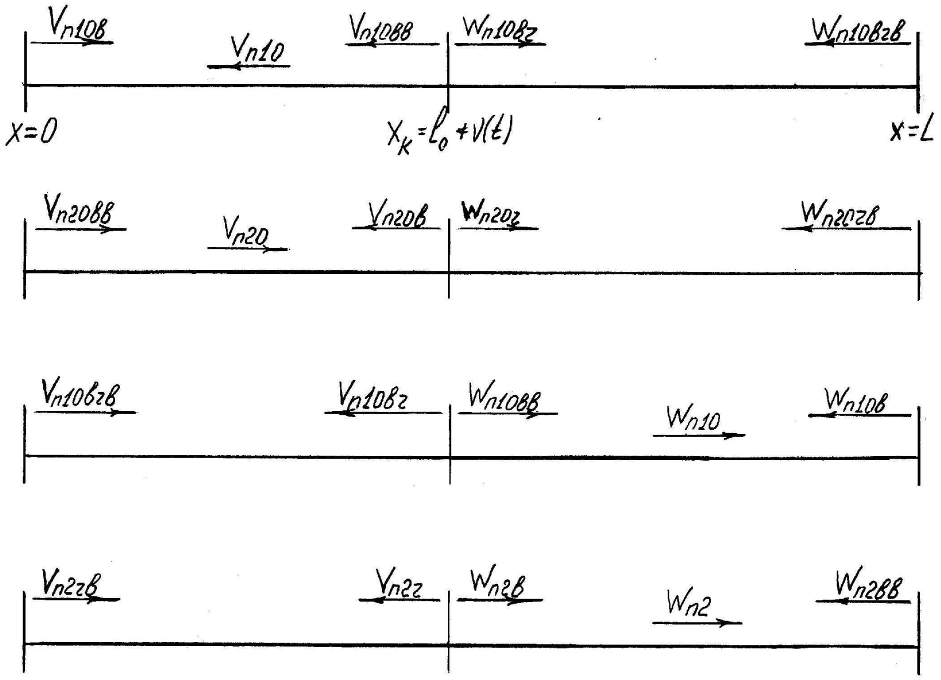

Figure 3. Secondary reflected (symbol b) and refracted (symbol r) waves

Thus, the solution of a problem on find of elastic displacements in a rope at lifting of loads develops of several components. First, this solution includes all primary waves $v_{n1}(x,t)$, $v_{n10}(x,t)$, $v_{n20}(x,t)$, $w_s(x)$, $w_{n10}(x,t)$ and $w_{n2}(x,t)$, disturbed in a rope at the moment of the beginning of its movement. Second, all reflected and refracted waves generated both primary waves, and the secondary reflected and refracted waves here should be included. Directions of movement of secondary waves are represented in Figure 3.

Therefore the solution of a considered problem will have structure (2.33), in which

$$\begin{aligned} v(x,t) &= v_{n1}(x,t) + v_{n10}(x,t) + v_{n10b}(x,t) + v_{n10bb}(x,t) + \cdots + \\ &\quad + v_{n20}(x,t) + v_{n20b}(x,t) + v_{n20bb}(x,t) + \cdots + v_{n10br}(x,t) + \\ &\quad + \cdots + v_{n2r}(x,t) + \cdots + \cdots \ ; \\ w(x,t) &= w_s(x) + w_{n10}(x,t) + w_{n10b}(x,t) + w_{n10bb}(x,t) + \cdots + \\ &\quad + w_{n2}(x,t) + w_{n20b}(x,t) + w_{n20bb}(x,t) + \cdots + w_{n10br}(x,t) + \\ &\quad + \cdots + w_{n20r}(x,t) + \cdots + \cdots \ . \end{aligned} \tag{3.33}$$

Here points designate the reflected and refracted waves arising at the subsequent reflections and refractions of already existing waves. Process of construction of the reflected and refracted waves needs to be continued up to a stop of a drum during some moment of time $t = t_k$. At $t > t_k$ movement of waves will proceed before their full attenuation. However in this case it is necessary to take into account that portable movement of a rope has stopped.

4. Conclusion

The exact solution of a problem on find of elastic displacements in a rope at lifting of loads is obtained. This solution consists of several components. First, this solution includes all primary waves $v_{n1}(x,t)$, $v_{n10}(x,t)$, $v_{n20}(x,t)$, $w_s(x)$, $w_{n10}(x,t)$ and $w_{n2}(x,t)$, disturbed in a rope at the moment of the beginning of its movement. Second, all reflected and refracted waves generated both primary waves, and the secondary reflected and refracted waves here should be included. It is very important to note that at every finite moment of time number of terms in formulas (3.33) will be finite as well.

References

1. *Dinnik A. N.* Articles on mining. (Russian)// Moskow: 1957.
2. *Florinsky F. V.* Dynamics of mining hoisting rope. (Russian)// Moskow: 1956.
3. *Ishlinsky A. Y.* On the equation of longitudinal motions of rope variable length. (Russian)// DAN USSR, Moskow: Vol. 3.
4. *Glushko M.F., Tchig A. A.* Basic equations of motion of the rope of variable length in hoisting plant with friction sheave. (Russian) //Applied mechanics, Moskow: Vol. 2, No. 8.
5. *Tchig A. A.* Integro-differential equations of oscillations of hoisting ropes of variable length. (Russian) // Mining journal, Moskow: No. 8.
6. *Goroshko O. A. Savin G. N.* Introduction in mechanics of one dimensional deformable bodies of variable length.(Russian) //Kiev: 1971.
7. *Panovko Y. G.* Introduction into theory of mechanical oscillations. (Russian)// Moskow: 1991.
8. *Panovko Y. G., Gubanova I. I.* Stability and oscillations of elastic systems. (Russian) // Moskow: 1987.
9. *Ostapenko V. A.* Dynamic field of displacements in rods of variable length. // Proceedings of 8th International Conference on Dynamical Systems Theory and Applications. Lodz: 316–323.
10. *Ostapenko V. A.* Exact solution of the problem for dynamic field of displacements in rods of variable length. //Archives of Applied Mechanics, Hamburg: Springer-Verlag, Vol. 77, 313–324.
11. *Ostapenko V. A.* Boundary problem without initial conditions for telegraph equation. (Russian) // The differential equations and their applications in physics. Dnepropetrovsk: 3–17.
12. *Ostapenko V. A.* The first initial-boundary value problem for region with mobile border. (Russian)/ V.A. Ostapenko // The differential equations and their applications in physics. Dnepropetrovsk: 4–14.
13. *Ostapenko V. A.* The second initial-boundary value problem for region with mobile border. (Russian) // The bulletin of the Dnepropetrovsk University, Mathematics, Dnepropetrovsk: Vol. 1, 3–21.
14. *Ostapenko V. A.* Dynamics of the waves in ropes of variable length. (Russian)/ V.A. Ostapenko // The bulletin of Poltava national technical university, Poltava: Issue 16 216–220.

15. *Ostapenko V. A.* Initial-boundary value problem for a rod of variable length, perturbed from the mobile top end. (Russian)// Visnyk DNU. Series: Mechanics, Dnipropetrovsk: DNU, Issue 10, No. 2, Vol. 1, 182–198.
16. *Ostapenko V. A.,Polyakov N. V.* Dynamic displacement in ropes of variable length at perturbation of nonzero initial conditions. //Proceedings of 9th International Conference on Dynamical Systems Theory and Applications. Lodz: 347–354.
17. *Ostapenko V. A.* Displacement field in ropes of variable length perturbed by initial statement of the system. (Russian) // The bulletin of Kherson national technical university, Kherson: Issue 2, No. 35, 337–341.
18. *Ostapenko V. A.* The first initial-boundary value problem for for telegraph equation in bounded domain. (Russian) // Visnyk DNU. Series: Mathematical Modelling, Dnipropetrovsk: DNU, Issue 1, No. 8, Vol. 17, 149–161.
19. *O*stapenko V. A. Displacement field in rod of variable length perturbed by inertia of a load. (Russian) // Visnyk DNU. Series: Mechanics, Dnipropetrovsk: DNU, Issue 14, No. 5, Vol. 18, 53–65.
20. *O*stapenko V. A. First initial boundary value problem for telegraph equation in domain with variable border. (Russian) // Visnyk DNU. Series: Mathematical Modelling, Dnipropetrovsk: DNU, Issue 3, No. 8, Vol. 19, 30–54.

Надійшла до редколегії 30.11.2011