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Car Dynamics using Quarter Model and Passive Suspension, Part I: Effect of Suspension Damping and Car Speed

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Abstract:

Quarter-car model is in use for years to study the car dynamics. The objection of this paper is to examine the dynamics of a car passing a circular hump for sake maintaining ride comfort for the passengers. Passive suspension elements are considered with suspension damping coefficient in the range 1 to 15 kNs/m. Car speed in the range 5 to 25 km/h is considered when passing the hump. Important phenomenon evolved from the analysis of the car dynamics which is performed using MATLAB. The mathematical model of the quarter-car model is derived in the state form and the dynamics are evaluated in terms of the sprung mass displacement and acceleration. The effect of suspension damping and car speed on the sprung-mass displacement and acceleration is examined. The study shows that for ride comfort the car speed has not to exceed 6.75 km/h when passing a circular hump depending on the suspension damping. .

***Keywords* —Car dynamics , quarter model , Passive suspension system , Standard humps , Ride comfort.**

I. INTRODUCTION

Abdelhaleem and Crola (2000) descrfibed the analysis and design of a hydro pneumatic limited bandwidth active suspension system. They used a quarter-car model to compare the performance of the proposed suspension with both baseline passive system and an idealized fully active system [1]. Soliman (2001) proposed a hybrid control system to overcome the drawbacks of using active suspensions. He examined the performance of the dynamic system using a seat suspension coupled to a quarter-car model representing the general properties of vehicle ride dynamics [2]. Sammier, Sename and Dugard (2003) proved the benefits of controlled semi-active suspension compared to passive ones. They used a quarter-car model to which an H∞ control design was applied to improve comfort using exact nonlinear model of the suspension [3]. Smith and Wang (2004) made a comparative study of simple passive suspension struts containing at most one damper and inerter. They used a quarter-car and full-car models in their analysis [4]. Verros, Natsiavas and Papadimitriou (2005) presented a method for optimizing the suspension damping and stiffness parameters of a nonlinear quarter-car model subjected to random road excitation [5].

Litak et. al. (2006) used the Melnikov criterion to examine the chaos in a quarter-car model excited by the road surface profile [6]. Litak et. al. (2007) examined the strange chaotic attractor and its unstable periodic orbits for a one fegree of freedom nonlinear oscillator using a quarter-car model forced by the road profile [7]. Chi, He and Natere (2008) presented a comparative study of three optimization algorithms for the optimal design of vehicle suspension based on a quarter-car model. They neglected the tire damping and their sprung-mass acceleration was above 1 m/s^2 [8]. Hanafi (2009) presented the design of a quarter-car fuzzy logic control for passive suspension model. He identified the quarter-car model parameters using an intelligent system identification [9]. Gysen et. al. (2010) used an electromagnetic active suspension providing additional stability and maneuverability through the application of active roll and pitch control during cornering and braking. They applied their suspension system to a quarter-car setup [10].

Changizi and Rouhani (2011) applied the fuzzy logic technique to control the damping of an automotive suspension system using a quarter-car model. They compared the fresults with those using a PID controller showing the car dynamics in terms of displacement and velocity [11]. Agharkakli, Chavan and Phvithran (2012) obtained a mathematical model for passive and active suspensions for quarter-car model subject to excitation from a road profile using a LQR controller. They assested the system dynamics in terms of the sprung mass displacement and velocity [12]. Aly (2013) established a robust control technique of an active suspension for a quarter-car model. He compared using MATLAB simulation the quarter-car model response using robust suspension sliding fuzzy control and passive control [13]. Tiwari and Mishra (2014) studied the active control of a vehicle suspension based on a PID controller and a quarter-car model. They neglected the tire damping and used the bond graph model to simulate the dynamics of the system using the fourth-order Runge-Kutta method [14]. Gasemalizadeh et. al. (2014) implemented three control approaches to a quarter-car model using MATLAB/Simulink. They used comfort-oriented approaches including an acceleration driven damper, power driven damper and H∞ robust control. They neglected the tire damping. They compared the sprung-mass acceleration for the three control approaches and the passive design [15].

II. ANALYSIS

2.1 Quarter Car Model

Car models are divided into three categories: full-car model, half-car-model and quarter-car model [5,7,16-19].

A quarter-car model consists of the wheel and its attachments, the tire (of visco-elastic characteristics), the suspension elements and quarter the chassis and its rigidly connected parts. Fig.1 shows a line diagram of a car quarter physical model [20].

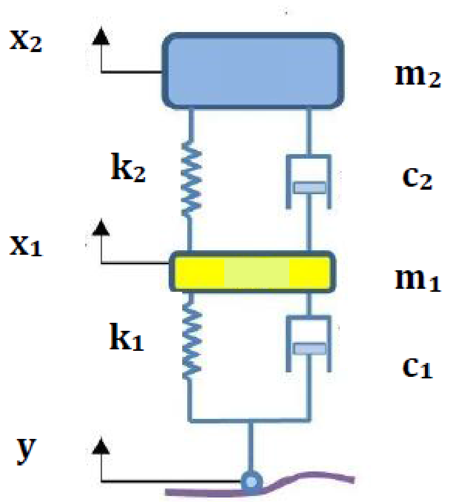

Fig.1 Quarter-car physical model [20].

The parameters of the quarter-car model according to Florin, Ioan-Cosmin and Liliana are considered in this analysis except for the suspension damping coefficient c_2. Their parameter are given in Table 1 [20].

TABLE 1
QUARTER-CAR MODEL PARAMETERS [20].

Parameters	**Description**	**Value**
k_1 (kN/m)	Tire stiffness	135
c_1 (kNs/m)	Tire damping coefficient	1.4
m_1 (g)	Un-sprung mass	49.8
k_2 (kN/m)	Suspension stiffness	5.7
m_2 (g)	Sprung mass	466.5

2.2 Model Input

The input is the irregularity of the road. It may take various shapes. It can be random roughness or standard humps to force drivers to reduce their vehicle speeds (say) in residential areas (speed hump). In this study a speed hump is considered as an input to the quarter-car model. Standard humps have well known designs. They may be circular, parabolic or flat-topped [21,22]. Fig.2 shows the three designs of speed hump [22].

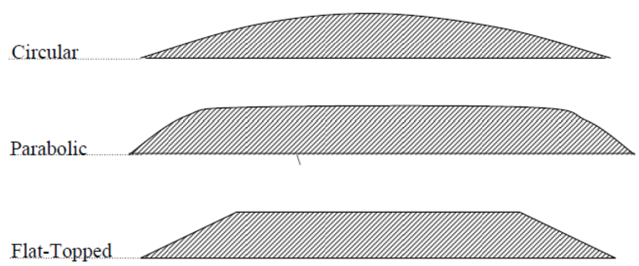

Fig.2 Three types of speed humps [22].

Only the circular type will be considered in this analysis. The optimal parameters of the circular speed hump are [21]:

Height (Y): 100 mm
Length (L): 5.2 m

The equation of the hump in the time domain depends on its length L, height Y and car speed V in km/h as follows:

$$y = Y\sin(\omega t) \qquad \text{for } 0 \le t \le T$$

where y is the vertical displacement over the hamp at time t from the starting point of the hump and ω is the angular frequency of hump assuming that it is a harmonic motion (half a cycle).

T is the time taken by the car to pass the hump. That is:

$$T = L / \{V*1000/3600\} \quad s$$

The period of this sine wave, τ is 2T. That is:

$$\tau = 2T = 2\pi / \omega$$

Giving; $\omega = \pi V / (3.6L)$ rad/s

2.3 Mathematical Model

Writing the differential equation of the unsprung and sprung masses of the quarter-car model yields the following two equations:

$$m_1x_1'' + (c_1+c_2)x_1' - c_2x_2' + (k_1+k_2)x_1 - k_2x_2 = k_1y + c_1y' \quad (1)$$

$$m_2x_2'' - c_2x_1' + c_2x_2' - k_1x_1 + k_2x_2 = 0 \quad (2)$$

The state model of the dynamic system is driven from Eqs.1 and 2 as follows:

- State variables: z_1, z_2, z_3 and z_4.

$$z_1 = x_1 \quad , \quad z_2 = x_1'$$
$$z_3 = x_2 \quad , \quad z_4 = x_2' \quad (3)$$

- Output variable:

The output variable of the quarter-car model is the sprung mass motion, x_2. It is related to the state variables through:

$$x_2 = z_3 \quad (4)$$

- State model:

Combining Eqs.1, 2 and 3 gives the state model of the quarter-car model as:

$$z_1' = z_2 \quad (5)$$
$$z_2' = (1/m_1)\{k_1y + c_1y' - (c_1+c_2)z_2 - (k_1+k_2)z_1 + k_2z_3\} \quad (6)$$
$$z_3' = z_4 \quad (7)$$
$$z_4' = (1/m_2)\{c_2z_2 - c_2z_4 + k_2z_1 - k_2z_3\} \quad (8)$$

- Suspension damper:

The damping coefficient of the suspension damper is assumed to be constant. To investigate its effect on the quarter-car model dynamics it was considered to be in the range:

$$c_2 = 1 - 15 \quad kNs/m \quad (9)$$

III. QUARTER-CAR MODEL DYNAMICS

- The state model of this dynamic problem is linear since the suspension parameters are assumed constant (linear characteristics).
- MATLAB is used to solve this problem using its command "ODE45" [23,24].
- The sprung mass motion is excited by the hump displacement only, i.e. zero initial conditions are set in the solution comment.
- Time span is set to twice the half-period, T of the hump.
- The car speed is changed in the range: 5 to 25 km/h when passing the hump.
- It was to emphasise the effect of damping on the sprung mass displacement and the ride comfort in terms of the maximum sprung-mass acceleration in m/s^2.

3.1 Sprung-mass Displacement

The displacement of the sprung-mass as generated by MATLAB for 1, 5, 10 and 15 kNs/m damping coefficient is shown for different hump passing speed as follows:

- 1 kNs/m damping coefficient : Figs.2 through 6.

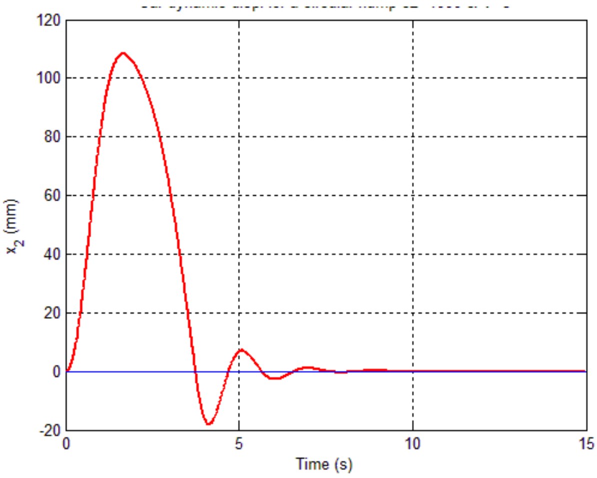

Fig.2 Sprung-mass displacement for c_2=1 kNs/m and V = 5 km/h.

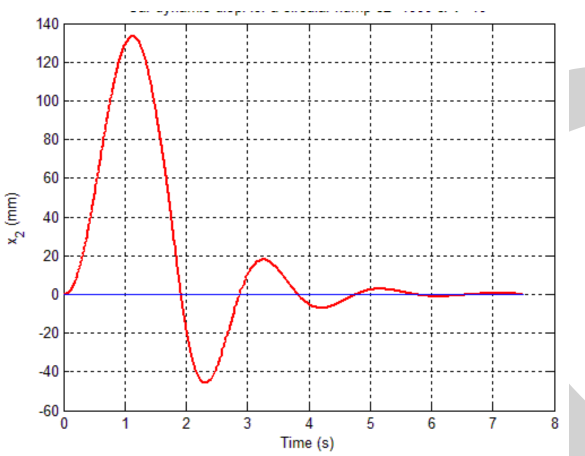

Fig.3 Sprung-mass displacement for c_2=1 kNs/m and V = 10 km/h.

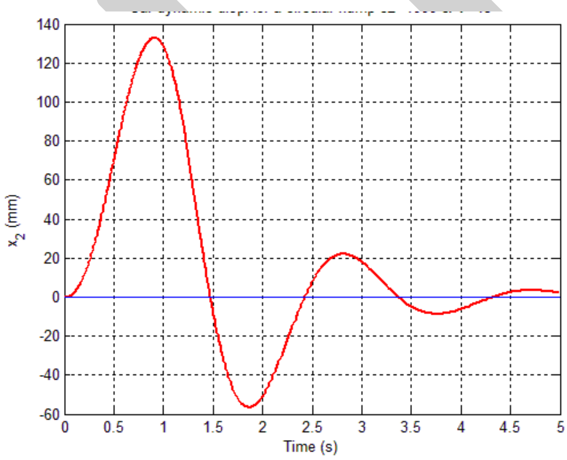

Fig.4 Sprung-mass displacement for c_2=1 kNs/m and V = 15 km/h.

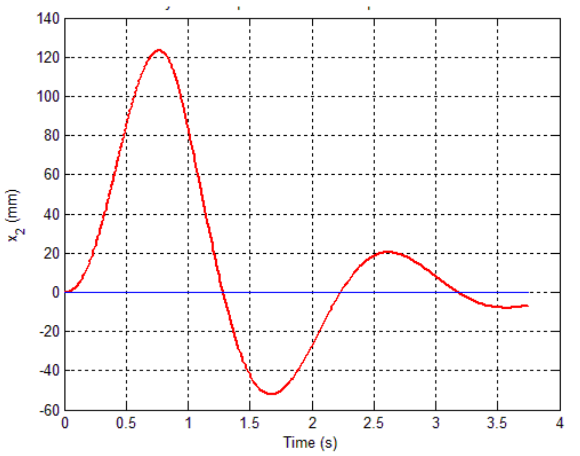

Fig.5 Sprung-mass displacement for c_2=1 kNs/m and V = 20 km/h.

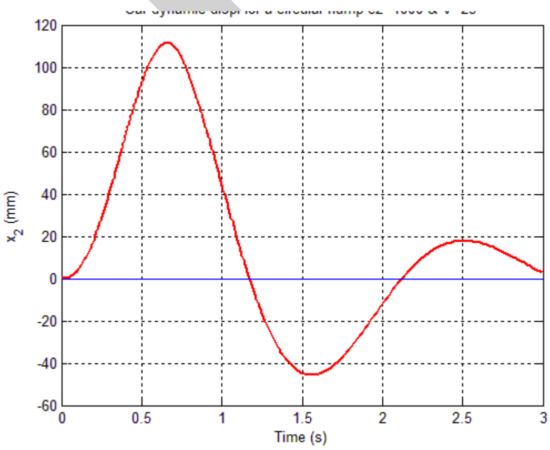

Fig.6 Sprung-mass displacement for c_2=1 kNs/m and V = 25 km/h.

- 5 kNs/m damping coefficient : Figs.7 through 11.

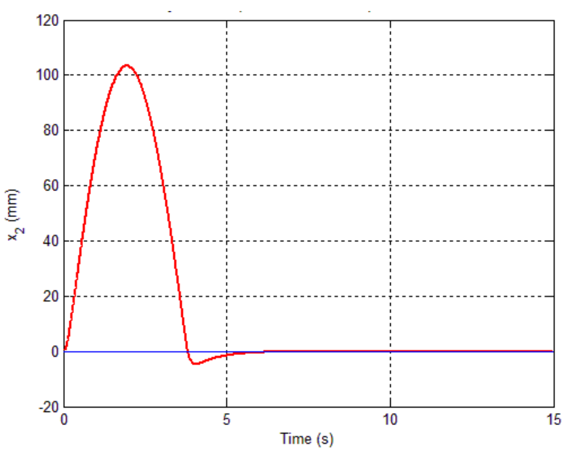

Fig.7 Sprung-mass displacement for c_2=5 kNs/m and V = 5 km/h.

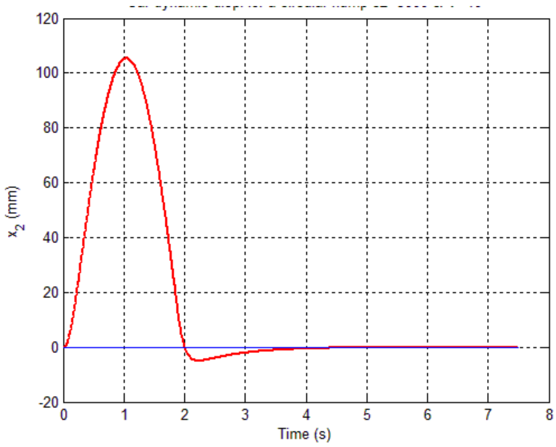

Fig.8 Sprung-mass displacement for c_2=5 kNs/m and V = 10 km/h.

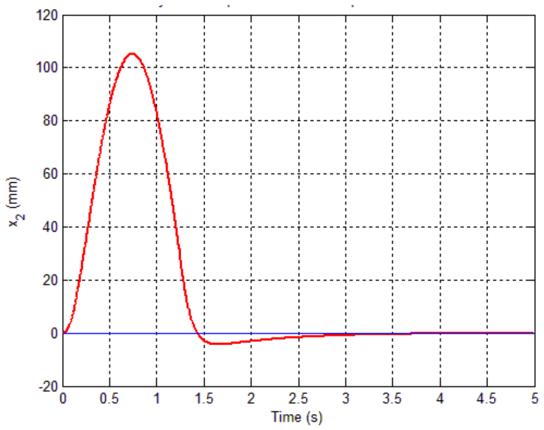

Fig.9 Sprung-mass displacement for c_2=5 kNs/m and V = 15 km/h.

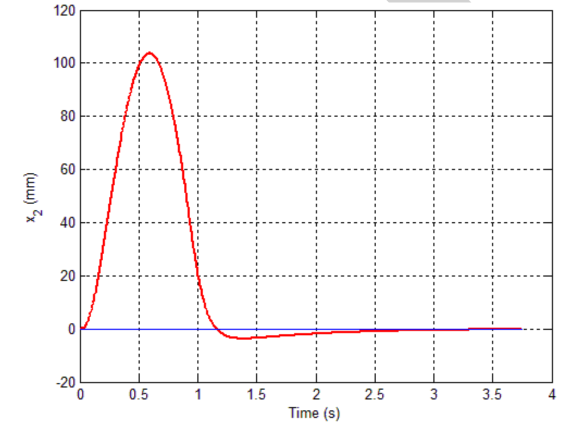

Fig.10 Sprung-mass displacement for c_2=5 kNs/m and V = 20 km/h.

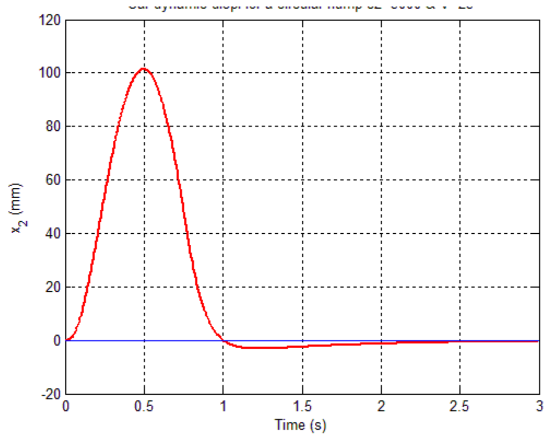

Fig.11 Sprung-mass displacement for c_2=5 kNs/m and V = 25 km/h.

- 10 kNs/m damping coefficient : Figs.12 through 16.

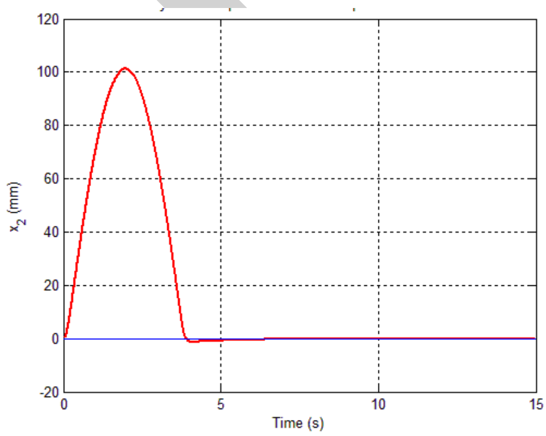

Fig.12 Sprung-mass displacement for c_2=10 kNs/m and V = 5 km/h.

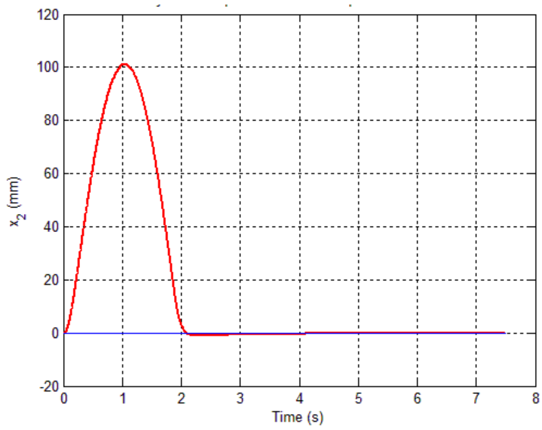

Fig.13 Sprung-mass displacement for c_2=10 kNs/m and V = 10 km/h.

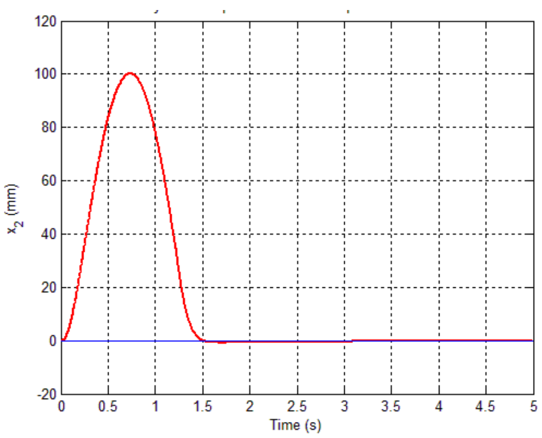

Fig.14 Sprung-mass displacement for c_2=10 kNs/m and V = 15 km/h.

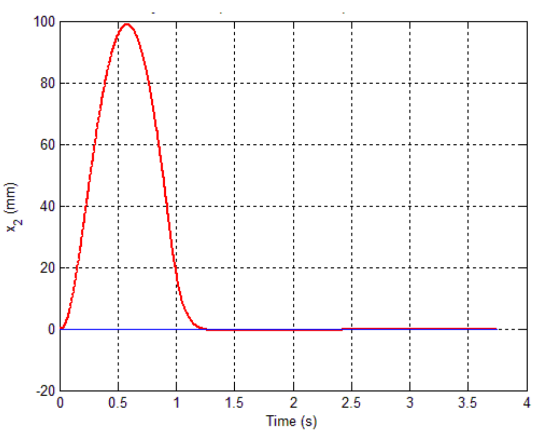

Fig.15 Sprung-mass displacement for c_2=10 kNs/m and V = 20 km/h.

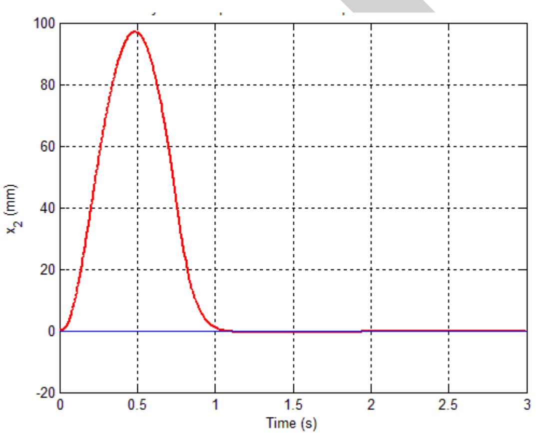

Fig.16 Sprung-mass displacement for c_2=10 kNs/m and V = 25 km/h.

- 15 kNs/m damping coefficient : Figs.17 through 21.

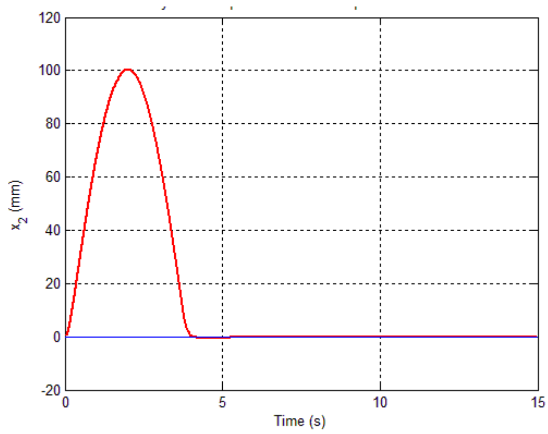

Fig.17 Sprung-mass displacement for c_2=15 kNs/m and V = 5 km/h.

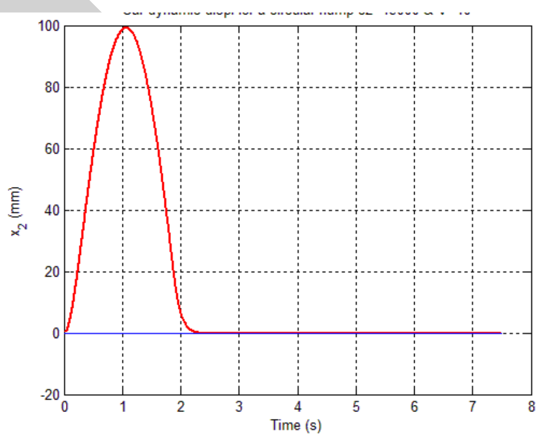

Fig.18 Sprung-mass displacement for c_2=15 kNs/m and V = 10 km/h.

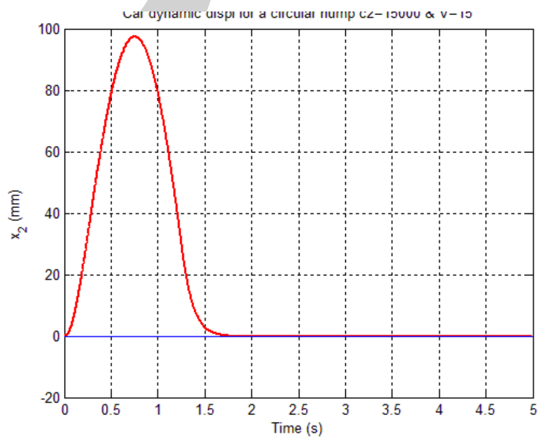

Fig.19 Sprung-mass displacement for c_2=15 kNs/m and V = 15 km/h.

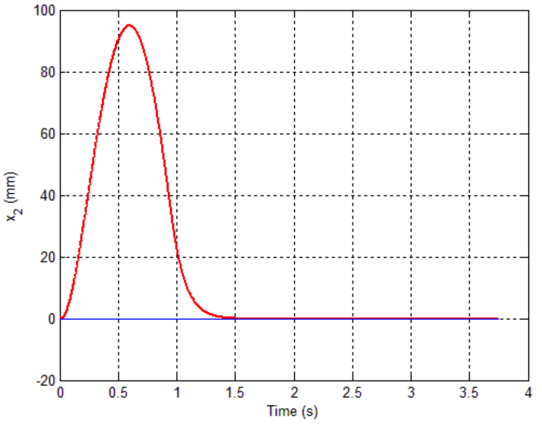

Fig.20 Sprung-mass displacement for c_2=15 kNs/m and V = 20 km/h.

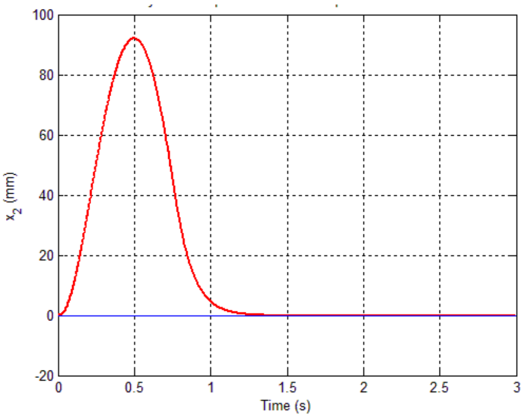

Fig.21 Sprung-mass displacement for c_2=15 kNs/m and V = 25 km/h.

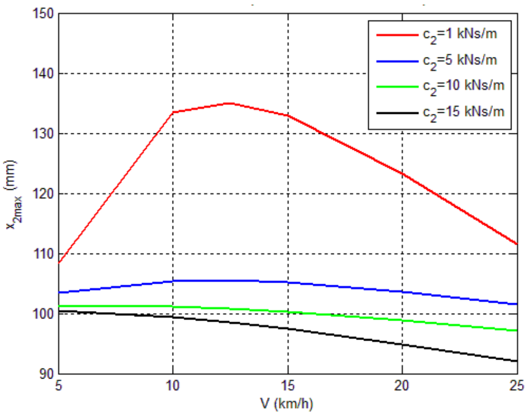

Fig.22 Sprung-mass maximum displacement.

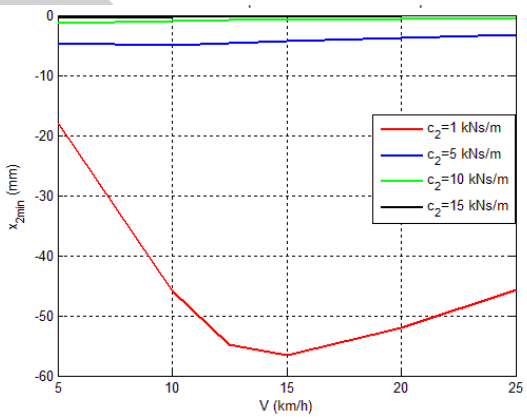

Fig.23 Sprung-mass minimum displacement.

3.2 Sprung-mass Maximum and Minimum Displacements

- As clear from all the sprung-mass response of the quarter model, the displacement reaches a maximum value then drops to a minimum value as the car passes the hump.
- The maximum and minimum displacements of the sprung-mass depend on both suspension damping and car speed.
- Figs.22 and 23 illustrate graphically this related obtained using the MATLAB commands "max" and "min" respectively.

3.3 Sprung-mass Acceleration

- The sprung-mass acceleration is the second derivative of its displacement with respect to time.
- The MATLAB command "diff" to differentiate the x2-t response twice producing the acceleration.
- Doing this, it didn't give any useful information.
- The author overcome this pug by fitting an 8th order polynomial to the displacement time response, then differentiated this polynomial analytically yielding the sprung-mass acceleration.
- A sample result of this procedure is shown in Fig.24 for c_2 = 15 kNs/m and V = 10 km/h.

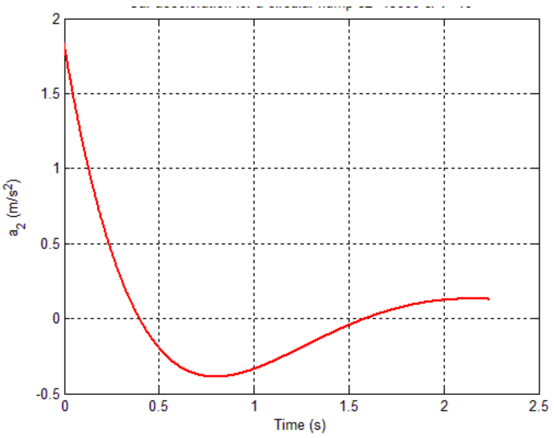

Fig.24 Sprung-mass acceleration for c2 = 15 kNs/m and V = 10 km/h..

- The maximum acceleration of the sprung-mass depends on the suspension damping coefficient and car speed.
- The maximum acceleration for a 15 kNs/m damping coefficient is shown in Fig.25 against car speed. The minimum level of the acceleration for ride confort is also drawn according to ISO 2631 [25].

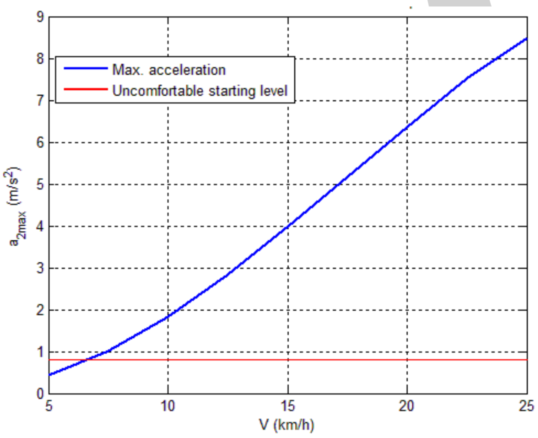

Fig.25 Maximum sprung-mass acceleration for c_2 = 15 kNs/m.

3.4 Maximum Car Speed for Ride Comfort

- According to ISO 2631, the ride comfort range starts from 0.8 m/s2 [25]. Imposing this limit on the car dynamics of a quadratic-car model when passing circular hump of the dimensions stated in section 2.2 gives an estimation for the maximum car speed when passing the hump for accepted ride comfort. This maximum car speed is given in Table II for the suspension damping coefficient cover in the present study.

TABLE II
MAXIMUM CAR SPEED FOR RIDE COMFORT

c_2 (kNs/m)	1	5	10	15
V_{max} (km/h)	5.46	6.60	6.70	6.75

IV. CONCLUSIONS

- A quarter-car model with passive elements was used in this study to investigate the car dynamics during passing a circular hump.
- The damping coefficient of the suspension was varied between 1 and 15 kNs/m to study its effect on the sprung-mass dynamics.
- Vehicle speed between 5 and 15 km/h was considered during passing the circular hump.
- For suspension damping coefficient ≤ 5 kNs/m, minimum sprung-mass displacement (undershoot) existed.
- The undershoot value increased with increasing the car speed for suspension damping coefficient < 5 kNs/m.
- The undershoot decreased as the suspension damping coefficient increased.
- The maximum sprung-mass displacement decreased with increased suspension damping coefficient, and with increased car speed for suspension damping coefficient ≥ 10 kNs/m.
- For a suspension damping coefficient ≤ 5 kNs/m, the maximum sprung-mass displacement increased with an increased car speed up to 12.5 km/h, then decreased for a car speed > 12.5 and ≤ 25 km/h.
- The sprung-mass acceleration was evaluated using a polynomial curve fitting and differentiation.
- The maximum sprung-mass acceleration occurred at the starting point of the circular hump.
- This maximum acceleration increased as the car speed increased from 0.43 to 8.5 m/s^2

corresponding to a speed range from 5 to 25 km/h at 15 kNs/m suspension damping coefficient.

- For a minimum ride comfort acceleration level of 0.8 m/s^2, the maximum car speed when passing the circular hump ranged from 5.4 km/h at 1 kNs/m suspension damping coefficient to 6.75 km/h at 15 kNs/m coefficient.

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BIOGRAPHY

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- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
- Published 10's of research papers in international journal and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.