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A modified member stiffness procedure for dynamic progressive collapse analysis of planar frames

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ABSTRACT

A computer program for progressive collapse analysis of planar frames is under development. The software has a capability to analyze structures after failures of members in which failure can occur at one or both ends of a member. When an end of a member fails, the failed end separates from the main structure and becomes discontinuous. In this paper, a modified member stiffness procedure with releases of end forces to track the response of a failed end is discussed. The procedure utilizes a condensation process of the element stiffness matrix of the failed member. An example in applying the modified member stiffness procedure is given to show that the assembly process for the stiffness matrix and the applied force vector of the main structure does not change. In addition, the Equation solver still determines the same number of unknown degrees of freedom. Accordingly, this approach provides a convenient, simple, yet efficient means of keeping track of all failed members for progressive collapse analysis of frame structures.

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1. Introduction

Progressive collapse is a type of collapse that can be defined as a chain reaction of failures initiated by a loss of one or more supporting elements. Thus far, an analysis technique known as the “Alternate Load Path” method has been employed for investigating the potential of progressive collapse in buildings. The method has been adopted by many current design codes and standards in USA (GSA 2000; IBC 2000; DOD 2001). Although the importance of considering dynamic effects has been shown (Pretlove et al., 1991; Kaewkulchai and Williamson, 2002) and some design provisions have suggested the use of dynamic analysis in conjunction with the alternate load path method (GSA 2000; DOD 2001), how to perform such analysis still at large relies on engineering judgment.

For the current research study being conducted, a computer program for dynamic progressive collapse analysis of planar frames is under development. The software has a capability to analyze structures after failures of members in which failure can occur at one or both ends of a member. When an end of a member fails, the failed end separates from the main structure and becomes discontinuous. In this paper, a modified member stiffness procedure with releases of end forces to track the response of a failed end is discussed. The procedure utilizes a condensation process of the element stiffness matrix of the failed member. An example in applying the modified member stiffness procedure is given to show that using the modified member stiffness approach can result in a simple, yet efficient analysis routine for analyses of frame structures after member failure.

2. A Computer Program

Two General description of the developed software is given in this section. A more detailed explanation of these subjects has been given elsewhere (Kaewkulchai and Williamson, 2004). The software utilizes the conventional direct stiffness method for the main analysis routine. An implicit direct integration scheme, the Newmark-beta method is employed to solve the governing

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equations of dynamic equilibrium coupled with Newton-Raphson iterations for carrying out the nonlinear analyses. The program assumes the use of a classical or proportional (Rayleigh) damping matrix along with the use of a lumped mass matrix. Geometric nonlinearity (P-Δ effect) is taken into account by using a simplified geometric stiffness matrix. For material nonlinearity, a lumped plasticity model for beam-column elements is applied in which inelasticity is assumed to occur only at element ends or hinges (Fig. 1). Effects of strength and stiffness degradation of members are modeled by means of a damage model. The damage model utilizing a damage index at each member end is used to determine the onset of member end failure.

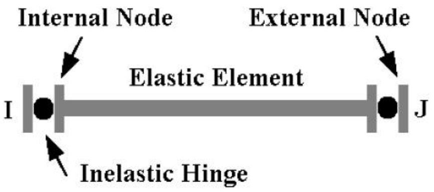

Fig. 1. Inelastic beam-column element.

The software has a capability to analyze frame structures after failures of members. When an end of a member fails, the failed end separates from the main structure and becomes discontinuous. To continue the analysis, an additional node at the failed end may be introduced. Because three new degrees of freedom associated with the new node are added to the structure, the system of equations becomes larger. Hence, the analysis requires more computational effort, particularly when there are many failed ends. In addition, changing the dimensions of all system matrices during the course of analysis is required, resulting in expensive computer time for transferring data between matrices. Also, new definitions for element connectivity must be established. As a result of the drawbacks associated with adding a new node to the definition of the structural model, in the current computer program the analysis continues in an efficient manner through the use of a modified member stiffness procedure with releases of end forces. Systematically, this approach provides a convenient means of keeping track of all failed members, and the main analysis routine is not greatly altered. The modified member stiffness procedure is described in the following section.

3. A Modified Member Stiffness Procedure

As mentioned in the previous section, the modified member stiffness procedure is employed to track the response of a failed end. The modified member stiffness approach utilizes a condensation process of the element stiffness matrix. Static condensation for a beam element is well established in the literature (e.g, Felton and Nelson, 1997). For the 2-D beam-column element under consideration (Fig. 2), all three degrees of freedom at one end of an element are released (u_1-u_3 or u_4-u_6) because of the failure of an end. When releasing one end, the element forces at that location become zero. Because these force values are known, the corresponding displacement quantities can be expressed in terms of the displacements at the other end of the element using matrix condensation.

Fig. 2. Element end displacements and forces.

Considering the response of a beam-column element in which one of the ends has failed, the incremental equilibrium equations of the failed element can be written as follows:

$$\Delta R = K\Delta_u + \Delta R_F \,. \tag{1}$$

For expressive reason, assume that failure takes place at the right end of the beam-column element in Fig. 2. Thus, two sets of matrix equations result and Eq. (1) can be rewritten as

$$\begin{Bmatrix} \Delta R_c \\ \Delta R_r \end{Bmatrix} = \begin{bmatrix} K_{cc} | K_{cr} \\ K_{rc} | K_{rr} \end{bmatrix} \begin{Bmatrix} \Delta u_c \\ \Delta u_r \end{Bmatrix} + \begin{Bmatrix} \Delta R_{Fc} \\ \Delta R_{Fr} \end{Bmatrix}, \tag{2}$$

where subscripts c and r refer to 'contracted' and 'released', respectively.

From Eq. (2), the contracted set consists of incremental force and displacement vectors corresponding to the element degrees of freedom 1 through 3 at the intact end. Similarly, the released set contains those for the element degrees of freedom 4 through 6 at the released end. Because the released element force vector ΔR_r is zero, the released displacement vector Δu_r can be written in terms of Δu_c as

$$\Delta u_r = -[K_{rr}]^{-1} \, [K_{rc}\Delta u_c + \Delta R_{Fr}] \,, \tag{3}$$

Accordingly, the incremental equilibrium equations for the contracted set can be expressed by

$$\Delta R_c = \bar{K}_{cc}\Delta u_c + \Delta\bar{R}_{Fc} \,, \tag{4}$$

where $\bar{K}_{cc} = [K_{cc} - K_{cr}\, K_{rr}^{-1}\, K_{rc}]$ is the modified member stiffness matrix and $\Delta\bar{R}_{Fc} = [\Delta R_{Fc} - K_{cr}\, K_{rr}^{-1}\, \Delta R_{Fr}]$ is the incremental modified fixed-end force vector.

It can be seen from Eq. (4) that the use of $\bar{K}_{cc}$ and $\Delta\bar{R}_{Fc}$ requires no special process in accounting for the new degrees of freedom associated with the failed end during the assembly process for the stiffness matrix and the applied force vector of the main structure in which the degrees of freedom associated with the failed end can be calculated using Eq. (3).

The relationships derived in Eqs. (3) and (4), however, are based on an assumption of static equilibrium of the element, and therefore cannot apply for dynamic analyses of frame structures. Nonetheless, because the

Newmark-beta method for solving the governing equations of dynamic equilibrium is employed, similar equations, which are valid for dynamic analyses, can be developed. In the Newmark-beta method, the governing equations of dynamic equilibrium can be cast in terms of unknown displacements. Accordingly, dynamic effects in the response are accounted for, and the procedure outlined previously can be used with only slight modification.

Let us consider the governing equations of motion for a dynamic system which can be described by

$$M_s U'' + C_s U' + K_s U = P \, , \tag{5}$$

where U, U' and U'' are the displacement, velocity and acceleration vectors; M_s, C_s and K_s are the system mass, damping and stiffness matrices; P is the external applied force vector.

Then, the incremental equations of motion combined with the Newmark-beta method can be written as

$$\Delta P_{eff} = K_{s\,eff} \Delta U \, , \tag{6}$$

$$K_{s\,eff} = A_1 M_s + A_4 C_s + K_s \, , \tag{7}$$

$$\Delta P_{eff} = \Delta P + M_s\{A_2 U' + A_3 U''\} + C_s\{A_5 U' + A_6 U''\}, \tag{8}$$

where A_1 through A_6 are the Newmark constants.

Because the effective system stiffness matrix, $K_{s\,eff}$ and the effective incremental applied force vector, ΔP_{eff} is a result from the assembly process of each beam-column element, one can express the incremental equilibrium equations of an element, similar to Eq. (1) as follows:

$$\Delta R_{eff} = K_{eff} \Delta u + \Delta R_{eff\,F} \, , \tag{9}$$

$$K_{eff} = A_1 M + A_4 C + K \, . \tag{10}$$

Because the fixed-end forces are the negative values of the applied forces, the effective incremental fixed-end force vector, $\Delta R_{eff\,F}$, similarly to ΔP_{eff} (see Eq. (8)), is given as

$$\Delta R_{eff\,F} = \Delta R_F - M[A_2 v + A_3 a] - C[A_5 v + A_6 a] \, , \tag{11}$$

where v and a are the velocity and acceleration vectors of the element, while M and C are the element mass and element damping matrices.

By assuming the right end of the element failed, two sets of matrix equations result in Eq. (9) and the released displacement vector Δu_r from Eq. (3) can be rewritten as

$$\Delta u_r = -\left[K_{eff\,rr}\right]^{-1} \left[K_{eff\,rc} \Delta u_c + \Delta R_{eff\,Fr}\right] . \tag{12}$$

The derived equation for Δu_r is now based on dynamic equilibrium, and therefore, inertial effects are accounted for by using K_{eff} and $\Delta R_{eff\,F}$. Furthermore, similar to Eq. (4), the incremental equilibrium equations for the contracted set can be given by

$$\Delta R_{eff\,c} = \bar{K}_{eff\,cc} \Delta u_c + \Delta \bar{R}_{eff\,Fc} \, , \tag{13}$$

where $\bar{K}_{eff\,cc}$ is the modified member stiffness matrix and $\Delta \bar{R}_{eff\,Fc}$ is the incremental modified fixed-end force vector for dynamic equilibrium.

$$\bar{K}_{eff\,cc} = \left[K_{eff\,cc} - K_{eff\,cr} K_{eff\,rr}^{-1} K_{eff\,rc}\right] , \tag{14}$$

$$\Delta \bar{R}_{eff\,Fc} = \left[\Delta R_{eff\,Fc} - K_{eff\,cr} K_{eff\,rr}^{-1} \Delta R_{eff\,Fr}\right] . \tag{15}$$

Based on the discussion above, together with Eqs. (10) to (15), the procedure for dynamic progressive collapse analysis with the modified member stiffness approach only involves modification of the stiffness matrix and fixed-end forces of a failed member. Thus, Eq. (13) can be employed with the modified member stiffness matrix, $\bar{K}_{eff\,cc}$ and the modified fixed-end force vector, $\Delta \bar{R}_{eff\,Fc}$. These matrices correspond to the contracted degrees of freedom at the intact end.

With $\bar{K}_{eff\,cc}$ and $\Delta \bar{R}_{eff\,Fc}$, analysis after member failure can continue with little modification to the main analysis routine because no new degrees of freedom are added to the system. At the end of a converged time step, the released displacement vector Δu_r at the failed end of the member can be obtained from the contracted displacement vector Δu_c using Eq. (12). Thus, using the approach just outlined, the assembly process for the stiffness matrix and the applied force vector of the main structure does not change. In addition, the equation solver still determines the same number of unknown degrees of freedom. Accordingly, applying the modified member stiffness approach results in a simple, yet efficient analysis routine for analyses of frame structures after member failure.

4. Analysis Example

An example in applying the modified member stiffness procedure is given in this section. Results obtained from a dynamic analysis using this approach are compared with those obtained from a conventional dynamic analysis. For illustrative purpose, a fixed-fixed beam shown in Fig. 3 is used. Note that the beam is assumed to have elastic-perfectly plastic behavior.

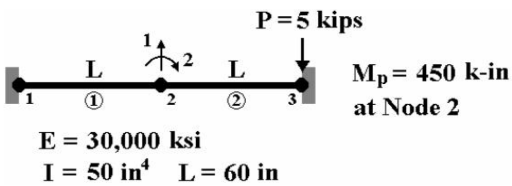

Fig. 3. A fixed-fixed beam modeled using two elements.

The fixed-fixed beam consists of two elements and two degrees of freedom as shown in Fig. 3. A point load, P of 5 kips acts at Node 3. Apparently, the system will not react because the point load is acting at a support. To illustrate the modified member stiffness method, the right end of member 2 is assumed to abruptly fail so that the support is no longer available to resist loads. For a conventional dynamic analysis, a new node (Node 3) would need to be introduced; therefore two new degrees of freedom are introduced into the system. Hence, after failure at Node 3, the system can be analyzed by using an equivalent system having four degrees of freedom with a suddenly applied force, *P* at Node 3 as shown in Fig. 4.

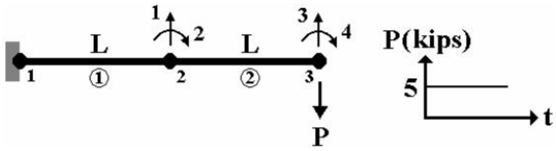

Fig. 4. An equivalent system with a suddenly applied force.

For a dynamic analysis using the modified member stiffness approach, however, only two degrees of freedom are required so that no new degrees of freedom are introduced to the system. The response, however, at the failed end can be obtained through Eq. (12). For the dynamic analyses performed, Δt = 0.01 sec and mass = 0.05 kips-s^2/in at each member end are used. In addition, rotational mass and damping are ignored. The results, obtained from a conventional dynamic analysis and a dynamic analysis using the modified member stiffness approach, are compared in Fig. 5.

Figs. 5(a-b) show the response of four degrees of freedom, 1 to 4. As seen in the graphs, the difference between the results obtained from the conventional dynamic analysis and the modified member stiffness approach is negligible. Similarly, bending moments and shear forces of members 1 and 2 obtained from the two approaches are nearly identical. Hence, applying the modified member stiffness approach results in a simple, yet efficient analysis routine for dynamic analyses of frame structures after member failure.

5. Conclusions

In recent years, progressive collapse has gained much interest due to severe building collapse. An analysis technique known as the "Alternate Load Path" method generally based on a static approach, has been employed for investigating the potential of progressive collapse in buildings by many current design standards. Although considering dynamic effects has been shown to be significant for the analysis of progressive collapse, how to perform such analysis is simply based on engineering judgment. For the current study, a computer program for dynamic progressive collapse analysis of planar frames is under development. The software has a capability to analyze structures after failures of members.

In this paper, a modified member stiffness procedure with releases of end forces to track the response of a failed end of members was discussed. An example in applying the modified member stiffness procedure was also given. The described approach to track the response of failed ends was simple, yet efficient to employ for dynamic collapse analyses of planar frame structures.

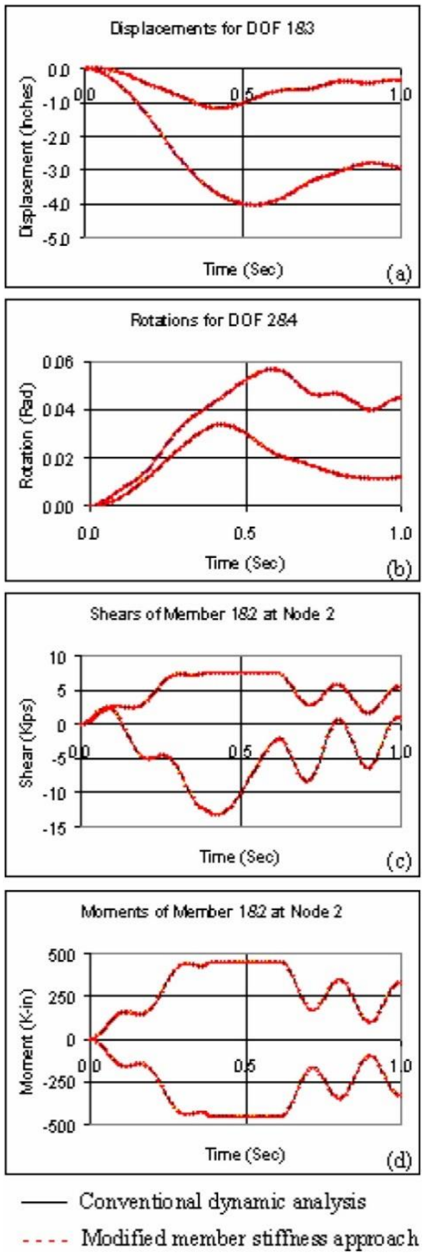

Fig. 5. Comparisons of the obtained results.

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