

# Mathematical Model for the Solution of the Boltzmann Transport Equation for Photons

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## ABSTRACT

In this article we adopted the Mathematical model of solution of the Boltzmann Transport equation (BTE) for photons. For the dose calculation of radiotherapy for cancer treatment we need the number of electrons which we get from the Boltzmann Transport equation for electrons. To solve this BTE for electrons we need the number of photons which we get by solving the Boltzmann Transport equation for photons.

**Keywords:** Boltzmann Transport equation for photons, Radiotherapy, Scattering cross section, Compton scattering, cancer treatment

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## INTRODUCTION

The high energy photon radiotherapy is very much useful in the present time for cancer treatment. So, it is most important to calculate the expected dose distribution, before start the treatment of the patient. If the dose of radiotherapy in the tumour tissue is not very low then we can expect a curative effect. But if the dose is so high then the many healthy tissue surrounding the tumour will be destroyed or they will not be able to protect or avoid the undesirable side effect from the high dose. Therefore, one of the main parts for a treatment plan is the perfect dose calculation before beginning the treatment for an effective real treatment.

The exact dose calculation for photon and electron radiation by well known physical principles of interaction of radiation with human tissue by the transport of energy into the

patients body that can be modeled and calculated by an appropriate Monte Carlo (MC) algorithm (Andreo, 1991). If we work carefully then its leads to exact results of the dose distribution in arbitrary geometries and nowadays highly developed MC codes for dose calculations are available but the computational time is very high in this case. Therefore this process is going to unattractive position day by day in clinical use.

There is an alternative approach to circumvent the drawback of the MC codes called kernel models (Ahnesjo and Aspradakis, 1999) offer a reliable and fast alternative for most types of radiation treatment. The pencil beam models are probably most in use and these models are based on the Fermi-Eyges theory of radiative transfer (Rossi and Greisen, 1941) and (Eyges, 1948). Originally introduced for pure electron radiation by Hogstramet Mills and Almond (Hogstrom *etel*, 1981) and later generalized to photon radiation by (Gustafsson *etel*, 1994) & (Ulmer and Harder, 1995) too. Although the result was good but this models fail in complicated setting like air cavities or other inhomogeneities.

The third access for dose calculation which is attracted in the last few years is the deterministic Boltzmann equation of radiative transfer based on the physical interactions of radiation in tissue. A mathematical model can be developed that allows in principle an exact dose calculation like as MC models. The resent studies for pure electron radiation were mostly done by (Borgers and Larsen, 1996). Electron and combined photon and electron radiation were studied by (Tervo *etal*, 1999). Tervo and Kolmonen (2002) in the context of inverse therapy planning and Zhengming *etel* (2004) restricted their model to one dimensional slab geometry.

In this paper we represent the mathematical model to solve the Boltzmann transport equation for photons.

## THE BOLTZMANN MODEL FOR PHOTON TRANSPORT

The photons move with high velocities so all the process can be regarded as time independent and the all calculations are done relativistic using the relativistic formulae for energy and fully relativistic scattering cross section. For convenience all energies are scaled by the rest energy of the electron  $mc^2 = 0.511$  MeV,  $m$  being the rest mass of the electron  $c$  is the velocity of light.

Let  $\psi_\gamma(r, \Omega_\gamma, \epsilon_\gamma) \cos \Theta dA d\Omega_\gamma d\epsilon_\gamma dt$  be the number of photons that move in time  $dt$  through area  $dA$  into the element of solid angle  $d\Omega_\gamma$  around  $\Omega_\gamma$  with an energy in the interval  $(\epsilon_\gamma, \epsilon_\gamma + d\epsilon_\gamma)$ .  $\Theta$  is the angle between direction  $\Omega_\gamma$  and outer normal of  $dA$ .  $\Omega_\gamma = (\sin \varphi_\gamma \cos \vartheta_\gamma, \sin \varphi_\gamma \sin \vartheta_\gamma, \cos \varphi_\gamma)^T$  where  $\varphi_\gamma$  is the zenith angle and  $\vartheta_\gamma$  is the polar angle in a Cartesian coordinate system.

The Boltzmann transport equation for photons is

$$\begin{aligned} \Omega_\gamma \cdot \nabla \psi_\gamma(r, \Omega_\gamma, \epsilon_\gamma) = & \rho_e(r) \int_0^\infty \int_{S^2} \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) \psi_\gamma(r, \Omega'_\gamma, \epsilon'_\gamma) d\Omega'_\gamma d\epsilon'_\gamma \\ & - \rho_e(r) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) \psi_\gamma(r, \Omega_\gamma, \epsilon_\gamma) \end{aligned} \quad (1)$$

where  $\rho_e$  is the electrons density of the medium and  $\tilde{\sigma}_{c,\gamma}$  is the scattering cross section of the photons, differential in angle and energy for comptions scattering of photons and  $\sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma)$  is the total compton scattering cross section of photons.

## RESOLUTION OF THE BTE OF PHOTONS

To find an approximate solution for the photons we decompose the photon fluency  $\psi_\gamma$  formally into a series of 0-times, 1-times, .....,  $i$ -times, ....., at least  $N$ -times scattered photons [?]. Then

$$\psi_\gamma(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) = \sum_{i=1}^N \psi_\gamma^{(i)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma). \quad (2)$$

where for  $i = 0, 1, \dots, N-1$ ,  $\psi_\gamma^{(i)}$  is the number of photons that are scattered  $i$ -times and  $\psi_\gamma^{(N)}$  is the number of photons that are scattered  $N$  or more times. This is only a formal decomposition, physically all photons are indistinguishable. Using this approach in the photon transport equation (1) one gets

$$\Omega_\gamma \cdot \nabla \sum_{i=0}^N \psi_\gamma^{(i)} + \rho_e \sigma_{C,\gamma}^{\text{tot}} \sum_{i=0}^N \psi_\gamma^{(i)} = \rho_e \int \int \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) \sum_{i=0}^N \psi_\gamma^{(i)}(\Omega'_\gamma, \epsilon'_\gamma) d\Omega'_\gamma d\epsilon'_\gamma. \quad (3)$$

As it is explained in (Hensel *et al*), interchanging the order of integration and summation and using the fact that photons are scattered ( $i - 1$ )-times can only act as a source for photons that are scattered  $i$ -times, one can uniquely decompose the integro differential equation into a system of differential equations and one integro differential equation. If we restrict the system of photons that are scattered at most  $M$  times we get the following set of  $M$  coupled partial differential equations with  $M < N$

$$\begin{aligned} \Omega_\gamma \cdot \nabla \psi_\gamma^{(0)} + \rho_e \sigma_{C,\gamma}^{\text{tot}} \psi_\gamma^{(0)} &= 0 \\ \Omega_\gamma \cdot \nabla \psi_\gamma^{(1)} + \rho_e \sigma_{C,\gamma}^{\text{tot}} \psi_\gamma^{(1)} &= \rho_e \int \int \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) \psi_\gamma^{(0)}(\Omega'_\gamma, \epsilon'_\gamma) d\Omega'_\gamma d\epsilon'_\gamma \\ \Omega_\gamma \cdot \nabla \psi_\gamma^{(M)} + \rho_e \sigma_{C,\gamma}^{\text{tot}} \psi_\gamma^{(M)} &= \rho_e \int \int \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) \psi_\gamma^{(M-1)}(\Omega'_\gamma, \epsilon'_\gamma) d\Omega'_\gamma d\epsilon'_\gamma. \end{aligned} \quad (4)$$

This set of differential equations can be solved with less effort than the exact original integro differential equation. The idea is to solve the above equations for  $\psi_\gamma^{(0)}, \dots, \psi_\gamma^{(M)}$  and compute  $\psi_\gamma$  via the approximation

$$\psi_\gamma(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) \approx \sum_{i=0}^M \psi_\gamma^{(i)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma). \quad (5)$$

Let  $Q \subset \mathbb{R}^3$  to be the spatial domain,  $Q$  is assumed to be non empty, open, bounded and convex. Again let  $\partial Q$  be the boundary of the domain  $Q$  and  $\bar{Q}$  be the closure of the domain. So,  $\bar{Q} = Q \cup \partial Q$ . Let  $\Gamma \subset \partial Q$  be the irradiated part and let  $\Lambda = \partial Q \setminus \Gamma$  be the non irradiated part. Here  $\mathbf{n}(\mathbf{r})$  is the outward unit normal at  $\mathbf{r} \in \partial Q$ .

The following are the boundary conditions needed for computing  $\psi_\gamma^{(0)}, \dots, \psi_\gamma^{(M)}$ .

$$\left. \begin{aligned} \psi_\gamma^{(0)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) |_{\mathbf{r} \in \Gamma} &= \psi_\gamma^{(0)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma); & \text{for } \mathbf{n}(\mathbf{r}) \cdot \Omega_\gamma < 0 \\ \psi_\gamma^{(0)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) |_{\mathbf{r} \in \Lambda} &= 0; & \text{for } \mathbf{n}(\mathbf{r}) \cdot \Omega_\gamma < 0 \\ \psi_\gamma^{(i)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) |_{\mathbf{r} \in \partial Q} &= 0; & \text{for } \mathbf{n}(\mathbf{r}) \cdot \Omega_\gamma < 0, 1 \leq i \leq M. \end{aligned} \right\} \quad (6)$$

### Obtaining $\psi_\gamma^{(i)}$ for $i = 0, \dots, M$

We get from the previous discussion

$$\sum_{i=0}^M \psi_\gamma^{(i)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) \approx \psi_\gamma(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma).$$

Now our objective is to obtain  $\psi_\gamma^{(i)}$  for  $i = 0, \dots, M$ .

### Obtaining $\psi_\gamma^{(0)}$

We begin to solve the first equation of (4) to get  $\psi_\gamma^{(0)}$ .

In particular given  $\mathbf{r}^* \in \partial Q$  and  $\Omega_\gamma \in S^2$  (photon direction) such that  $\Omega_\gamma \cdot \mathbf{n}(\mathbf{r}^*) < 0$ , we get an analytical expression for the value of  $\psi_\gamma^{(0)}$  on the line passing from  $\mathbf{r}^*$  in the direction  $\Omega_\gamma$ . Naturally it is the part of the line which is contained in the domain  $Q$  (see Figure 1).

The equation of the segment is

$$\mathbf{r}(\lambda) = \mathbf{r}^* + \lambda \Omega_\gamma, \tag{7}$$

with  $\lambda \in [0, L]$  for some positive value of  $L$ . Here  $\lambda \geq 0$  such that  $\mathbf{r}(\lambda) \in Q$ . This Figure 1 represents the path of the photon.

$$\Omega_\gamma \cdot \nabla \psi_\gamma^{(0)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) + \rho_\epsilon(\mathbf{r}) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) \psi_\gamma^{(0)}(\mathbf{r}, \Omega_\gamma, \epsilon_\gamma) = 0 \tag{8}$$

implies

$$\frac{d\psi_\gamma^{(0)}(\mathbf{r}(\lambda), \Omega_\gamma, \epsilon_\gamma)}{d\lambda} + \rho_\epsilon(\mathbf{r}(\lambda)) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) \psi_\gamma^{(0)}(\mathbf{r}(\lambda), \Omega_\gamma, \epsilon_\gamma) = 0. \tag{9}$$

If we define

$$y(\lambda) = \psi_\gamma^{(0)}(\mathbf{r}(\lambda), \Omega_\gamma, \epsilon_\gamma),$$

$$f(\lambda) = -\rho_\epsilon(\mathbf{r}(\lambda)) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma)$$

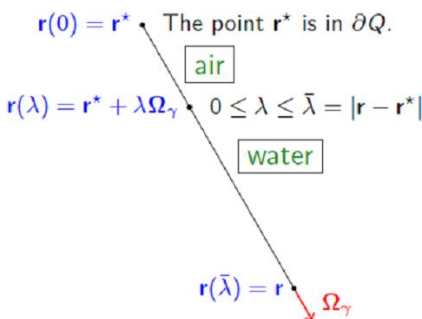


Figure 1

and

$$y_0 = \psi_\gamma^{(0)}(\mathbf{r}^*, \Omega_\gamma, \epsilon_\gamma)$$

then our problem (9) is formulated as follows

$$\left. \begin{aligned} y'(\lambda) &= f(\lambda)y(\lambda) \\ y(0) &= y_0 \end{aligned} \right\}. \quad (10)$$

The solution of equation (10) is clearly

$$y(\lambda) = y_0 e^{\int_0^\lambda f(s) ds}. \quad (11)$$

Hence the solution of the first equation of (4) is

$$\psi_\gamma^{(0)}(\mathbf{r}(\lambda), \mathbf{\Omega}_\gamma, \epsilon_\gamma) = \psi_\gamma^{(0)}(\mathbf{r}^*, \mathbf{\Omega}_\gamma, \epsilon_\gamma) e^{-\rho_e(\mathbf{r}(s)) \int_0^\lambda \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) ds} \quad (12)$$

which takes the value 0 if  $\mathbf{r}^* \in \Lambda$ . This process can be performed for each value of  $\epsilon_\gamma > 0$  (energy of photon) according to (6).

## Obtaining $\psi_\gamma^{(i)}$ for $i = 1, \dots, M$

For known  $\psi_\gamma^{(0)}$  we will have to find  $\psi_\gamma^{(i)}$ ,  $i = 1, \dots, M$  given the similarity to the equation for  $\psi_\gamma^{(0)}$ , proceed in a similar way.

For  $i \geq 1$  the  $i^{\text{th}}$  equation of (4) is

$$\mathbf{\Omega}_\gamma \cdot \nabla \psi_\gamma^{(i)} + \rho_e \sigma_{C,\gamma}^{\text{tot}} \psi_\gamma^{(i)} = \rho_e \int \int \tilde{\sigma}_{C,\lambda}(\epsilon'_\gamma, \epsilon_\gamma, \mathbf{\Omega}'_\gamma \cdot \mathbf{\Omega}_\gamma) \psi_\gamma^{(i-1)}(\mathbf{r}(\lambda), \mathbf{\Omega}'_\gamma, \epsilon'_\gamma) d\mathbf{\Omega}'_\gamma d\epsilon'_\gamma. \quad (13)$$

If we write on the line  $\mathbf{r}(\lambda)$  for fixed  $\mathbf{\Omega}_\gamma$  and  $\epsilon_\gamma$  then the equation becomes the following:

$$\begin{aligned} & \frac{d\psi_\gamma^{(i)}(\mathbf{r}(\lambda), \mathbf{\Omega}_\gamma, \epsilon_\gamma)}{d\lambda} + \rho_e(\mathbf{r}(\lambda)) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) \psi_\gamma^{(i)}(\mathbf{r}(\lambda), \mathbf{\Omega}_\gamma, \epsilon_\gamma) \\ &= \rho_e(\mathbf{r}(\lambda)) \int_0^\infty \int_{S^2} \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \mathbf{\Omega}'_\gamma \cdot \mathbf{\Omega}_\gamma) \psi_\gamma^{(i-1)}(\mathbf{r}(\lambda), \mathbf{\Omega}'_\gamma, \epsilon'_\gamma) d\mathbf{\Omega}'_\gamma d\epsilon'_\gamma. \end{aligned} \quad (14)$$

As we did in the homogeneous case, we write our problem by using simpler notation:

$$\begin{aligned} y(\lambda) &= \psi_\gamma^{(i)}(\mathbf{r}(\lambda), \mathbf{\Omega}_\gamma, \epsilon_\gamma) \\ f(\lambda) &= -\rho_e(\mathbf{r}(\lambda)) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) \\ g(\lambda) &= \rho_e(\mathbf{r}(\lambda)) \int_0^\infty \int_{S^2} \tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \mathbf{\Omega}'_\gamma \cdot \mathbf{\Omega}_\gamma) \psi_\gamma^{(i-1)}(\mathbf{r}(\lambda), \mathbf{\Omega}'_\gamma, \epsilon'_\gamma) d\mathbf{\Omega}'_\gamma d\epsilon'_\gamma. \end{aligned} \quad (15)$$

Thus equation (14) is written

$$y'(\lambda) = f(\lambda)y(\lambda) + g(\lambda) \quad (16)$$

and therefore

$$\begin{aligned} y(\lambda) &= \left\{ \int_0^\lambda g(t) e^{-\int_0^t f(s) ds} dt \right\} e^{\int_0^\lambda f(s) ds} \\ &= \int_0^\lambda \left\{ g(t) e^{\int_t^\lambda f(s) ds} \right\} dt. \end{aligned} \quad (17)$$

Then the solution of (16) is;

$$\psi_\gamma^{(i)}(\mathbf{r}(\lambda), \mathbf{\Omega}_\gamma, \epsilon_\gamma) = \int_0^\lambda g_{\epsilon_\gamma, \mathbf{\Omega}_\gamma}^{(i-1)}(t) e^{-\left(\int_t^\lambda \rho_e(\mathbf{r}(s)) ds\right) \sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma)} dt, \quad (18)$$

where we use the notation

$$g_{\epsilon_{\gamma}, \Omega_{\gamma}}^{(i-1)}(\lambda) = \rho_{\epsilon}(\mathbf{r}(\lambda)) \int_0^{\infty} \int_{S^2} \tilde{\sigma}_{C,\gamma}(\epsilon'_{\gamma}, \epsilon_{\gamma}, \Omega'_{\gamma} \cdot \Omega_{\gamma}) \psi_{\gamma}^{(i-1)}(\mathbf{r}(\lambda), \Omega'_{\gamma}, \epsilon'_{\gamma}) d\Omega'_{\gamma} d\epsilon'_{\gamma}. \quad (19)$$

## CONCLUSION

By solving the equation (18) we get the number of photons. we have to use this number of photons to solve the BTE for electrons to get the number of electrons. This number of electrons are needed for the dose calculation of radiotherapy for cancer treatment. This paper represented only the mathematical model to calculate the Boltzmann Transport equation for photons.

## REFERENCES

- Ahnesjo and Aspradakis M M1999 Dose calculation for external photon beams in radio-therapy Phys.Med.Biol., 44 R99-R155
- Andreo P 1991 Monte Carlo Techniques in medical radiation physics Phy.Med.Biol., 36 861-920
- Borgers and E.W.Larsen, Asymptotic derivation of the Fermi pencil-beam approximation, Nuclear Sci. Eng., 2123(1996), pp. 343-357
- Das, S. (2014). Lovejoy and Osburn's Overpartitions. *Engineering International*, 2(2), 63-71. Retrieved from <http://journals.abc.us.org/index.php/ei/article/view/4.1>
- Eyges L , 1948 Multiple scattering with energy loss Phys. Rev., 74 1534-35
- Gustafsson A, Lind B K and Brahme A, 1994 A generalized pencil beam algorithm for optimization of radiation therapy Med.Phys., 21 343-56
- Hartmut Hensel, Rodrigo Iza-Teran, Norbert Siedow, Deterministic model for dose calculation in photon radiotherapy,
- Hogstrom K R, Mills M D and Almond P R 1981 Electron beam dose calculation Phys.Med.Biol., 26 445-59
- Mohajan HK. Upper Limit of the Age of the Universe with Cosmological Constant, *International Journal of Reciprocal Symmetry and Theoretical Physics*. 2014;1(1):43-68.
- Rossi B and Greisen K 1941 cosmic-ray theory Rev.Mod.Phys., 13 340-309 Fermi's work given on pp 265-68
- Talukder, M. (2013). Derivation of Non-Einsteinian Relativistic Equations from Momentum Conservation Law. *Asian Journal Of Applied Science And Engineering*, 2(1), 69-79. Retrieved from <http://journals.abc.us.org/index.php/ajase/article/view/2.8%28G%29>
- Talukder, M., & Ahmad, M. (2013). Wave Particle Dualism for Both Matter and Wave and Non-Einsteinian View of Relativity. *Asian Journal Of Applied Science And Engineering*, 2(1), 80-91. Retrieved from <http://journals.abc.us.org/index.php/ajase/article/view/2.9G%26M>
- Tervo J and Kolmonen P, 2002 Inverse radiotherapy treatment planning model applying Boltzmann-transport equation Math.Models Methods Appl. Sci. 12 109-41
- Tervo J, Kolmonen P, Vauhkonen M, Heikkinen L M and Kaipio J P , 1999 A nite-element model of electron transport in radiation therapy and related inverse problem Inverse Problems 15 1345-61
- Ulmer W and Harder D, 1995 A triple Gaussian pencil beam model for photon beam treatment planning Z. Med.Phys., 5 25-30
- Zhengming L, Chengjun G and Zhangwen W, 2004 Characteristic line theory of photon transport Proc. of the 14th Int. Conf. on the Use of Computers in Radiation Therapy (Seoul) 658-63.

## APPENDICES

We have used the Compton scattering cross section in our Boltzmann model for solving the system of photon equation. The differential scattering cross section is differential in energy and in solid angle. The Compton scattering cross section can be decomposed into a product of a cross section that is only differential in solid angle or energy and a Dirac delta function. Total cross section is calculated by integrating the double differential cross section with respect to energy and solid angle. Because the Delta functions one integral is always trivial.

To represent the cross-section we have used the quantities with a prime for incoming particles and the quantities without prime for outgoing particles. We have used the following symbols;

- (a) for incoming energy we have used  $\epsilon'_\gamma$  ;
- (b) for outgoing energy we have used  $\epsilon_\gamma$  ;
- (c) for incoming direction of photon we have used  $\Omega'_\gamma$  ;
- (d) for outgoing direction of photon we have used  $\Omega_\gamma$  ;

### Differential cross section for Compton scattering of photons

Literature: (C.M.Davisson, R.D.Evans, 1952)

$$\tilde{\sigma}_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) = \sigma_{C,\gamma}(\epsilon'_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) \delta_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma) \quad (1)$$

with

$$\sigma_{C,\gamma}(\epsilon'_\gamma, \Omega'_\gamma \cdot \Omega_\gamma) = \frac{r_e^2}{2} \left[ \frac{1}{1 + \epsilon'_\gamma(1 - \cos \vartheta_\gamma)} \right]^3 \left[ 1 + \cos^2 \vartheta_\gamma + \frac{\epsilon_\gamma'^2 (1 - \cos \vartheta_\gamma)^2}{1 + \epsilon'_\gamma(1 - \cos \vartheta_\gamma)} \right] \quad (2)$$

$$\delta_{C,\gamma}(\epsilon'_\gamma, \epsilon_\gamma) := \delta \left( \epsilon_\gamma - \frac{\epsilon'_\gamma}{1 + \epsilon'_\gamma(1 - \cos \vartheta_\gamma)} \right) \quad (3)$$

### Total cross section for Compton scattering of photons

Literature: (Davisson and Evans, 1952)

$$\sigma_{C,\gamma}^{\text{tot}}(\epsilon_\gamma) = 2\pi r_e^2 \left[ \frac{1 + \epsilon_\gamma}{\epsilon_\gamma^2} \left( \frac{2(1 + \epsilon_\gamma)}{1 + 2\epsilon_\gamma} - \frac{1}{\epsilon_\gamma} \ln(1 + 2\epsilon_\gamma) \right) + \frac{1}{2\epsilon_\gamma} \ln(1 + 2\epsilon_\gamma) - \frac{1 + 3\epsilon_\gamma}{(1 + 2\epsilon_\gamma)^2} \right] \quad (4)$$

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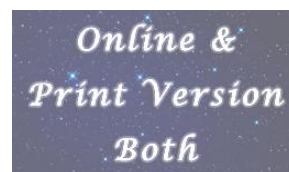
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