RESEARCH ARTICLE

# Nomogram-Based Synthesis of Complex Planar Mechanisms, Part I: 6 Bar – 2 Sliders Mechanism

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## Abstract:

This research paper presents a new technique for the synthesis of complex planar mechanisms. The author calls this technique 'nomogram-based synthesis' since it depends on a kinematic nomogram facilitating the synthesis of complex planar mechanisms without need to complex optimization procedures. A procedure of five steps is presented to synthesize the mechanism under study for time ratio up to 4.3, normalized stroke up to 3.33. The nomogram based synthesis can maintain transmission angle to be within a range from 94 to 120 degrees indicating the effectiveness of the synthesis approach presented.

*Keywords* — Nomogram-based mechanism synthesis, 6 bar-2 sliders planar mechanism, successful mechanism performance.

## I. INTRODUCTION

Practicing mechanical engineers and designers are in need to simple tools facilitating the selection of machine structures for specific purposes. Here, a new simple technique is presented without need to the solution of nonlinear equations nor the application of advanced optimization techniques.

Hongying, Zhixing, Dewi and Jiansheng (2003) studied in details a numerical comparison method of planar six-bar dwell mechanism synthesis. They obtained dwell angle range from 25 to 140 degrees and rocker angle of oscillation from 10 to 160 degrees [1]. Srinath and Rao (2005) presented a method to compare chains from the structural error point of view. They applied their method on Stephenson and Watt 6-bar chains [2]. Vashista et. al. (2007) described the analysis of a new mechanism for the sheep shearing machine. They presented a number of planar mechanisms including a 6-bar mechanism and a 4-bar spherical mechanism [3].

Shen, Lee and Sodhi (2008) formulated and demonstrated a motion generation method for the synthesis of the Watt I 6-bar mechanism. They demonstrated the synthesis of a finger mechanism to achieve a prescribed grasping pose sequence [4]. Cruz et. al. (2010) presented a genetic algorithm

based approach for optimization of the kinematic analysis of a 6-bar Watt type mechanism for prosthetic knee application. The generated trajectory illustrated how it naturally evolved from an erratic solution to a smooth fitted curve [5]. Soh, Ying and McCarthy (2012) considered the problem of designing planar 6-bar linkages driven by prismatic joint. They demonstrated the synthesis process with the design of a wheelchair seat [6].

Mohammad and Kumar (2013) developed a 6-bar linkage using MATLAB for eight precision points traced by the mechanism coupler. They used Stephenson III six-bar mechanism and presented a numerical example and have drawn the mobility of the mechanism from initial to primed positions [7]. Wasnik, Sonpimple and Undirwade (2014) studied the solution o methods of optimal synthesis of a path generation linkage using non-conventional approach. They divided the path to some sections and found the minimum error between desired points and design points. They coud decrease the path error [8]. Plecnik and McCarthy (2015) presented a synthesis method for designing planar 6-bar linkages to move a trace point through eleven points of a specified path. They inverted the Stephenson II function generator into either a Stephenson II or Stephenson III path generator [9]. Larochelle, Surdram and Zimmerman (2015)

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presented the kinematic synthesis of Watt II six-bar mechanisms for simultaneously guiding two bodies through four prescribed positions. They included a case study to illustrate the application of the synthesis algorithm [10]. Hassaan (2015) presented the optimal synthesis of a 6 bar - 3 sliders planar mechanism for motion generation with maximum time ratio. He could synthesize the mechanism for a normalized stroke up to four, a time ratio up to 4.1and keeping the transmission angle within the recommended range during a full crank rotation [11].

## II. MECHANISM

The 6 bar - 2 sliders mechanism was synthesized before by the author for optimal assignment of its dimensions. The line diagram of the mechanism is shown in Fig.1.

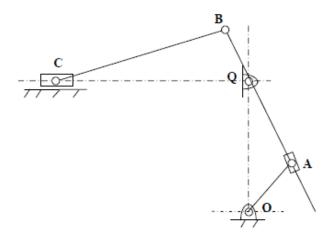


Fig.1 Line diagram of the mechanism [12].

A fully rotating crank OA drives an oscillating lever QB using a slider A which in turn drives a coupler BC. The coupler drives the output slider C. Such mechanism structure with number of links more than four is expected to give time ratio greater than 1.5 [13].

## III. MECHANISM ANALYSIS

The kinematic performance of any planar mechanism is judged by a number of functions which are:

- Mechanism time ratio, TR.
- Mechanism stroke, S.
- Minimum transmission angle, TA<sub>min</sub>.
- Maximum transmission angle, TA<sub>max</sub>.

To assign those functions for the mechanism under study, the mechanism is drawn in its two limiting positions as shown in Fig.2 [12].

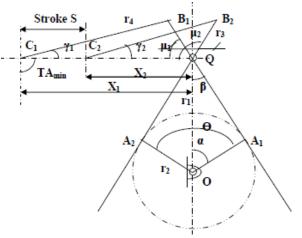


Fig.2 Limiting positions of the mechanism [12].

Using the normalized dimensions,  $r_{1n}$ ,  $r_{3n}$  and  $r_{4n}$ , the performance functions of the mechanism are given by [12]:

### Time ratio:

$$TR = (360 - \Theta) / \Theta \tag{1}$$

The crank angle  $\Theta$  is twice the angle  $\alpha$  as shown in Fig.2. That is:

 $\Theta = 2 \alpha$ (2)
Where  $\alpha = \cos^{-1}(1/r_{1n})$ 

## Stroke, S:

The stroke S is the distance between the two extreme positions of the output slider at C. With  $X_1$  is slider C position relative to Q in the first limiting position and  $X_2$  is its position relative to Q in the second extreme position, S becomes:

Where  

$$S = X_{1} - X_{2} \qquad (3)$$

$$X_{1} = r_{3} \sin(180 - \mu_{1} - \gamma_{1}) / \sin\gamma_{1}$$

$$X_{2} == r_{3} \sin(180 - \mu_{2} - \gamma_{2}) / \sin\gamma_{2}$$

$$\mu_{1} = \alpha$$

$$\mu_{2} = 180 - \alpha$$

$$\gamma_{1} = \sin^{-1} \{r_{3n} \sin\mu_{1} / r_{4n}\}$$

$$\gamma_{2} = \sin^{-1} \{r_{3n} \sin\mu_{2} / r_{4n}\}$$

Using Eq.3, the normalized mechanism stroke  $S_n$  is given by:

$$S_n = S/r_2 = r_{3n} \sin(180 - \mu_1 - \gamma_1) / \sin\gamma_1 - r_{3n} \\ \sin(180 - \mu_2 - \gamma_2) / \sin\gamma_2$$
(4)

## Minimum transmission angle, TA<sub>min</sub>:

The minimum transmission angle is shown in Fig.2. It is the angle between coupler  $B_1C_1$  and the vertical through the output slider. It is given by:

$$TA_{min} = \gamma_1 + 90$$
 degrees (5)

## Maximum transmission angle, TA<sub>max</sub>:

The maximum transmission angle occurs when the oscillating lever is exactly vertical (Fig.1). In this case,  $TA_{max}$  is given by:

 $TA_{max} = tan^{-1}(r_{3n}/r_{4n}) + 90$  degrees (6)

## IV. PARAMETRIC MECHANISM PERFORMANCE

In a previous work the author showed that the optimal synthesis of the mechanism reveals the following mechanism normalized dimensions range [12]:

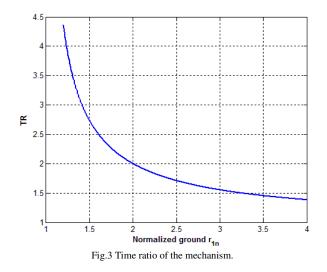
| Ground, r <sub>1n</sub> :                    | $1.2\leqr_{1n}\leq4$     |     |
|--|--------------------------|-----|
| <b>Oscillating lever</b> , r <sub>3n</sub> : | $1.2\leqr_{3n}\leq2$     | (7) |
| <i>Coupler</i> , r <sub>4n</sub> :           | $3.4~\leq~r_{4n}~\leq~4$ |     |

Therefore, the coupler dimension is kept unchanged at a level of 4. The other two dimensions are changed as between their levels in Eq.7.

The graphs of the mechanism functions governing the performance of the mechanism are plotted using Eq.1 for the time ratio, Eq.4 for the normalized stroke, Eq.5 for the minimum transmission angle and Eq.6 for the maximum transmission angle. They are as follows:

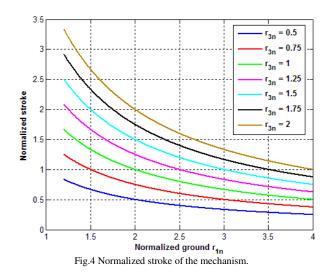
## Time ratio, TR:

The time ratio of the mechanism is independent of the oscillating lever and coupler dimensions. Therefore, it is drawn in Fig.3 against the normalized ground  $r_{1n}$  only.



## Normalized stroke, S<sub>n</sub>:

The normalized stroke is function of both  $r_{1n}$  and  $r_{3n}$ . This relationship is illustrated graphically in Fig.4.



## Minimum transmission angle, TA<sub>min</sub>:

For accepted performance of the mechanism, its minimum transmission angle has to be  $\geq 45$ degrees [14]. This is checked through the plotting of the minimum transmission angle within the mechanism dimensions range in Eq.7. The result is presented graphically in Fig.5.

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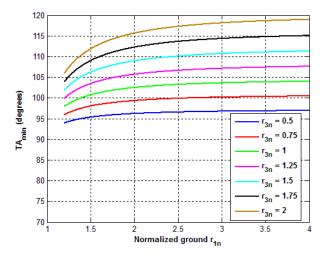


Fig.5 Minimum transmission angle of the mechanism.

## Maximum transmission angle, TA<sub>man</sub>:

The maximum transmission angle for an accepted performance of the mechanism has to be  $\leq 135$  degrees [14]. For the mechanism in hand, the maximum transmission angle is function only of the oscillating lever and coupler dimensions. The relation is shown graphically in Fig.6 as TA<sub>max</sub> is drawn against r<sub>3n</sub>.

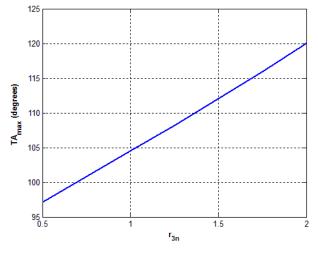


Fig.6 Maximum transmission angle of the mechanism.

The minimum and maximum values of the performance functions against the normalized oscillating lever dimension r3n are given in Table I.

TABLE I MINIMUM AND MAXIMUM VALUES OF THE MECHANISM PERFORMANCE FUNCTIONS

| r <sub>3n</sub> | S <sub>n,min</sub> | S <sub>n,max</sub> | TA <sub>min,min</sub><br>(deg.) | TA <sub>min,man</sub><br>(deg.) | TA <sub>max</sub><br>(deg.) |
|-----------------|--------------------|--------------------|---------------------------------|---------------------------------|-----------------------------|
| 0.50            | 0.250              | 0.833              | 93.962                          | 96.952                          | 97.181                      |
| 0.75            | 0.375              | 1.250              | 95.949                          | 100.460                         | 100.807                     |
| 1.00            | 0.500              | 1.667              | 97.943                          | 104.008                         | 104.477                     |
| 1.25            | 0.625              | 2.083              | 99.947                          | 107.612                         | 108.210                     |
| 1.50            | 0.750              | 2.500              | 101.964                         | 111.290                         | 112.024                     |
| 1.75            | 0.875              | 2.917              | 103.995                         | 115.063                         | 115.945                     |
| 2.00            | 1.000              | 3.333              | 106.045                         | 118.955                         | 120.000                     |

#### V. MECHANISM SYNTHESIS NOMOGRAM

The four figures for time ratio, normalized stroke, minimum transmission angle and maximum transmission angle are collected in one nomogram as shown in Fig.7. The nomogram is used for the purpose of mechanism synthesis as follows:

- 1. Assign the desired time ratio according to the application.
- 2. Draw a horizontal line from the desired time ratio in the graph in the left corner of Fig.7 to intersect the curve in the corresponding value of the normalized ground,  $r_{1n}$ .
- 3. Move to the second graph in the right corner and draw a horizontal line at the desired normalized stroke. This will intersect with a vertical line in the same graph at the normalized ground  $r_{1n}$  obtained in step 2. The intersection is at a curve corresponding to the normalized oscillating lever dimension  $r_{3n}$ .
- 4. Dropping a line from  $r_{1n}$  of the first graph down to the third graph in the nomogram will intersect the curve corresponding to r3n in the value of the minimum transmission angle, TA<sub>min</sub>.
- 5. Dropping a line from the third graph to the fourth graph at the bottom right corner at the value of  $r_{3n}$  assigned in step 3 will locate the corresponding maximum transmission angle, TA<sub>max</sub>.

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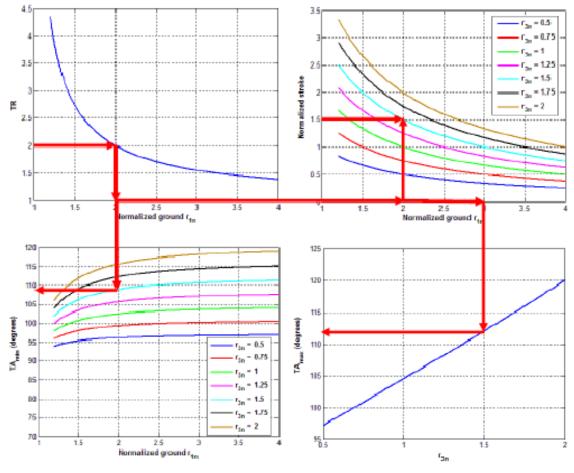


Fig.7 Synthesis nomogram of the mechanism.

#### VI. APPLICATION

It is required to synthesize a 6 bar-2 sliders planar mechanism such that it has a time ratio of 2 and a normalized stroke of 1.5. The nomogrambased procedure is used as follows:

- 1. Step 2 of the procedure gives  $r_{1n} = 2$  for a time ratio of 2.
- 2. Step 3 gives  $r_{3n} = 1.5$  for a normalized stroke of 1.5.
- 3. Step 4 gives the minimum transmission angle of the mechanism as 108.5 degrees.
- 4. Step 5 gives the maximum transmission angle of the mechanism as 112 degrees.
- 5. The mechanism displacement using the parameters defined by the approach

presented in this work is shown in Fig.8 for two revolutions of the mechanism crank.

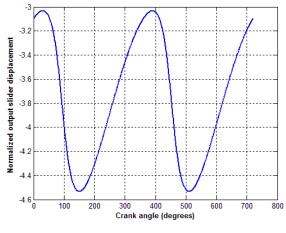


Fig.8 Normalized output slider displacement ..

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## VII. CONCLUSIONS

- A new approach for the synthesis of complex planar mechanisms using nomogram-based procedure was presented.
- This new approach did not require the solution of nonlinear equations or using optimization techniques.
- A nomogram was constructed using the four mechanism functions: time ratio, normalized stroke, minimum transmission angle and maximum transmission angle.
- A five steps procedure was outlined for the mechanism synthesis for a required time ratio and stroke.
- The procedure guaranteed the transmission angle to be within the recommended range.
- An application example was presented to examine the effectiveness of the proposed approach.

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