APPROXIMATION OF IMPLICITLY EXPRESSED OPTIMALITY CRITERIA BY POZYNOM AND ANALYSIS OF THEIR SENSITIVITY

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Authors consider the possibility and feasibility of a given tabular function approximation by pozynom. Authors have shown that if the function is approximated by binomial pozynom, the solution of direct and inverse problems of sensitivity is obtained in an analytical form.

Keywords: given tabular function, approximation, pozynom, direct and inverse problems system.

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Characteristic feature of the problemes of systems states optimal control is that optimality criteria in mathematical models are set in algorithmic form, since they do not have analytical expression, and can be determined only by numerical methods. This complicates the problem of analysis of optimal solutions regarding sensitivity. Inability to perform the complete analysis of optimal solutions, as a result of which place and role of separate control devices (CD) in the process of system states optimization are determined, significantly reduces the value of the time-consuming optimization calculations. The example of such systems is electrical power system, in which optimality criterion (power losses during its transportation) has no analytical expression relatively parameters of CD [1]. The dependence of such optimality criterion on the parameters of CD can be defined and set in the form of the Table.

One of the possible ways to solve optimal control problems without analytically set efficiency function is approximation of optimality criterion in order to obtain analytical dependence on CD parameters. It should be taken into account that analytical dependence must be obtained in a form that would provide rapid and efficient technical and economic analysis of optimal solutions (sensitivity) by means of rather simple software. This is particularly important for on-line control of the systems states.

The aim of the paper is to present tabular set function in analytical form, convenient for technical-economical sensitivity analysis of optimal solution.

Approximation of tabular set function by pozynom

Taking into account the factors, influencing the efficiency of practical realization of optimal states of the system the control task is formulated in the following way:

$$F(\mathbf{x},\mathbf{u}) \Rightarrow \min, \qquad (1)$$

under the conditions

$$W(\mathbf{x},\mathbf{u}) = 0; \qquad \mathbf{x} \in \mathbf{M}_x; \qquad \mathbf{u} \in \mathbf{M}_u, \quad (2)$$

where **x** – are the parameters of system state; **u** – are the parameters of regulating devices; W(**x**, **u**) –is constraint equation of controlling **u** and controlled variables **x**; $\mathbf{M_x}$, $\mathbf{M_u}$ - are regions of admissible values of variables **x** and **u**.

The dependence F(**x**, **u**) for feasibility analysis of optimal solutions is expedient to construct in relative units when optimal varient is taken as the basic one [2]. However, since dependences $F_*=f(u_{*j})$ in analytical form can not be obtained from the equation of system state, then computational experiment on the computer is carried out, as a result of this experiment data, needed for approximation, are accumulated. Search of approximating formula is performed among the pozynoms of the following form:

$$F_* = \sum_{j=1}^{p} \sum_{i=1}^{m} \left(a_{ij} u_{*j}^{\alpha_{ij}} + b_{ij} u_{*j}^{\beta_{ij}}\right) \quad (3)$$

where $F_*=F_j/F_0$, $u_{*j}=u_j/u_{0j}$ – are relative values of efficiency function and control variables; F_j, u_j, F_0, u_{0j} –are current and basic values of the function and control variables; a_j, b_j, α_j, β_j – are constants that form the character of the dependence and level of u_{*j} impact on the value of F_*; p – is a number of control variables; m – is a number of members of approximating pozynom.

The values of coefficients a_j, b_j, α_j, β_j for the j-th CD in (3) we obtain applying the least squares method. Then in general form the approximation of F_*, for example, by binomial pozynom is reduced to the problem:

$$\left\{ R(a,b,\alpha,\beta) = \sum_{l=1}^{n} \left(F_{*l} - \overline{F}_{*l}\right)^2 \right\} \Rightarrow \min$$

or

$$\left\{ R(a,b,\alpha,\beta) = \sum_{l=1}^{n} \left(F_{*l} - a u_{*l}^{\alpha} - b u_{*l}^{\beta}\right)^2 \right\} \Rightarrow \min$$

(4)

where F_{*l}, $\overline{F}_{*l}$ – are experimental and calculated values of the function in the l-th point; n – is a number of experimental points, which are defined by the range and degree of regulation of CD.

After simple transformations of minimization conditions of the function R(a, b, α, β) we obtain the system of nonlinear equations:

$$\frac{\partial R(r)}{\partial a} = f_1(r) = \sum_{i=1}^{n} F_i k_i^{\alpha} - \alpha \sum_{i=1}^{n} k_i^{2\alpha} - b \sum_{i=1}^{n} F_i k_i^{\alpha+\beta} = 0;$$

$$\frac{\partial R(r)}{\partial b} = f_2(r) = \sum_{i=1}^{n} F_i k_i^{\beta} - \alpha \sum_{i=1}^{n} k_i^{\alpha+\beta} - b \sum_{i=1}^{n} F_i k_i^{2\beta} = 0;$$

$$\frac{\partial R(r)}{\partial \alpha} = f_3(r) = a \cdot \ln a \sum_{i=1}^{n} F_i k_i^{\alpha} + a \sum_{i=1}^{n} F_i k_i^{\alpha} \cdot \ln k_i^{\alpha} - a^2 \cdot \ln a \sum_{i=1}^{n} k_i^{2\alpha} -$$

$$-a^2 \sum_{i=1}^{n} k_i^{2\alpha} \cdot \ln k_i^{\alpha} - ab \cdot \ln a \sum_{i=1}^{n} k_i^{\alpha+\beta} - ab \sum_{i=1}^{n} k_i^{\alpha+\beta} \cdot \ln k_i^{\alpha} = 0;$$

$$\frac{\partial R(r)}{\partial \beta} = f_4(r) = b \cdot \ln b \sum_{i=1}^{n} F_i k_i^{\beta} + b \sum_{i=1}^{n} F_i k_i^{\beta} \cdot \ln k_i^{\beta} - b^2 \cdot \ln b \sum_{i=1}^{n} k_i^{2\beta} -$$

$$-b^2 \sum_{i=1}^{n} k_i^{2\beta} \cdot \ln k_i^{\alpha} - ab \cdot \ln b \sum_{i=1}^{n} k_i^{\alpha+\beta} - ab \sum_{i=1}^{n} k_i^{\alpha+\beta} \cdot \ln k_i^{\beta} = 0,$$

where r = [a, b, α, β] – is vector of variables.

Regarding the variables a, b, α, β it is solved by Newton method according to the scheme:

where k – is iteration number.

$$\begin{bmatrix} \frac{\partial f_1(r^k)}{\partial a} & \frac{\partial f_1(r^k)}{\partial b} & \frac{\partial f_1(r^k)}{\partial \alpha} & \frac{\partial f_1(r^k)}{\partial \beta} \\ \frac{\partial f_2(r^k)}{\partial a} & \frac{\partial f_2(r^k)}{\partial b} & \frac{\partial f_2(r^k)}{\partial \alpha} & \frac{\partial f_2(r^k)}{\partial \beta} \\ \frac{\partial f_3(r^k)}{\partial a} & \frac{\partial f_3(r^k)}{\partial b} & \frac{\partial f_3(r^k)}{\partial \alpha} & \frac{\partial f_3(r^k)}{\partial \beta} \\ \frac{\partial f_4(r^k)}{\partial a} & \frac{\partial f_4(r^k)}{\partial b} & \frac{\partial f_4(r^k)}{\partial \alpha} & \frac{\partial f_4(r^k)}{\partial \beta} \end{bmatrix} \cdot \begin{bmatrix} a^{k+1}-a^k \\ b^{k+1}-b^k \\ \alpha^{k+1}-\alpha^k \\ \beta^{k+1}-\beta^k \end{bmatrix} + \begin{bmatrix} f_1(r^k) \\ f_2(r^k) \\ f_3(r^k) \\ f_4(r^k) \end{bmatrix} = 0 \qquad (5)$$

According to (5) the algorithm of the process of determination the coefficients of approximating pozynoms is developed

Evaluation of the sensitivity of optimality criterion

Having the expression for the efficiency function in relative units, in the form of pozynom (3), we can determine the sensitivity of optimality criterion F relatively control variables u, especially, in case of deviation of their values from optimal ones. Fig. 1 shows the dependence $F_*(u_{*j}) = a_j u_{*j}^{\alpha_j} + b_j u_{*j}^{\beta_j}$ for the j-th CD. The additional increase of -optimality criterion in case of discrepancy between the value of the control variable and its optimal value is determined:

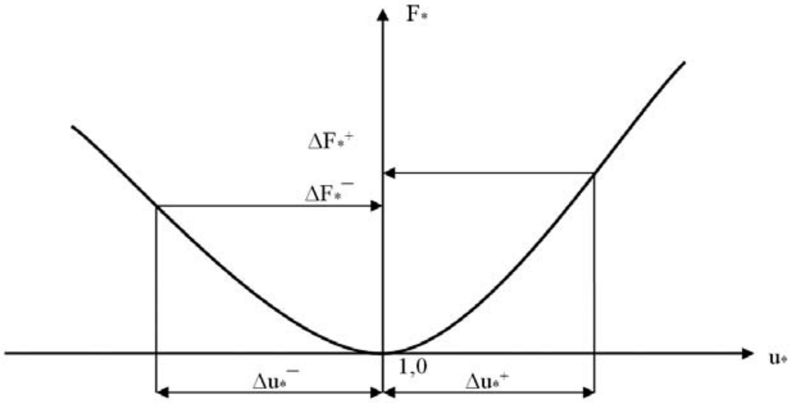

Fig. 1. Direct problem of sensitivity

$$\Delta F_{*j} = a_j u_{*j}^{\alpha_j} + b_j u_{*j}^{\beta_j} - 1 . \qquad (6)$$

This is so-called direct problem of sensitivity. If necessary, additional increase of optimality criterion can be defined in units, in which functions are measured: $\Delta F_j = \Delta F_{*_j} \cdot F_{min}$;

Inverse problem of sensitivity is solved when admissible deviation of optimality criterion δF_*, from its optimal value is set and corresponding admissible deviations from optimal value of control variables δu_* must be found. This problem is illustrated in Fig. 2.

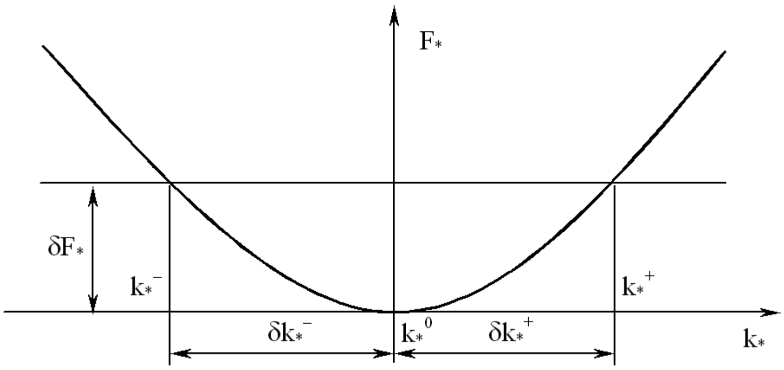

Fig. 2. Inverse problem of sensitivity (δu_*^-, δu_*^+ – lower and upper admissible deviation of control variable from its optimum value)

The equation for determining boundary values of control variables is:

$$a_j u_{*j}^{\alpha_j} + b_j u_{*j}^{\beta_j} - 1 - \delta F_* = 0 ,$$

or

$$1 + \delta F_* = a_j u_{*j}^{\alpha_j} + b_j u_{*j}^{\beta_j} . \qquad (7)$$

Since one δF_* can correspond several δu_*, then the inverse problem of sensitivity refers to the ill-posed problems. Let us solve it as follows. Divide both parts of equation (7) into $1+\delta F_*$.

We obtain:

$$1=\frac{a_j}{1+\delta F_*}u_{*j}^{\alpha_j}+\frac{b_j}{1+\delta F_*}u_{*j}^{\beta_j} . \qquad (8)$$

In (8) $\pi_1=\frac{a_j}{1+\delta F_*}u_{*j}^{\alpha_j}$ and $\pi_2=\frac{b_j}{1+\delta F_*}u_{*j}^{\beta_j}$ – are relative portions of the components of the efficiency function or similarity criteria of optimal control process [3, 4]. As it can be seen from (8)

$$\pi_1+\pi_2=1. \qquad (9)$$

The last equation is the expression of normalization condition. Similarity criteria or weight coefficients π represent the fraction of each component of the efficiency function in its optimal value.

From (8) it follows that between π and u_{*j} there exists direct connection:

$$u_{*j}^{+} = \left(\pi_2 \frac{1+\delta F_*}{b_j}\right)^{1/\beta_j},$$

$$u_{*j}^{+} = \left(\pi_2 \frac{1+\delta F_*}{b_j}\right)^{1/\beta_j}. \qquad (10)$$

Similarity criteria π_1 and π_2 can be found by using the results of the solution of the dual problem [4]

$$d(\pi)=\left(\frac{a_j}{\pi_1}\right)^{\pi_1}\cdot\left(\frac{b_j}{\pi_2}\right)^{\pi_2} \to \max \qquad (11)$$

under the conditions

$$\left.\begin{aligned} &\alpha_j\pi_1+\beta_j\pi_2=0; \\ &\pi_1+\pi_2=1; \\ &\pi_1>0;\ \pi_2>0. \end{aligned}\right\} \qquad (12)$$

From the system (12) we have

$$\pi_1 = \frac{-\beta_j}{\alpha_j - \beta_j};$$

$$\pi_2 = \frac{\alpha_j}{\alpha_j - \beta_j} \qquad (13)$$

Having substituted in (10) the value of similarity criteria from (13) we finally obtain

$$u_{*j}^{-} = \left[\frac{-(1+\delta F_*)\beta_j}{a_j(\alpha_j - \beta_j)} \right]^{1/\alpha_j},$$

$$u_{*j}^{+} = \left[\frac{(1+\delta F_*)\alpha_j}{b_j(\alpha_j - \beta_j)} \right]^{1/\beta_j} \qquad (14)$$

Thus, at the set value of insensitivity zone of optimality criterion δF_* the area of equieconomic values of control variables is within the limits of $u_{*j}^{-} \div u_{*j}^{+}$ for the j-th CD. Solution of the inverse problem is obtained in analytical form due to approximation of efficiency function by pozynom.

Conclusions

1. Functions that have no analytical expression, but can be set in tabular form, can be approximated by pozynom. Pozynom coefficients are determined by the method of least squares using standard procedures for solution of nonlinear equations systems.

2. Functions approximation by pozynom has certain advantages. In particular, if it is used to analyze the sensitivity of optimal solution. Thus, the solution of the inverse problem of sensitivity is obtained in analytical form when admissible deviation of optimality criterion from its optimal value is set and it is necessary to determine corresponding values of control variables.

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